PERFORMANCE OF A SINGLE STAGE VAPOUR COMPRESSION REFRIGERATION CYCLE AT LOW EVAPORATION TEMPERATURES

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ABSTRACT

At relatively high evaporating temperature, the single stage vapour compression cycle is an efficient method of refrigeration. As the evaporating temperature is reduced, the volumetric efficiency decreases, the coefficient of performance decreases and the power input per KW of refrigeration increases as well as the compressor discharge temperature.

The principal demerits of single stage refrigeration cycle operating at low evaporating temperatures are low refrigerating efficiency, low compressor volumetric efficiency (for reciprocating compressors) and a high compressor discharge temperature.

Decrease of compressor volumetric efficiency imposes a practical limitation on the single stage cycle and makes staging of the compressors necessary at low evaporating temperatures.

Using a multistage vapour compression cycle with intercooling of liquid and vapour refrigerant largely overcomes the disadvantages of the single stage cycle.

INTRODUCTION

The theoretical single stage vapour compression refrigeration cycle is a deviation from the carnot refrigeration cycle based on the replacement of the expansion engine by a simple irreversible expansion valve and the use of one compressor instead of two in the compression process.

This deviation has led to a reduction in the coefficient of performance achieved by the single stage cycle compared with the carnot cycle for the same temperature limits. This paper briefly reviews the theoretical single stage vapour compression refrigeration cycle and examines the performance of the cycle at low evaporating temperatures. Generally, it is accepted that the single stage cycle will perform excellently well at relatively high evaporating temperatures, but as this temperature is reduced, a number of parameters that determine the performance of this cycle will change. Prominent among these parameters are:

i The volumetric efficiency - which reduces as the evaporating temperature reduces.

ii The coefficient of performance suffers a reduction too as the evaporating temperature reduces, so also the refrigerating efficiency follows the same trend.

iii The power input per kilowatt of refrigeration as well as the compressor discharge temperature increase as the evaporating temperature is reduced.

The paper will carry out some analytical calculation to determine the effect of the parameters mentioned above and others on the system as the evaporation temperature is reduced.

A computer programming method will be applied to the analytical calculations in order to determine the values of these parameters at the various temperatures, thus making the work load on the numerous computations light. Furthermore, an analysis of the multistage vapour compression refrigeration cycle will be made in comparison with the single stage cycle to show the variation in their performance characteristics.

1.1. CARNOT REFRIGERATION VERSUS SINGLE STAGE CYCLE

The theoretical single stage refrigeration cycle is a deviation from the carnot refrigeration cycle since the carnot cycle itself is not feasible in practice. The carnot cycle is the ideal cycle and a model of perfection for the refrigeration cycle because it transfers energy from a low level of temperature to a high level of temperature.

The diagram of the equipment and the property diagram on temperature - entropy coordinates for the carnot refrigeration cycle are shown in Fig 1. The processes that constitute the cycle are also included.

All the process in the carnot cycle are thermodynamically reversible hence processes 1-2a and 3-4 are isentropic. Because of this reversibility of processes, its performance has been found to be higher than could be achieved by an actual cycle.

A measure of performance in refrigeration systems is called the coefficient of performance (COP) which for the carnot cycle is a ratio of the absolute values of the operating temperatures.

Thus the COP for the carnot refrigeration cycle is defined thus:

$$COP = \frac{T_e}{T_c - T_e} \quad (1)$$

where T_e = Evaporating temperature

T_c = Condensing temperature

The single stage cycle is the actual cycle that makes compromises even though reduces the COP, but allows responsible construction and operation. Practical considerations

cause a deviation to be made from the carnot cycle to give

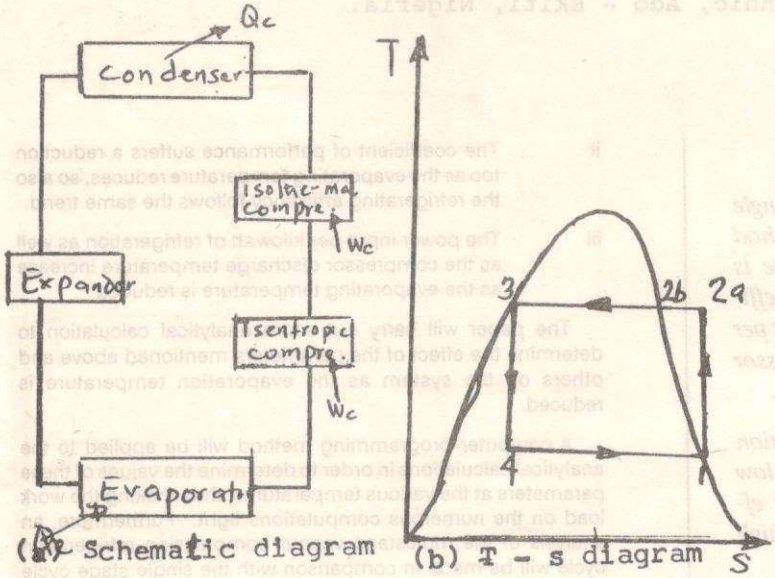


Fig 1 Carnot Refrigeration Cycle

the single stage cycle. This deviation causes the expander (turbine or engine) to be replaced by a single irreversible expansion valve thus relinquishing attempt to utilize any work output. Furthermore, this deviation allows a single compressor to be used instead of two in the compression process. These two compromises in design make the entire cycle feasible to construct.

2.2 THE THEORETICAL SINGLE STAGE CYCLE

The single stage vapour compression refrigeration cycle operates with a single compressor pumping the saturated refrigerant vapour from the evaporator at low pressure to the condenser in the high pressure side of the system. The heat absorbed in the evaporator (Qref) is rejected in the condenser and a saturated liquid is throttled down via an expansion valve to the lower evaporator pressure.

Fig 2 shows a schematic diagram of the single stage refrigeration cycle. Fig 2(a) shows the flow diagram through the equipments used. Figs 2(b) & 2(c) show the property diagram on T-S and p-h co-ordinates respectively. The processes as shown are:

1-2 Throttling or expansion process

2-3 Isothermal heat absorption in the evaporator.

2-4 Isentropic compression

4-1 Isothermal heat rejection

By applying the steady flow energy equation and on one kilogram of refrigerant.

Heat absorbed in the evaporator $Q_{ref} = h_3 - h_2$

Work input to the compressor $w_c = h_4 - h_3$

Heat rejected in the condenser $Q_c = h_4 - h_1$

Thus the coefficient of performance COP

$$COP = (Q_{ref} / w_c) = (h_3 - h_2) / (h_4 - h_3) \quad (2)$$

Stocker and Jones (1) defined the compressor power per kilowatt of refrigeration as the reciprocal of the coefficient of performance. James Threlkeld (2) defined the theoretical piston displacement in m^3/sec per KW of refrigeration as

$$PD = (3.516 \times V_3) / (h_1 - h_2) \quad (3)$$

Where V_3 is the volumetric discharge from the evaporator

2..3 OPERATION AT LOW EVAPORATING TEMPERATURES

The operation of a single stage vapour compression refrigerating system at low evaporating temperatures has the following draw backs:

- low refrigerating efficiency, low volumetric efficiency (reciprocating compressors)
- high discharge temperatures.

