PARAMETER PLANE TECHNIQUE FOR CHOOSING DIGITAL CONTROLLER PARAMETERS FOR DEAD TIME SYSTEMS

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ABSTRACTS

An investigation is described in this paper on the benefits that can be gained by the application of parameter plane technique in assessing the system stability region and provide the process engineer the confidence of choosing the controller parameters. The technique is a unique approach that utilizes the graphical procedure for facturing characteristic polynomials in the plane with two parameters as co-ordinates. Stability boundary plots for a two-term controller parameters for a dead time process is presented and the system responses to the typical sets of these parameters are demostrated. It is shown that the technique provides a viable alternative to established stability criteria when dealing with systems with significant dead time.

Key words - Parameter plane, control system, dead time, digital system.

1. INTRODUCTION

The central problem of feedback control systems could be identified as a stability problem and its analysis is known to be a critical problem in processes with significant amount of dead time. The system stability region for such systems is difficult to assess because the dead time introduces an increase in the order of the characteristic equation for discrete systems while the characteristic equation for analogue systems becomes a transcendental function. The well established stability criteria like Routh-Hurwitz and Nyquist do not consititute a complete satisfactory theory for design of control algorithms for processes with significant dead time.

The Root-locus method which permits the designer to obtain readily information about stability and salient features of the system fails to provide simple and explicit correlation between the root locations and adjustable controller parameters which appear on the coefficients of the characteristic equation. The introduction by the dead time of increased order of characteristic equation for discrete systems and exponential term in the characteristic equation for the analogue systems makes the application of Root-locus more difficult.

The theoretical work introduced by Mitrovic [9] and developed by Siljak [13] makes available to the engineer an alternative method to that of the classical technique of Root-locus for analysis and synthesis of feedback control systems. The work [9, 13] which is known as parameter plane technique consists of plotting certain diagrams into the parameter plane and enables the simultaneous evaluation of all characteristic roots to be performed without any calculation for different values of system parameters. The importance of the technique lies in the simple and direct use of algebraic and graphical operations. The introduction of Chebyshev functions greatly simplfies the proposed procedure and makes simulation possible. The axes set of this plane are the controller parameters to be chosen by the design engineer. Typically, a region of chosen relative stability in the z- and s-plane would be maped to the parameter plane and this region would represent a specific solution domain from which to select controller parameter values.

2. SYSTEM MODEL

The plant model used by the authors is the raw feed system of a sinter plant [10] and is considered to be a typical industrial plant that is most likely to suffer from dead time. The total dead time has been estimated from the sinter model [10] to be 6mins. In its simplest form the plant model is shown in Fig. 1. The raw feed system is represented by the first order transfer function with a dead time, and the surge hopper by a simple intergrator. The parameters C_1 and C_2 are respectively the strand speed and surge hopper extraction rate propotionalities.

The appropriate plant model is:

$$G(s) = \frac{Kpe^{-\Delta s}}{(1+s\tau 1)} \times \frac{1}{(s+{}^{C}{}_{1}{}^{C}{}_{2})} \quad (1)$$

The gain K_p is estimated to be 1.0 and Eqn. (1) reduces to

$$G(s) = \frac{\tau 2e^{-\Delta s}}{(1+s\tau_1)(1+s\tau_2)} \quad (2)$$

where $\tau_2 \frac{1}{C_1C_2}$ The numerical values of the time constants could be obtained from Ref. [10] and Eqn. (2) becomes

$$G(s) = \frac{10/3e^{-\Delta s}}{(1+2/3s)(1+10/3s)} \quad (3)$$

The time unit in Eqn. (3) is the minute and the discrete equivalent of Eqn. (3) under a sampling time of 3 mins and with zero-order hold (ZOH) is given by

$$G(z) = \frac{Z^{-2}(1.648z + 0.308)}{Z^2 - o.4177z + 0.0045} \quad (4)$$

The traditional process controller used extensively in industry employs the P+I algorithm. The impact on the numerical values of the coefficients associated with this control law is presented for a process containing large inherent transportation lag. The control law is