2.3.1 REFRGERATING EFFICIENCY

Faye Mcquiston & Jerald Parker (3) defined refrigerating efficiency η_R as the ratio of the coefficient of performance of an actual cycle or system to that of an ideal cycle or system.

$$\eta_R = (COP)_{actual} / (COP)_{ideal} \quad (4)$$

it should be recalled that the COP_{actual} is defined thus:

COP_{actual} = (Refrigerating effect, Q_{ref}) / (Heat input, w_c)

and COP_{ideal} is defined thus:

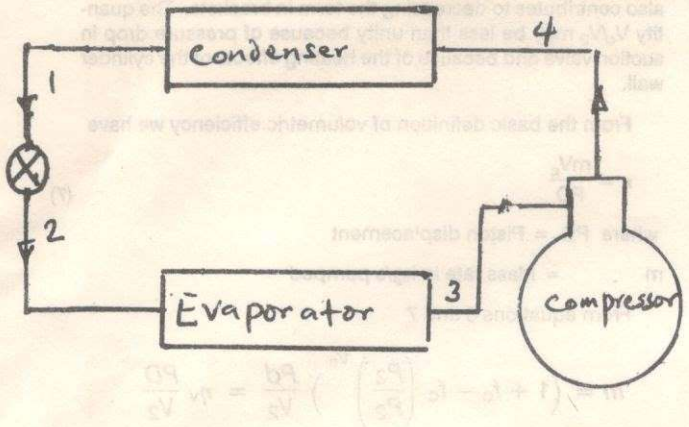
$COP_{ideal} = T_e / (T_c - T_e)$

Where Te = evaporating temperature:

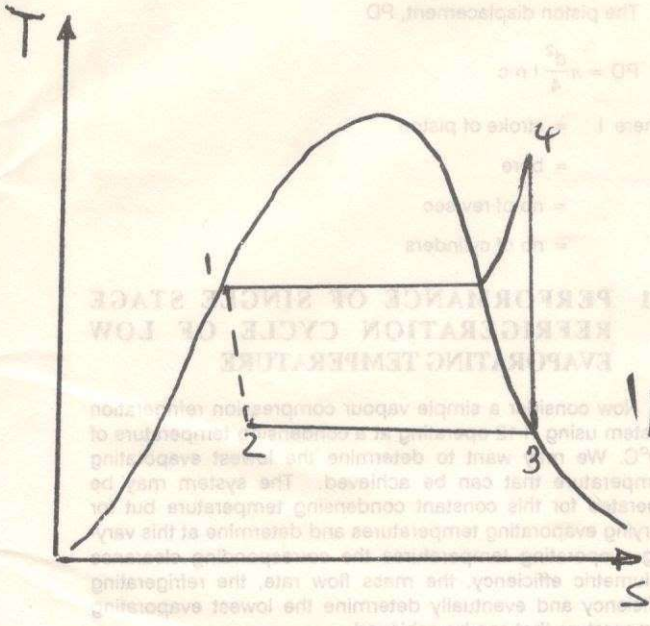
Tc = condensing temperature.

2.3.2. COMPRESSOR VOLUMETRIC EFFICIENCY

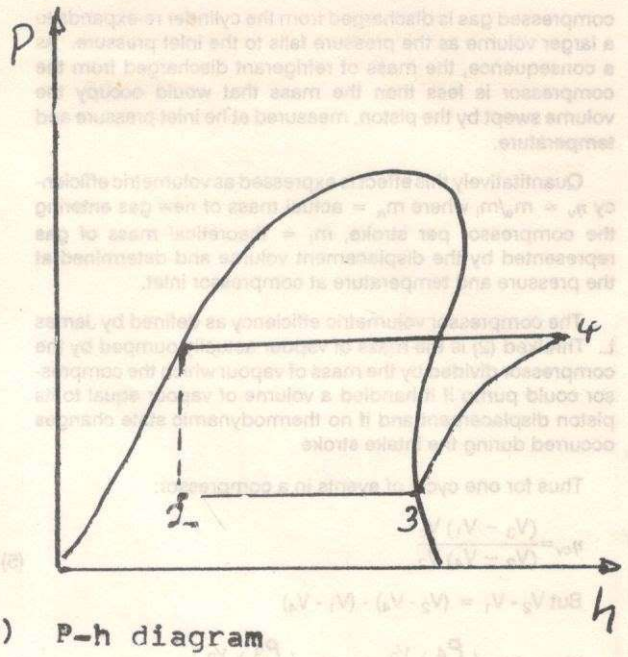
ASHRAE (4) defined volumetric efficiency as a measure of how well piston displacement, (size) of the compressor is utilized in moving refrigerant vapour through the cycle. It further discussed the effect of the clearance volume on the efficiency of the compressor. The gas remaining in the clearance volume (between the piston and the cylinder head when he piston is in the top dead centre position) after the



(a) Flow diagram



(b) T-s diagram



(c) P-h diagram

Fig 2: single stage vapour compression refrigeration cycle

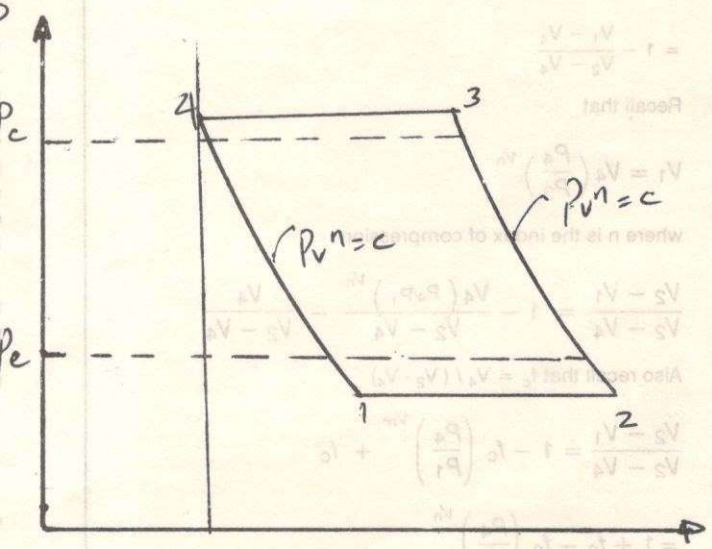


Fig 3. Schematic indicator diagram for a reciprocating Compressor

compressed gas is discharged from the cylinder re-expands to a larger volume as the pressure falls to the inlet pressure. As a consequence, the mass of refrigerant discharged from the compressor is less than the mass that would occupy the volume swept by the piston, measured at he inlet pressure and temperature.

Quantitatively this effect is expressed as volumetric efficiency $\eta_v = m_a/m_i$ where m_a = actual mass of new gas entering the compressor per stroke, m_i = theoretical mass of gas represented by the displacement volume and determined at the pressure and temperature at compressor inlet.

The compressor volumetric efficiency as defined by James L. Threlked (2) is the mass of vapour actually pumped by the compressor divided by the mass of vapour which the compressor could pump if it handled a volume of vapour equal to its piston displacement and if no thermodynamic state changes occurred during the intake stroke

Thus for one cycle of events in a compressor:

$$\eta_{cv} = \frac{(V_3 - V_1)\,V_c}{(V_2 - V_4)\,V_2} \qquad (5)$$

But $V_2 - V_1 = (V_2 - V_4) - (V_1 - V_4)$

$$V_1 = V_4 \left(\frac{P_4}{P_1}\right)^{1/n} = V_4 \left(\frac{P_3}{P_2}\right)^{1/n}$$

Defining a clearance factor f_c, as

$f_c = V_4 / (V_2 - V_4)$

Therefore

$$\frac{V_2 - V_1}{V_2 - V_4} = \frac{(V_2 - V_4) - (V_1 - V_3)}{V_2 - V_4}$$

$$= 1 - \frac{V_1 - V_4}{V_2 - V_4}$$

Recall that

$$V_1 = V_4 \left(\frac{P_4}{P_3}\right)^{1/n}$$

where n is the index of compression

$$\frac{V_2 - V_1}{V_2 - V_4} = 1 - \frac{V_4 \left(P_4/P_1\right)^{1/n}}{V_2 - V_4} - \frac{V_4}{V_2 - V_4}$$

Also recall that $f_c = V_4 / (V_2 - V_4)$

$$\frac{V_2 - V_1}{V_2 - V_4} = 1 - f_c \left(\frac{P_4}{P_1}\right)^{1/n} + f_c$$

$$= 1 + f_c - f_c \left(\frac{P_4}{P_1}\right)^{1/n} \qquad (6)$$

This expression takes care of the principal factors which affect compressor volumetric efficiency like the re-expansion of the vapour, pressure drop in suction, and discharge valves and heating of the vapour on the intake stroke. If f_c is assumed to be zero, the terms in the brackets become unity. On the other hand if f_c is greater than zero, the valve pressure drop also contributes to decreasing the term in brackets. The quantity V_c/V_2 may be less than unity because of pressure drop in suction valve and because of the heating effects of the cylinder wall.

From the basic definition of volumetric efficiency we have

$$\eta = \frac{mV_c}{PD} \qquad (7)$$

where PD = Piston displacement

m = Mass rate in kg/s pumped

From equations 6 and 7

$$m = \left(1 + f_c - f_c \left(\frac{P_3}{P_2}\right)^{1/n}\right) \frac{Pd}{V_2} = \eta_v \frac{PD}{V_2}$$

$$m = \eta \frac{PD}{V_2} \qquad (8)$$

The piston displacement, PD

$$PD = \pi \frac{d^2}{4} l\, n\, c$$

where l = stroke of piston

d = bore

n = no of rev/sec

c = no of cylinders

3.1 PERFORMANCE OF SINGLE STAGE REFRIGERATION CYCLE OF LOW EVAPORATING TEMPERATURE

Now consider a simple vapour compression refrigeration system using R-12 operating at a condensing temperature of 40°C. We may want to determine the lowest evaporating temperature that can be achieved. The system may be operated for this constant condensing temperature but for varying evaporating temperatures and determine at this varying evaporating temperatures the corresponding clearance volumetric efficiency, the mass flow rate, the refrigerating efficiency and eventually determine the lowest evaporating temperature that can be achieved.

For a machine of this data bore of 50mm, stroke of 50mm, four cylinders and a rotating speed of 1800 rpm with 5% clearance and assuming no pressure drop i.e. $V_c = V_2$ taking the index of compression to be 1.13 the analysis can be as follows:

Considering evaporating temperatures of

0°C, - 10°C, - 20°C, - 30°C, - 40°C, - 50°C

Given that no pressure drop occurs i.e $P_s = P_d = 0$; $V_C = V_2$ then Clearance Volumetric efficiency

$$\eta_{cv} = \left(1 + f_c - f_c \left(\frac{P_4}{P_3}\right)^{1/n}\right) \frac{V_1}{V_2}$$

where f_c = clearance factor

p_4 = condensing pressure

p_3 = evaporating pressure

n = polytropic exponent

The condensing and evaporating pressures are read from Refrigerant tables = (R-12) Rogers and Mayhew (5). Thermodynamics and Transport properties of fluids.

Given the condensing temperature of 40°C, and for the

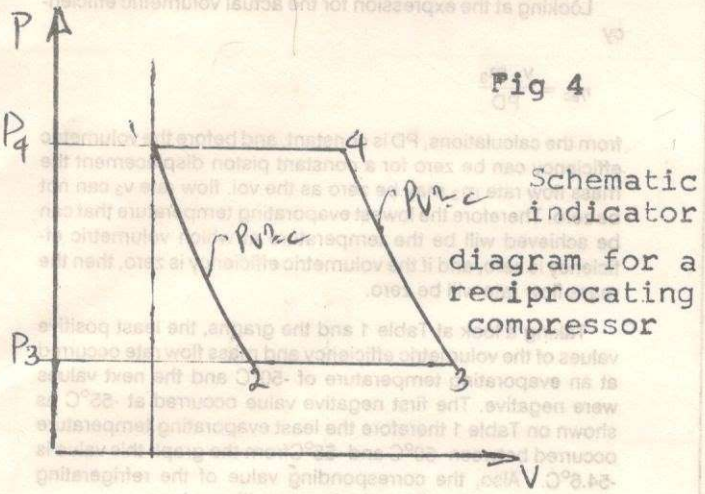


Fig 4

Schematic indicator diagram for a reciprocating compressor

various evaporating temps the corresponding values of the other parameters can be computed. Such parameters include:

(i) Clearance Volumetric efficiency

(ii) Mass flow rate

(iii) Refrigerating effect, Qref

(iv) Refrigerating efficiency

(v) compressor power input

(iv) power per kw of refrigeration

Evaporating temperature 0°C

Condensing temperature = 40_oC

Evaporating pressure p_3 = 3.086 bar

Condensing pressure p_4 = 9607 bar

(1) Clearing Volumetric Efficiency

$$\eta_{cv} = (1 + f_c - f_c \left(\frac{P_4}{P_3}\right)^{1/n}) \frac{V_1}{V_2}$$

But $V_c = V_2$

$$\eta_{cv} = (1 + f_c - f_c \left(\frac{P_4}{P_3}\right)^{1/n})$$

From the refrigerant table, at evaporating temperature of 0_oC, the properties are:

Evaporating pressure = 3.086 bar

$V_3 = V_g = 0.0554 m_3/kg$

$h_3 = h_g = 107.53 kj/kg$

$s_3 = s_g = 0.6966 kj/kg^k$

Given that the clearance factor

$f_c = 5\% = 0.05$ and n = 1.13

$\eta = 1 + 0.05 - 0.05\ (9.607/3.006)^{1/1.13}$

$\eta = 1.05 - 0.1366$

$\eta = 0.91341 = 91.34\%$

(ii) Mass Flow Rate

Clearance volumetric eff.