$$U(t) = K_c\left[e(t) + \frac{1}{T_i}\int_o^t e(\tau)\,d\tau\right] \quad (5)$$

where U (t) is the manipulated variable, K_C the proportional terms, T_i the integral term and e (t) is the error signal between the set-point and the system output. Eqn. (5) can be transformed to the difference equation given by

$$U(z)[1-z^{-1}] = K_c[E(z) - z^{-1}E(z) + \frac{T_s}{T_i}E(Z)] \quad (6)$$

The controller transfer function is

$$D(z) = \frac{U(z)}{E(z)} = \frac{\alpha z + \beta}{z - 1} \quad (7)$$

where

$$\alpha = K_c\left[1 + \frac{T_s}{T_i}\right] \text{ and } \beta = -K_c$$

The system characteristic equation is

$$1 + D(z)\,G(z) = 0$$

$$z^5 - 1.4177z^4 + 0.4222z^3 + (1.648\alpha - 0.0045)z^2 + (0.308\alpha + 1.648\beta)z + 0.308\beta = 0 \quad (8)$$

It is necessary to determine all values of parameters a and b for which the Eqn. (8) has all the roots located on the left of the constant damping line z = 0.5 of the contour of Fig. 2 and its descrete equivalent in Fig. 3.

3. THEORY

The characteristic equation given in eqn. (8) can be written as

$$\sum_{k=0}^{m} \varphi_i z^k = 0 \quad (9)$$

Where φ (i = 0,1,...,m) are real coefficients and Z is defined as

$$Z = \omega z\zeta z + j\omega z\sqrt{1-\zeta^2 z} \quad (10)$$

The term ω_z and ζ_z can be related to the undamped natural frequence (ω_n) and damping coefficient (ζ) by the relations;

$$\omega_z = e^{-\omega_n \zeta T_s}$$

$$\zeta_z = \cos \omega_n T_s \sqrt{1-\zeta^2} \quad (11)$$

Substituting Eqn. (10) into Eqn.. (9), and independently equating the real and imaginary parts to zero, gives the two simultaneous equations,

$$\sum_{k=0}^{m} \varphi_k \omega^k{}_z T_k(\zeta z) = 0$$

$$\sum_{k=0}^{m} \varphi_k \omega^k{}_z U_k(\zeta z) = 0 \quad (12)$$

where $T_k(\zeta_z)$ and $U_k(\zeta_z)$ are Chebyshev functions (14) of the first and second kind respectively and their relationship with trigonometric functions are

$$T_k(\zeta_z) = \cos\ (k \cos^{-1}(1\zeta_z)) \qquad (13)$$

$$U_k(\zeta_z) = \frac{\sin\ (k \cos^{-1}(\zeta_z))}{\sin\ (\cos^{-1}\zeta_z)}$$

If the function $T_k(z_z)$ is expressed in terms of the function

$$U_k(\zeta_z) \text{ and } U_{k-1}(\zeta_z), \text{ i.e.}$$

$$T_k(\zeta z) = \zeta z\ U_k(\zeta_z) - U_{k-1}(\zeta_z)$$

Then Eqn. (12) can be written in terms of one Chebyshev function only, viz

$$\sum_{k=0}^{m} \varphi_k \omega^k{}_z U_{k-1}(\zeta_z) = 0 \qquad (14)$$

$$\sum_{k=0}^{m} \varphi_k\, \omega^k{}_z\, U_k(\zeta_z) = 0$$

Simplifying the computation. Defining the coefficients of the characteristic equation as linear functions of the variable system parameters α and β yields

$$\varphi_k = b_k\ \alpha + c_k\ \beta + d_k \qquad (15)$$

where b_k, c_k and d_k are constants. Substituting φ_k into Eqn. (14) leads to the expressions

$$\begin{aligned} \alpha B_1 + \beta C_1 + D_1 &= 0 \\ \alpha B_2 + \beta C_2 + D_2 &= 0 \end{aligned} \qquad (16)$$