$$\eta_{cv} = 1 + f_c - f_c \left(\frac{P_4}{P_3}\right)^{1/n} \quad (A)$$

Actual volumetric efficiency

$$\eta_{ac} = \frac{m_3 V_c}{PD} \quad (B)$$

within some limits of approximation, equation (A) & (B)

$$\frac{m_3 V_c}{PD} = 1 + f_c - f_c \left(\frac{P_4}{P_3}\right)^{1/n}$$

$$m_3 = (1 + f_c - f_c \left(\frac{P_4}{P_3}\right)^{1/n}) \frac{PD}{V_c}$$

or $m_3 = \eta_{cv} \frac{PD}{V_c}$

where PD is the piston displacement

$$PD = \pi \frac{D^2 L\ N\ C}{4}$$

Where: D = cylinder bore = 50mm = 0.05m

L = stroke = 50mm = 0.05m

N = rpm = 1800

C = No of cylinders = 4

$$PD = \pi \frac{D^2}{4} L\ N\ C$$

$$PD = \pi \frac{0.05^2}{4} (0.05)\ (1000/60)\ (4)$$

$PD = 0.01178\ m^3/s$

$$m_3 = \eta_{cv} \times \frac{PD}{V_c}$$

PD and η_{cv} have been calculated and V_C will be read from the refrigerant Table, thus at the evaporating temperature of 0_oC, $v_c = v_g = 0.0554 m/kg$

$$m_3 = \eta_{cv} \times \frac{PD}{V_c}$$

$= 0.9134 \times (0.01178 / 0.0554)$

$m_3 = 0.1912 kg/s$

the mass flow rate m3 = 0.1942kg/sec.

(iii) Refrigerating Effect Qref

$Qref = m_3 (h_3 - h_2)$

$h_3 = h_g$ at 0°C = 187.53 KJ/kg

$h_2 = h_1 = h_f$ at 40°c = 74.59 KJ/kg

Qref = 0.1942 (kg/s) (187.53 - 74.50) KJ/kg

= 0.1942 x 112.94 KJ/s

Qref = 21.93 KJ/s

Refrigerating effect = 21.93KW

(iv) The refrigeration efficiency

$= COP_{actual} / COP_{ideal}$

$COP_{ctual} = (Qref / Power\ Input) = m_3(h_3 - h_2)/m_3(h_4 - h_3)$

= 0.1942 (208.27 - 107.53)

= 4.03 KW

Power per kw of refrigeration is the reciprocal of the COP

$= Power = (Input / Q_{ref}) = 4.03 / 21.93 = 0.1837$ kw/kw

Power per kw refrigeration = 0.1873

(v) Compressor Discharge Temperature ($T_{c,d}$)

This is the temperature of the superheated vapour at the exit of the compressor and can be read directly from the refrigerant - 12 chart or interpolated from the refrigerant - 12 table (5).

By interpolation the discharge temperature has been calculated from the various evaporating temperatures and this parameter increases from 45.9°C at T_e = 0°C to about 70°C at T_e = 60°C. This is shown on Table 1

3.2 COMPUTER PROGRAMMING METHOD

The computation from (i) to (v) can be repeated for all the chosen evaporating temperatures, thus a table will be produced which will show the variations of these parameters with the evaporating temperatures.

With these computations done for all the given temperatures of 0°C, -10°C, -20°C, -30°C, -40°C and -50°C a table was produced to show these variations. (See Table 1.)

Rather than use the manual and lengthy computational method as used for the 0°C temperature for other temperatures a computer programming approach had been developed which is faster and more accurate. The programme has been written in the BASIC language which computes and prints out all the parameters at each evaporating temperature. (Table 1.) This table is attached.

Cottfried S. Byrons (6) gave a simple approach to writing programmes in BASIC, a method which was made use of in this paper

3.3 THE LOWEST EVAPORATING TEMPERATURE ($T_{e,min}$)

The lowest evaporating temperature than can be achieved will be that temperature at which the volumetric efficiency will be zero. From the results as given by Table 1 and fig 6 it can be deduced that as the evaporating temperature is reduced for a given condensing temperature, so the volumetric efficiency reduces, therefore the lowest evaporating temperature that can be achieved will be when the volumetric efficiency reduces to zero.

Looking at the expression for the actual volumetric efficiency

$$\eta_{ac} = \frac{v_3 m_3}{PD}$$

from the calculations, PD is constant, and before the volumetric efficiency can be zero for a constant piston displacement the mass flow rate m_3 may be zero as the vol. flow rate v_3 can not be zero. Therefore the lowest evaporating temperature that can be achieved will be the temperature at which volumetric efficiency is zero, and if the volumetric efficiency is zero, then the mass flow rate will be zero.

Taking a look at Table 1 and the graphs, the least positive values of the volumetric efficiency and mass flow rate occurred at an evaporating temperature of -50°C and the next values were negative. The first negative value occurred at -55°C as shown on Table 1 therefore the least evaporating temperature occurred between -50°C and -55°Cfrom the graph this value is -54.6°C. Also, the corresponding value of the refrigerating effect at this value shows the least positive values.

Analytically this lowest achievable evaporating temperature can be computed using the expression for the clearance volumetric efficiency η_{cv}

$$\eta_{cv} = \left(1 + fc - fc\left(\frac{P_4}{P_3}\right)\right)^{1/n}$$

By equating η_{cv} to zero and calculating from the evaporating pressure P_3 the corresponding saturation temperature can be calculated. This temperature will be the lowest evaporating temperature that can be achieved.

$$\eta_{cv} = \left(1 + fc - fc\left(\frac{P_4}{P_3}\right)\right)^{1/n}$$

$0 = 0 + 1.05 - 0.05\ (9.507 / P_3)^{1/1.13}$

$0 = 1.05 - 0.05\ (9.507 / P_3)^{0.885}$

$(9.607/P_3) = (1.05/0.05)^{1/0.885}$

$(9.6070/P^3) = 31.197$

$P_3 = 9.6070/31.197$

$P_3 = 0.3080$ bar

and by interpolation from the R-12 refrigerant table, the corresponding saturation temperature is -54.6°C. Thus the lowest evaporating temperature achievable, T_e is -54.6°C at a pressure of 0.308 bar.

This agrees perfectly with the value read from the graph, -54.6°C.