where

$$B_1 = \sum_{k=0}^{m} b_k\ \omega^k{}_z\ U_{k-1}\ (\zeta_z)$$

$$B_2 = \sum_{k=0}^{m} b_k\ \omega^k{}_z\ U_k\ (\zeta_z)$$

$$C_1 = \sum_{k=0}^{m} c_k\ \omega^k{}_z\ U_{k-1}\ (\zeta_z)$$

$$C_2 = \sum_{k=0}^{m} c_k\ \omega^k{}_z\ U_k\ (\zeta_z)$$

$$D_1 = \sum_{k=0}^{m} d_k\ \omega^k{}_z\ U_{k-1}\ (\zeta_z)$$

$$D_2 = \sum_{k=0}^{m} d_k\ \omega^k{}_z\ U_k\ (\zeta_z)$$

solving Eqn. (16) for α and β gives

$$\alpha = \frac{C_1D_2 - C_2D_1}{B_1C_2 - B_2C_1} \qquad (1$$

$$\beta = \frac{B_2D_1 - B_1D_2}{B_1C_2 - B_2C_1}$$

The variables α and β are the axes set in parameter plane and Eqn. (17) represent general loci of points corresponding to boundary of a chosen closed region in the com z-plane.

If the boundaries of the region in the com plane are positioned by the limits of the con roots and the limits of the real roots, the perir of this region can be mapped in the para plane for the complex roots using Eqn. (17 for the real roots by substituting the value limiting real root into Eqn. (19), i.e.

$$\alpha = \sum_{k=0}^{m} b_k\, \sigma^k{}_z + \beta \sum_{k=0}^{m} c_k\, \sigma^k{}_z + \sum_{k=0}^{m} d_k\, \sigma^k{}_z =$$

For a given value of s_z, Eqn represents a straight line in the parameter The ability to map boundaries in the z-p the parameter plane does guarantee th chosen set of controller parameter values, of the characteristic equation are locat known bounded region of the complex pl

4. STABILITY ANALYSIS

The stability criteria proposed by Routh Nyquist and Jury for control systems p interpretation of the absolute stability by the existence of roots of the characteristi in the appropriate regions of s-plane an Figures 2 and 3 show the contours w been suggested. Mapping these cont different plane provides the informatio existence of root and the parame technique provides a viable solution of stability problem to be obtained in a c effective manner.

The parameter plane plots for the sinter plant controller are given in Fig. 4. The shading of these loci is on the left of the trajectory when facing in the direction of increasing frequency if the denominator of Eqn. (17) is positive. If the denominator of Eqn. (17) is negative the shading is on the right of the parameter plane loci.

Fig. 4 shows clearly the significant reduction, with increasing values of dead time, that takes place in the size of the stable region and hence the region in which the controller parameters can be selected. A change in dead time from 2 to 4 minutes reduces the size of the stability region by 65% and for the change 4 to 6 minutes gives a further reduction of 45%. These changes indicate very clearly why the controller gain settings must, generally, be reduced on many plants below the values recommended by the equipment suppliers. The variations in surge hopper level for the three arbitrary selected locations marked on Fig. 4 are given in Fig. 5 for 10% step change in set point and in fig. 6 for a 10% step change in strand speed.

5. CONCLUSIONS

This paper presents a method for the analysis and synthesis of digital control systems in the parameter plane. The increase in the order of characteristic equation introduced by the presence of dead time do not present any problem to the proposed method.

A significant advantage of the parameter plane technique is that the characteristic equation is considered in its general form and the graphical procedure is independent of the system configuration. Process Engineers may advantageously apply the technique with confidence without the cost of trial and observation procedures often used in the choice of controller parameters for dead time systems.

6. REFERENCES

[1] Bennett, S. and Linkens, D. A. (1982). Computer Control of Industrial Processes. IEE Control Engineering series 21, Peter peregrinus Ltd.

[2] Ball, D.F. Dartnell, J., Davison, J., Grieve, A. and Wild, R. (1973). Agglomeration of Iron Ores. heinemann Educational books Ltd. London.

[3] Cappel, F. (1968). "Present State and Prospects of Sinter Plant Automation. Proceedings AIME -Iron Making. Vol. 27, PP. 29-35.

[4] Dreher, P. (1970). Process Control Computer for Sintering Plant. Siemens Review XXXVII, No. 6, PP. 337-341.

[5] Dipoggio, E. Richardi, C. and Casadio, G. (1972). "How to Increase the Automation of Industrial Plants having Specific Charateristics", 12th International Automation and Instrumentation Conference. Vol. 22-24, November, PP. 487-496.