TABLE 1: VARIATION OF PARAMETERS WITH EVAPORATION TEMPERATURE FOR A SINGLE STAGE VAPOUR COMPRESSION CYCLE

EVAP mp dept	Volumetric Efficency %	Mass Flow Rate	Refr. Effect kw	Compressor Power kw	COP actual ratio	COP ideal ratio	Refr Effcy %	Poower/kw ref kw/kw	Compre Discharge Temp deaC
0	91.34	0.19	21.94	4.83	5.45	4.83	79.79	0.18	45.88
-10	86.50	0.13	14.45	3.60	4.02	5.26	76.35	0.25	48.13
-20	79.27	0.09	0.94	2.91	3.07	4.22	72.92	0.33	50.92
-30	68.10	0.05	5.01	2.08	2.41	3.47	69.52	0.41	54.38
-40	50.17	0.02	2.32	1.20	1.93	2.91	66.20	0.52	59.18
-50	20.10	0.01	0.56	0.36	1.56	2.48	62.85	0.64	68.10
-55	-2.52	-0.00	-0.05	-0.04	1.40	2.29	61.17	0.71	68.10
-60	-32.96	-0.01	-0.52	-0.41	1.27	2.13	59.49	0.79	70.10

Table 2: EVAPORATION TEMPERATURE AND THEIR CORRESPONDING SATURATION PRESSURES

Evap Temp (deg C)	Sat Pressure (bar)
-60	.2262
-55	.2998
-50	.3915
-40	.6417
-30	1.004
-20	1.509
-10	2.191
0	3.086
LOWEST EVAPORATION TEMP. AND PRESSURE	
-54.5556	.30795

3.4 COMPUTER APPROACH TO LOWEST EVAPORATING TEMPERATURE T_e, 1

The analytical method has been formulated into a computer program in which the evaporating temperatures and their corresponding pressures have been read into the computer memory and stored. The computation for the lowest evaporating temperature that can be achieved based on the zero value of the volumetric efficiency has been used to determine the lowest evaporating pressure from the formula

$$\eta_{cv} = \left(1 + fc - fc\left(\frac{P_4}{P_3}\right)\right)^{1/n}$$

and finally the computer does the interpolation to get the corresponding saturation temperature for the calculated pressure.

A print out of the various evaporating temperatures and corresponding pressure is shown on table 2 and the lowest evaporating temperature achievable is found to be -54.6°C at a pressure of 0.308 bar. This agrees with the analytical and the graphical values accurately. Dwyer Thomas and Critchfield Marget (7) gave on approach to the use of personal computers for problem of this nature.

3.5 ANALYSIS OF MULTISTAGE VAPOUR COMPRESSION CYCLES

The performance of single-stage cycle at low evaporating temperatures has three principal disadvantages viz:

low refrigerating efficiency

low compressor volumetric efficiency (for reciprocating compressors and

High compressor discharge temperature

By using a multi-stage vapour compression cycle with intercooling of liquid and vapour refrigerant, the disadvantages of the single stage cycle may be largely overcome. It could be highly necessary at low evaporating temperatures to employ multi-stage compressors to avoid reduced volumetric efficiency and prevent excessive discharge temperature. At the same time, this arrangement will allow significant improvement in coefficient of performance. Refrigerants like ammonia and R-22 are well suited for this multi-stage arrangements. Fig 5(a) shows a two stage cycle suitable for use with R-12 and ammonia The system incorporating a flash intercooler as well as water intercooler.

The flash intercooler is a pressure vessel in which a fixed liquid level is maintained by a float valve which serves as an expansion valve. Saturated liquid at the intermediate pressure is withdrawn from the flash intercooler and is expanded through an expansion valve to the evaporator pressure. The vapour leaving the low-pressure compressor is cooled first by the water intercooler, then is further cooled by direct contact with the very cool liquid in the flash intercooler. The flash vapour formed from the throttling process in the float valve is sent to the suction line of the high pressure compressor. This

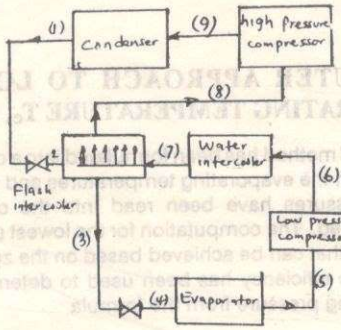


Fig 5 (a) Schematic diagram for cycle with two stage compression, water intercooler and flash intercooler.

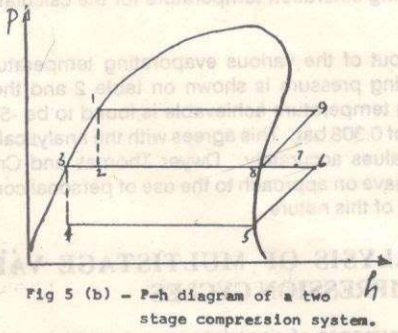


Fig 5 (b) – P-h diagram of a two stage compression system.

vapour includes the flash vapour formed in the float valve, refrigerant circulated through the evaporator vapour formed by evaporation of liquid in the inter cooler.

Analytically a comparison between a single stage cycle and a multistage cycle can be investigated to show the performances of these cycles. For such cycle, assume a low evaporating temperature say-50°C, a condensing temperature of 40°C, a compressor clearance of 5%, and R-12 as the refrigerant. For the two stage cycle, assume that the intermediate saturation temperature is -10°C and that the vapour leaves the water inter cooler at 40°C. This analysis will be shown to calculate for each cycle:

(i) the coefficient of performance

(ii) the max. temperature

(iii) the volumetric efficiency

(iv) the piston displacement

(v) the refrigeration efficiency

Considering first the single stage cycle using Fig 6(a)

(i) C.O.P. $= (h_3 - h_2)/(h_4 - h_3) = (164.95 - 74.59)/(22298 - 164.95) = (90.36/57.07)$

$= \underline{1.56}$

(ii) Compressor Discharge Temp./maximum Temperature, t_4

This can be read directly from the R-12 chart or interpolated from R-12 table.

At this condensing temperature in the superheat region $t_4 = 64.90°C$.

(iii) The volumetric efficiency η_v

$$\eta_{cv} = \left(1 + fc - fc\left(\frac{P_4}{P_3}\right)\right)^{1/n}$$

$= 1 + 0.05 - 0.05\,(960.7/39.15)^{1/1.13}$

$= 0.2010$

$\eta_{cv} = 0.201$ or 20.1%

(iv) Piston Displacement P.D.

$$PD = \frac{3.516 \times v_3}{(h_3 - h_2)\,\eta_{cv}}$$

$= (3.516 \times 0.3831)/[(164.95 - 74.59)(0.2010)]$

$= 1.347/18.162 = 0.0742\ m^3/sec.$

$= 267.12\ m^3/hr$

(v) Refrigeration Efficiency η_R

$\eta_R = (COP_a/COP_r) \times 100\%$

$COP_a = 1.56$

$COP = T_e/(T_c - T_e) = 223/(313 - 223)$

$= 223/90 = 2.48$

$\eta_R = 1.56/2.48 \times 100\% = 62.9\%$

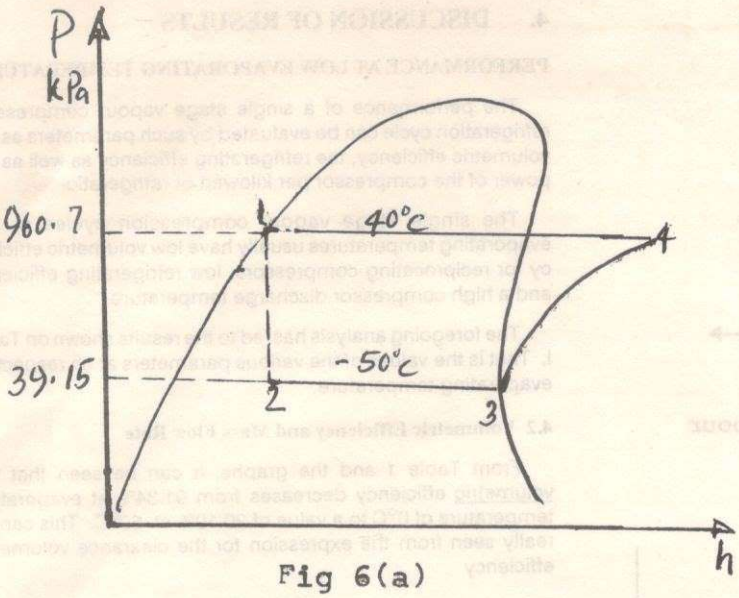


Fig 6(a)

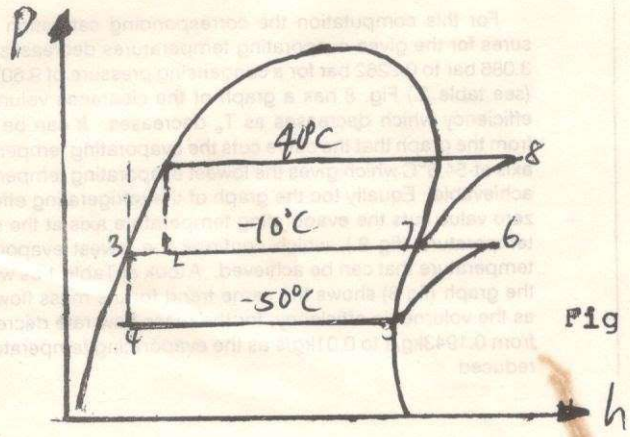


Fig 6(b)

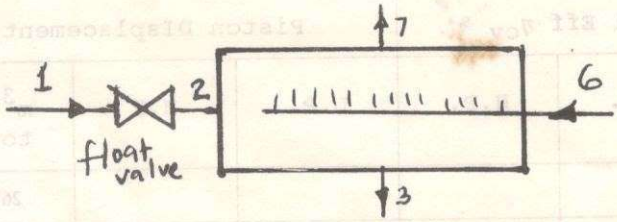


Fig.6(c)

For the two stage cycle the property data of the state points can be read from R-12 table and charts thus:

$h1 = h2 = 74.59kj/kg, = h3 = h4 = 26.87kj/kg$

$h_5 = 164.95kj/kg\ h_6 = 193.56kj/kg$

$h_7 = 183.19kj/kg\ h_8 = 209.46kj/kg\ t_8 = 50°C$

COP

Since the mass flow rates through the two loops of the system are different, the COP may not be expressed in terms of the enthalpy differences alone.

$COP = m_3(h_5 - h_4)/[m_3(h_6 - h_5) + m_7(h_8 - h_7)]$

Considering the flash intercooler as an isolated thermodynamic system and performing energy and material balance using the steady flow energy equation.

$m_2h_2 + m_6h_6 = m_3h_3 - m_7h_7$

$m_2 + m_6 = m_3 - m_7$ for this cycle

Thus $m_7(h_7 - h_2) = m_3(h_6 - h_3)$

$m_7/m_3 = (h_6 - h_3)/(h_7 - h_2)$

for the low pressure loop

$m_3 = 3.516/(h_5 - h_4)$

$= 3.516/(164.95 - 26.07) = 0.0255kg/sec(kw)$

Recall the expression for the COP as derived earlier

$COP = m_3(h_5 - h_4)/[m_3(h_6 - h_5) + m_7(h_8 - h_7)]$

$COP = (h_5 - h_4)/[(h_6 - h_5) + (m_7/m_3)(h_8 - h_7)]$

Recall $m_7/m_3 = (h_5 - h_8)/(h_7 - h_2) = (193.56 - 26.87)/(183.19 - 74.59) = 1.535$

$COP = (164.95 - 26.87)/[(193.56 - 164.95) + 1.535(209.46 - 187.19)]$

$= 138.08/[29.61 - 1.535(22.27)]$

$= 138.08/62.79 = 2.199$ or 2.20

the discharge temperature, $t_8 = 50°C$

C Volumetric Efficiency

For the low pressure compressor

$\eta_c = 1.05 - 0.05(2.191/.3915)^{1/13}$

$= 0.82 = 82\%$

and piston displacement

$PD = m_3v_5/\eta_{cv}$

where $v_5 = 0.3831\ m_3/kg$

$PD = (0.0255 \times 0.3831)/0.82 = 0.0119\ m^3sec = 42.84m^3/hr$

for the high temperature compressor

$\eta_{cv} = 1.05 - 0.05(9.607/2.191)^{1/13}$

$\eta_{cv} = 0.865$ or $0.87 = 87\%$

$m_7 = m_3\ (h_6 - h_3)/(h_7 - h_2)$

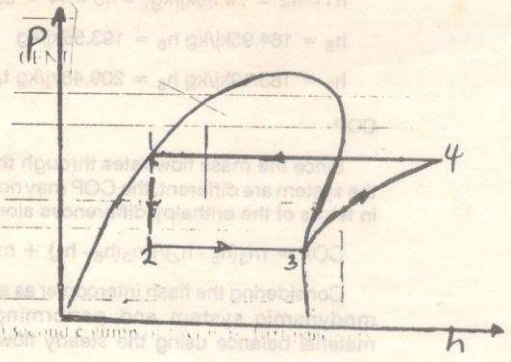


Fig 7

p-h diagram of a single stage vapour compression refrigeration cycle.

$= 0.0255(193.56 - 26.87)/(187.19 - 74.59)$

$m_7 = 0.038 kg/m^3$

thus: $PD = m_7 v_7/\eta_{cv} = (0.038 \times 0.0766)/0.87$

$PD = 0.0033 m^3/sec = 11.88 m^3/hr$

Refrigerating efficiency η_R

$\eta_R = COP_a/COP_r = (2.20/2.48) \times 100\% = 88.71\%$

The previous analysis has shown a comparison between the single stage and the multi-stage cycles. The summary of the analysis as given on Table 3, shows an increased of about 41% in the COP of the two-stage cycle over the single stage cycle and the refrigerating efficiency increases from 64.9% for single stage to 88.71% for the two-stage cycle. The volumetric efficiency also increases from 20.1% for the single stage to 82% for the two stage cycle. Also the discharge temperature decreases from 64°C to 50°C. Thus for the same evaporating and condensing temperatures, the multistage system offers a better performance than the single stage.

4. DISCUSSION OF RESULTS

PERFORMANCE AT LOW EVAPORATING TEMPERATURE

The performance of a single stage vapour compression refrigeration cycle can be evaluated by such parameters as the volumetric efficiency, the refrigerating efficiency as well as the power of the compressor per kilowatt of refrigeration.

The single stage vapour compression cycles at low evaporating temperatures usually have low volumetric efficiency for reciprocating compressors, low refrigerating efficiency and a high compressor discharge temperature.

The foregoing analysis has led to the results shown on Table I. That is the values of the various parameters at eh respective evaporating temperature.