[6] Goodwin, G.C. and Payne, R.L. (1977). "Dynamic System Identification; Experiment Design and Data Analysis". Academic Press.

[7] Hang, C.C. (1980). "DDC Algorithms - Present and Future", Inst. of Engineers Singapore, (Vol. 20, No. 2 PP. 15-27 for Automatic Control of Production.

[8] Miyagi, T. (1980). "Process Control System of Sintering Plant". The Sumitomo Search (Japan) No. 24, PP. 83-93.

[9] Mitrovic, D. (1959). "Graphical Analysis and Synthesis of Feedback Control Systems - Theory and Analysis", AIEE Trans. PP. 567-496, Jan. 1959.

[10] Onogu M. I. (1985) "Self-tuning Algorithm Design for Plants Containing Significant Amount of Dead Time", Ph.D. Thesis Mechanical Engineering Department Leeds University, U.K.

[11] Rowen, H.E. (1956). "Development of the Dwight-Lloyd Sintering Process", Journal of Metals vol. 8 PP. 28-33.

[12] Smith, C. L. (1972). "Digital Computer Control Systems", McGraw-Hill Book Company Inc.

[13] Siljak, D.D. (1964), "Analysis and Synthesis of Feedback Control Systems in the Parameter Plane: (1) Linear Continuous Systems. (II) Sampled-Data Systems. (III) Nonlinear Systems. IEEE Trans. Applications and Industry, vol. 83 PP. 449- 473, Nov. 1964.

[14] Taicahashi, Y., Rabins, M.J. and Auslander, D.M. (1970). "Control ano Dynamic Systems", Addison-Wesley Publishing Company.

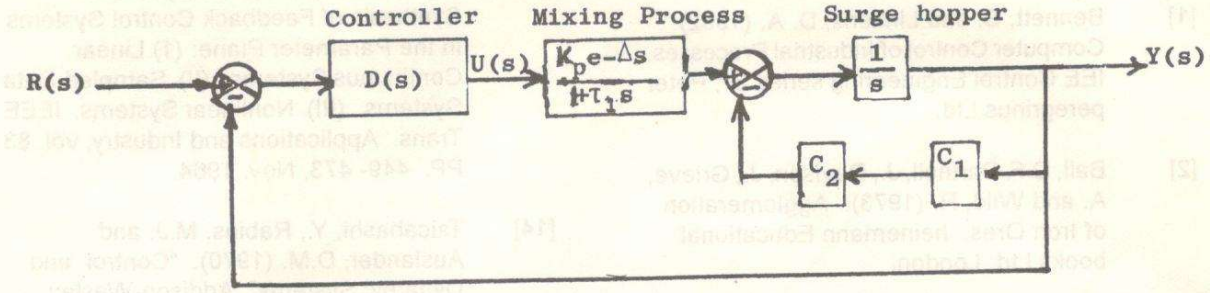
[15] Williams, R.V. (1983). "Control and Analysis in Iron and Steel Making" Butterworths.

[16] Moore, C.F., Smith, C.L. and Murrill, P.W. (1969). "Simplifying Digital Control Dynamics for Controller Tuning and hardware Lag Effects". Instrument Practice, vol. 23, No. 1, pp 45-49.

[17] Franklin, G.F. and Powell, J. D. (1980). "Digital Control of Dynamic Systems". Addisson - Wesley Publishing Company.

[18] Benjamin C. Kuo (1980). "Digital Control Systems". Holt - saunders International Editions.

[19] Paul, K. (1981). "Digital Control using Microprocessors". Prentice - Hall International.