4.2 Volumetric Efficiency and Mass Flow Rate

From Table 1 and the graphs, it can be seen that the volumetric efficiency decreases from 91.34% at evaporating temperature of 0°C to a value of 20.10% at -50°C. This can be really seen from the expression for the clearance volumetric efficiency

$$\eta_{cv} = \left(1 + fc - fc\left(\frac{P_4}{P_3}\right)\right)^{1/n}$$

For this computation the corresponding saturation pressures for the given evaporating temperatures decreases from 3.086 bar to 0.2262 bar for a condensing pressure of 9.607 bar. (see table 2.) Fig. 8 has a graph of the clearance volumetric efficiency which decreases as T_e decreases. It can be seen from the graph that the curve cuts the evaporating temperature axis at-54.6°C which gives the lowest evaporating temperature achievable. Equally too the graph of the refrigerating effect at zero value cuts the evaporating temperature axis at the same temperature (fig 8.), which confirms the lowest evaporating temperature that can be achieved. A look at Table 1 as well as the graph (fig 8) shows the same trend for the mass flow rate as the volumetric efficiency, for the mass flow rate decreases from 0.1943kg/s to 0.01kg/s as the evaporating temperature is reduced.

Table 3 Result of Single STage versus Multistage

Cycle	COP	Ref Eff η_R	Discharge temp °C	Vol Eff η_{cv} %		Piston DIsplacement		
				L.P.	H.P.	L.P.	H.P	m^3/hr total
Single Stage	1.56	62.96%	64.9	20.1				267.12
Two Stage	2.20	88.71%	50.	82	87	42.84	11.88	54.72

4.3 Refrigeration effect, Qref and Coefficient of Performance

(COP)

Another effect on the performance of a single stage cycle at low evaporating temperature is the low refrigerating effect. Table 1 and the graph Fig 9 show that the Qref reduces from 21.94kw at $T_e = 0_oC$ to 0.56 kw at $T_e = -50_oC$. The COP which is a measure of the amount of energy of refrigeration removed per power of compressor reduces as the evaporating temperature decreases. This is evident from the expression for the COP which is Refrigerating effect divided by the power of compressor i.e.

$$COP = Q_{ref}/W$$

From Table 1, Qref decreases with decrease in T_e, so also W decreases therefore it should be expected that COP will decrease with reduction in T_e.

Reference Fig 7, Qref = h_3-h_2, as T_e decreases h_3 decreases for constant value of h_2, see Table 4 for value of h_3.

4.4 Refrigerating efficiency

This parameter is a measure of the deviation of the actual refrigeration cycle form the ideal refrigeration cycle. Table 1 shows that the refrigeration efficiency decreases from 79.8% for $T_e = 0°C$ to 59.49% for $T_e = -60°C$. This trend can best be explained from the expression for the parameter i.e.

Ref Efficiency η_R = (actual COP / ideal COP) x 100%

Table 1 shows reduction in both actual and ideal COPs that define the refrigerating efficiency. Obviously there will be reduction in the refrigerating efficiency as T_e decreases.

The ideal COP is the carnot COP measured in terms of the absolute values of evaporating and condensing temperature i.e.

Ideal COP = $T_e/(T_c - T_e)$

Fig 9 contains the graphs showing the variation of COP and refrigerating efficiency with the evaporating temperature. The coefficient of performance curve shows a sharp drop as Te decreasos while the refrigerating efficiency graph is a straight line one sloping to the left thus confirming a gradual decrease in the efficiency as T_e decreases. (see Table 1)

4.5 Compressor Power per kw of Refrigeration and the Compressor Discharge Temperature

The power of compressor per kw of refrigerator is the amount of energy required by the compressor for every kilowatt of energy of refrigeration to be removed. Or more directly, this is the inverse of the COP. With decrease in T_e, Table 1 shows an increase in the power of compressor per kw of refrigeration. The table shows a value of 0.104kw/kw for Te of 0°C and 0.789kw/ kw for $T_e = -55°C$.

Fig 9 presents the graphs showing the variation of the power per kw of refrigeration with evaporating temperature. It will be seen that the trend of the graph is such that the power per kw of refrigeration increases as T_e decreases. The same trend holds for the compressor discharge temperature curve which tends to a high value as T_e decreases. See Fig 9.

4.6 MULTISTAGING

The analysis of a multistage vapour compressor cycle has shown a tremendous improvement over a single stage vapour compression cycle. The improvement over the single stage cycle shows a higher refrigerating efficiency, high compressor volumetric efficiency and a lower discharge temperature. This indicates that as the evaporating temperature is decreasing there will be the need for multistaging for abetter performance. Table 3 shows a comparison of the single stage cycle with the two stage cycle of low evaporating temperatures. The COP, refrigerating efficiency as well as volumetric efficiency of the two stage cycle show appreciable increases over the single stage cycle.

5. CONCLUSIONS

The foregoing analysis has shown the performance of a single stage vapour compression refrigeration cycle at low evaporating temperatures, which has shown to have low volumetric efficiency, low refrigeration efficiency and higher discharge temperatures as this temperature decreases.

Decreases of compressor volumetric efficiency imposes practical limitation on the single stage cycle and makes staging of compressors necessary in low temperature systems irrespective of other considerations. This has been confirmed by the multistage analysis.

It has been shown through the analysis and from the graph of the variation of volumetric efficiency with the evaporating temperatures that the lowest possible evaporating temperature with this system is-54.6$_o$C. Fig 9 shows the graphs of the COP and refrigerating efficiency, both of which show a dropping trend thus confirming that it is not very efficient to operate this system at low evaporating temperatures.

Operating of this system at low evaporating temperature causes the system to have a high value of power input to the compressor per kw of refrigeration, thus the COP will suffer as seen from the graph on Fig 9.

Conclusively, operating a single stage vapour compression cycle at low evaporating temperatures will lead to loss of efficiencies of the system and this is not desirable. Hence it might make staging of the compressor necessary.

NOTATION

C:	number of cylinder
COP:	actual coefficient of performance
COP_r:	ideal coefficient of performance
d:	cylinder bore, m
f_c:	clearance factor
h:	enthalpy of refrigerant, kj/kg
l:	stroke of the engine, m
m:	mass flow rate, kg/s
n:	index of compression
$p_c p_4$:	Condensing pressure, bar
p_e, p_3:	evaporating pressure bar
p_s:	compressor suction pressure, bar
pd:	compressor discharge pressure bar
PD:	piston displacement, m^3/s
Qref:	effect, kw

Table 4. Some Properties of R-12 at the various phases for each Evaporating Temperature

Evaporating Temperature T_e ^oC	Saturation Pressure P_3 bar	Specific Vol. of Saturated vapour v_3 m^3/kg	Enthalpy of Saturated vap h_3 KJ/kg	Enthalpy of superheated vapour h_4 KJ/kg	Enthalpy of Saturated liquid $h_2 = h$ KJ/kg
-60	0.2262	0.6379	160.29	227.92	74.59
-55	0.2998	0.4010	162.02	225.34	"
-50	0.3915	0.3831	164.95	222.98	"
-40	0.6417	0.2419	169.60	218.82	"
-30	1.004	0.1994	174.20	215.47	"
-20	1.509	0.1088	178.73	212.60	"
-10	2.191	0.0766	183.38	210.23	"
0	3.086	0.0554	187.53	200.27	"

T_c: Condensing temperature °C

Te: Evaporating temperature, $_oC$

Te,l: lowest evaporating temperature, °C

V specific volume m^3/kg

V_c: Work of compression, kj/kg

η_{av} actual volumetric efficiency, %

η_{cv} clearance volumetric efficiency %

η_R Refrigerating efficiency,%

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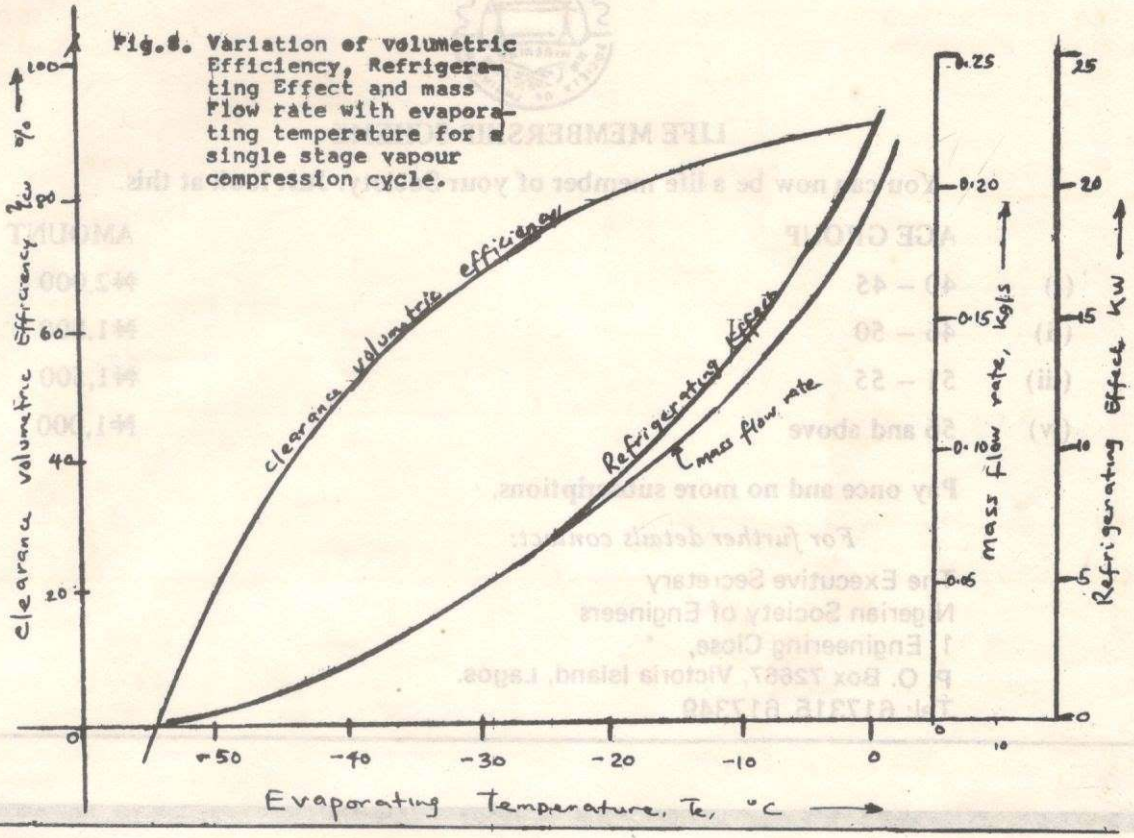


Fig.8. Variation of volumetric Efficiency, Refrigerating Effect and mass flow rate with evaporating temperature for a single stage vapour compression cycle.

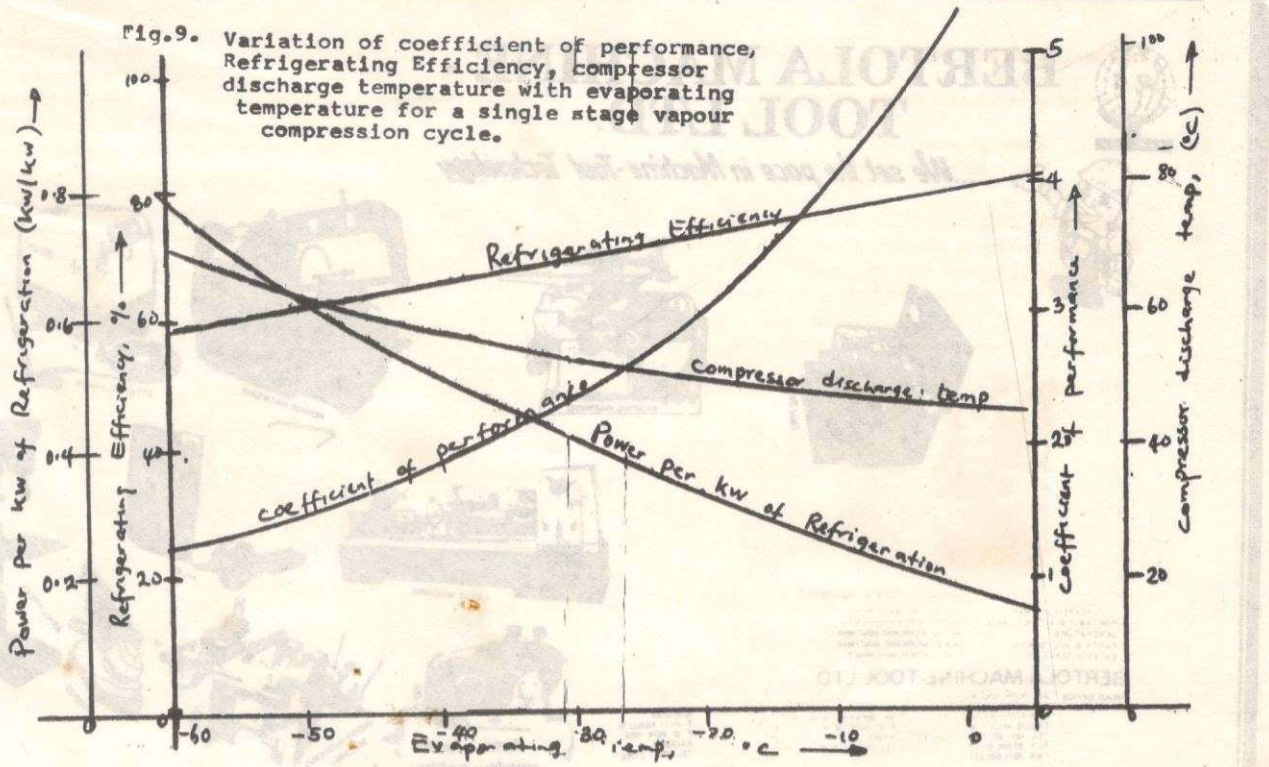


Fig.9. Variation of coefficient of performance, Refrigerating Efficiency, compressor discharge temperature with evaporating temperature for a single stage vapour compression cycle.