C_1: Strand Speed proportionality

C_2: Surge hopper extraction rate

Fig. 1 System block diagram

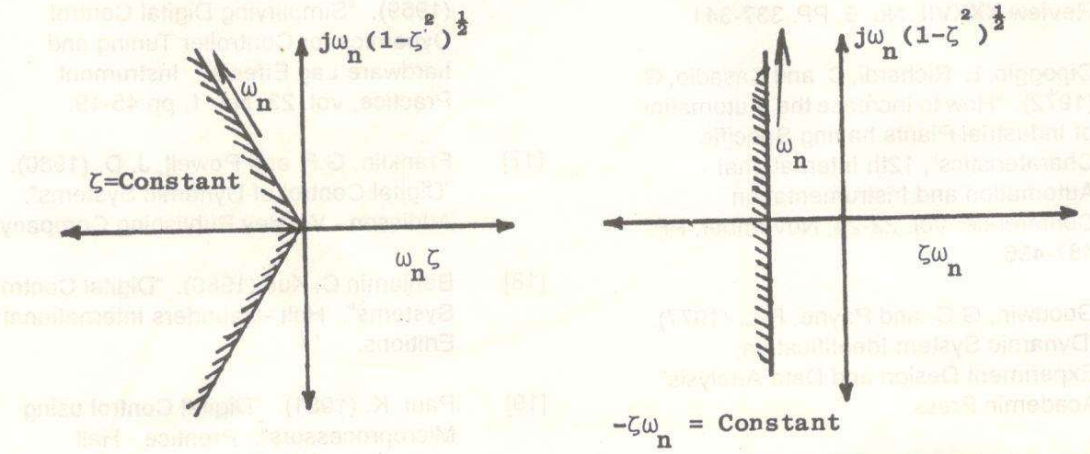


Fig. 2 The s-plane contours

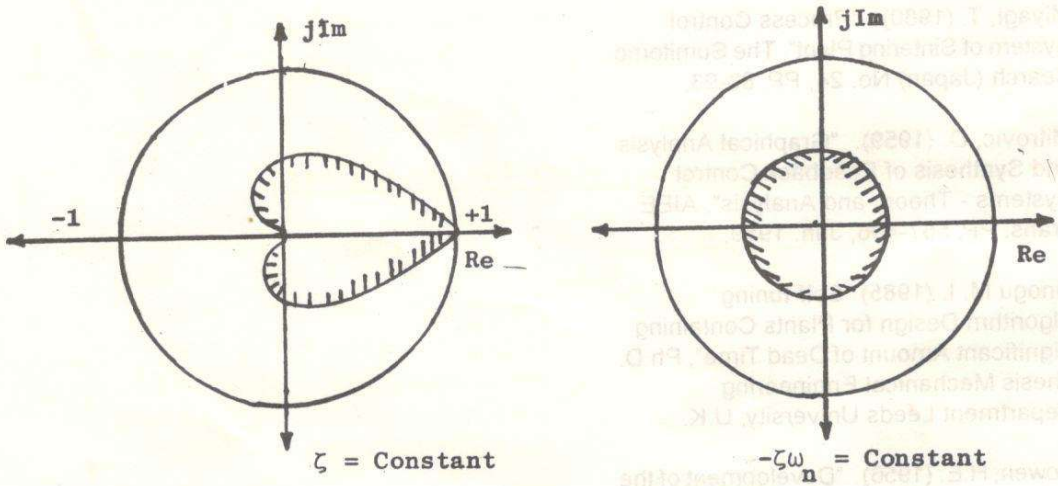


Fig. 3 The z-plane contours

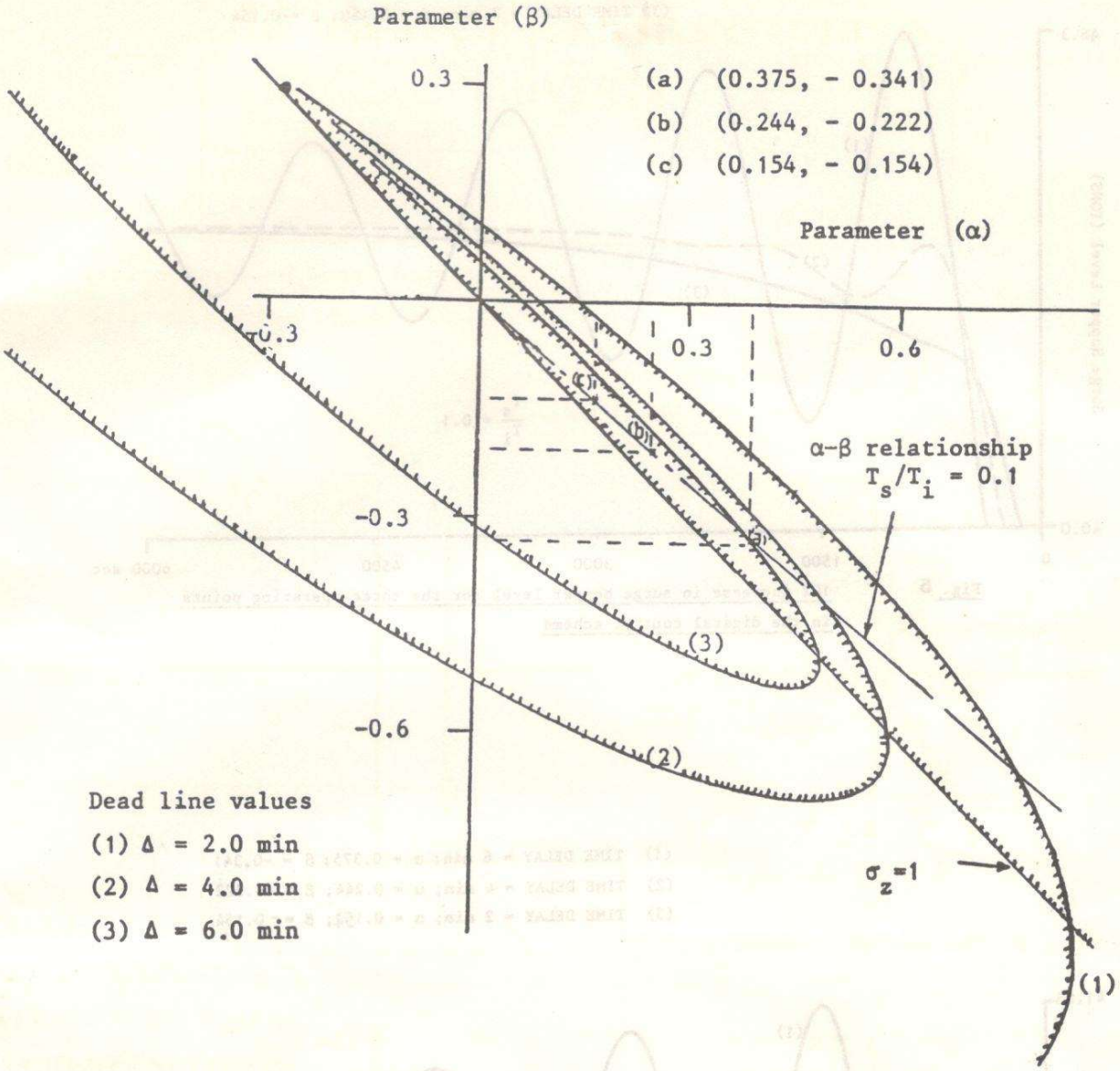


Fig. 4 Stability boundary plots for the digital two-term controller

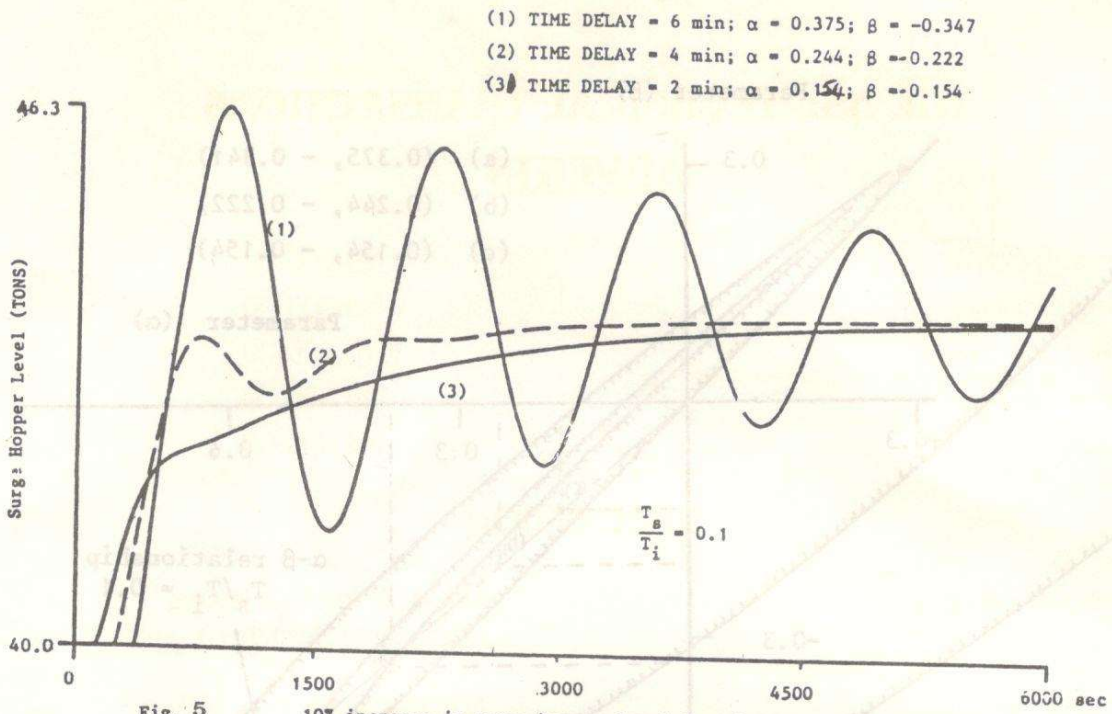


Fig. 5 10% increase in surge hopper level for the three operating points in the digital control scheme

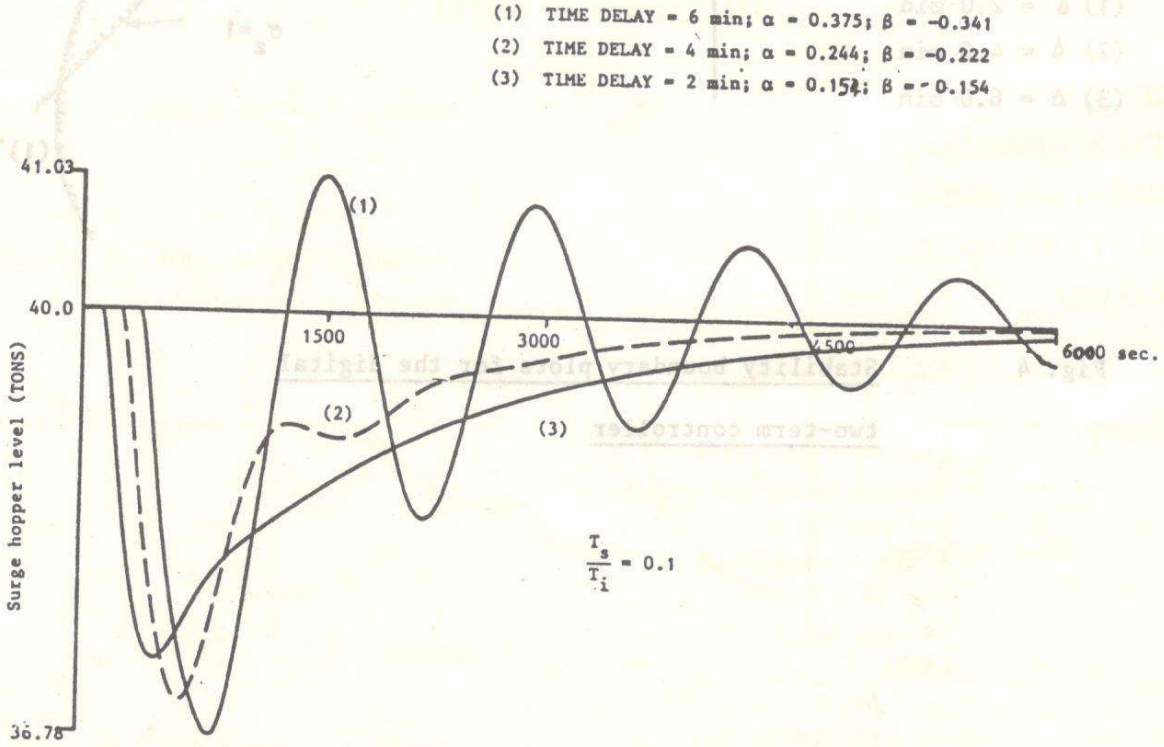


Fig. 6 10% increase in strand speed for the three operating points in the digital control scheme