VELOCITY AND MIXING CHARACTERISTICS IN A LAMINAR JET FLAME

DR. D. P. S. ABAM
Department of Mechanical Engineering
University of Port Harcourt Nigeria.

ABSTRACT

This paper describes a semi-empirical method for determination of velocity and mixing characteristics in a laminar jet flame. Empirical correlations of a conserved mixing property and temperature are derived from similarity analysis of the conservation equations of enthalpy and mass. These are then used in transforming the conserved equation of the mixing property otherwise called mixture fraction, to obtain explicit expression for the velocity field of the jet flame, which is then computed from known properties.

It is observed that the density function and the mass transport coefficient exert strong influence on the velocity field through their strong temperature dependence. It is further observed that the interface region between the fuel and air is characterised by high viscous dissipation as a result of the high rate of mixing. The combined effect of buoyancy through the density function and viscous dissipation is the increase in flow velocities and acceleration of the flow from initial conditions by two orders of magnitude, through the flame.

KEY WORDS: Laminar Flow, Density Function, Viscous Dissipation Similarity Analysis, Empirical Correlation, Mixing Property, Conservation Equations.

INTRODUCTION

The structure of diffusion flames has been extensively studied in recent years. This is as consequence of the increasing use of diffusion flames in modern power plants and the environmental question of harmful emissions from energy conversion systems. The theoretical frameworks for describing the diffusion flame are founded on two models. The first model is the classical work of Burke and Schumann (1), which deals with gross physical attributes of the flame. The second model is due to Shvab-Zeldovich (2) and it appropriately describes experimentally observed phenomena relating to flame structure. However, none of these two models is readily amenable to complete analytical solution without recourse to over simplifying assumptions. In view of this, the applicability of analytical solution is severely limited, particularly in obtaining correct information on the physico-chemical properties of the flame. Fortunately, Computer methods in fluid flow and heat transfer have advanced to the point where numerical solution of complex fluid dynamic and thermodynamic phenomena are readily supplied. Patankar and Spaiding (3) developed a numerical procedure for calculating heat, mass and momentum transfer in three-dimensional parabolic flows. Moore and Moore (4), the author (5) and others (6, 7, 8) have independently, at various times adapted the numerical system to investigate various features of diffusion flames.

There is general understanding that, although numerical simulation remains a powerful computational tool, the large numerical effort could be drastically reduced if the velocity field is known apriori. Furthermore prior knowledge of the velocity field would eliminate the uncertainties associated with assumed flow parameters such as pressure gradient and the transport coefficients. The work presented in this paper describes a method for computing the velocity field. The method originates from transformation of the diffusion equation from the cylindrical (r, x) coordinate system to a single co-ordinate system defined by the similarity principle.

2. THE SHVAB-ZELDOVICH SPECIES: MIXTURE FRACTION

The mixture fraction (G) is a conserved mixing property enunciated by Shvab-Zeldovich (2) and subsequently, explicitly derived by Bilger (9) from similiarity considerations. For methane - air diffusion flames, the mixture fraction is defined by the following functions:

i) mass fraction of oxgyen in all forms
ii) mass fraction of carbon in all forms
iii) mass fraction of hydrogen in all forms
iv) mass fraction of nitrogen in all forms
v) mass enthalpy function.

The above functions independently satisfy the diffusion equation for a conserved species. In cylindrical coordinates, the equation takes the form:

$$\frac{\partial G}{\partial t} = \frac{1}{\rho U r}\frac{\partial}{\partial r}\left(r\rho D\frac{\partial G}{\partial r}\right) = 0 \quad \text{---------- (1)}$$

The author (5) has shown that by scaling the conserved functions defined above and reducing them to common boundary and initial conditions, they become equal to one another. These conditions appropriate to the Jet Flame are as follows:

Initial conditions: G= 1; $(0 \frac{r}{a} \leq 1)$ (Fuel Stream)

G= 0; $(0 \leq \frac{r}{a} \leq \frac{R}{a}$ (Air stream)

Boundary conditions: $\frac{\partial G}{\partial r} = 0$(r =0 and r= R)

3. DETERMINATION OF MIXTURE FRACTION (G) FROM MEASURED SPECIES CONCENTRATIONS

There is general agreement from available data that the stable species composition of methane - air diffusion flame consists of CH_4, CO CO_2, H_20, H_2, O_2 and N_2. Given this composition, the mixture fraction is determined from the species mass fractions as:

$$G = m_{CH4} + \frac{12}{44}m_{CO2} + \frac{12}{28}m_{CO} + \frac{12}{18}m_{H2O} + m_{H2} \text{ --------- (2)}$$

In terms of molar concentrations, the mixture fraction is given by:

$$G = \frac{1}{M}[16f_{CH4} + 12f_{CO2} + 12f_{CO} + 2f_{H20} + 2f_{H2}] \text{------ (3)}$$

It is worth noting that the local equivalence ratio (EQ) is related to mixture fraction thus:

$$EQ = \frac{17.24G}{1 - G} \text{---------- (4)}$$

It follows from equation (4) that at stoichiometric, corresponding to EQ = 1, the mixture fraction has the unique value of G = 0.0548.

4. SPATIAL CORRELATION OF MIXTURE FRACTION

The mixture fraction is correlated with the cylindrical coordinates from measured data obtained from jet flames of various geometries in the form:

$$G = G_1\left(\frac{x}{L}\right).G_2\left(\frac{r}{a}\right) \text{---------- (5)}$$

where $G_1 = \frac{1}{\left(1 + \alpha\frac{x}{L}\right)}$; $G_2 = EXP\left[\beta\left(\frac{r}{a}\right)^2\right]$

(α and β) are determined from the correlations as $\alpha = \frac{941}{59}$ and $\beta = 2.08$)

Details of the flames covered in the study and from which the generalised correlation of equation (5) is obtained are given in Table 1. The variational profiles of G are presented in figures 1.1, 1.2 and 1.3.

5. TEMPERATURE - MIXTURE FRACTION CORRELATION

The enthalpy function H is related to the mixture fraction in the form:

$$\frac{H}{H_F} = G \text{ ---------- (6)}$$

Using a standard thermodynamic base corresponding to temperature, T=298K and pressure p= I atmosphere, equation (6) simplifies to:

$$\int_{298}^{T} C_p dT = \left(G - m_{CH_4}\right)H_F - m_{CO}H_{CO} - m_{H_2}H_{H_2} \text{ ------ (7)}$$

The consistency of equation (7) is established by the Author (5), using the measure temperature and stable species concentrations.

This temperature - mixtured fraction correlation is presented in Figure 2. The profile reveals the occurrence of a local temperature maxima at a fixed value of mixture fraction (G = 0.059). The luminous flame boundary is in this neighbourhood. Using this observation the maximum temperature boundary is then determined by the expression.

$$r^2_{STOIC} = \frac{a^2}{\beta}ln\left(\frac{0.0549}{G_1(\frac{x}{L})}\right) \text{ ---------- (8)}$$

The corresponding stoichiometric boundary defined by EQ - 1, is determined from:

$$r^2_{STOIC} = \frac{-a^2}{\beta} In\left(\frac{0.0548}{G_1(\frac{x}{l})^{\frac{1}{2}}}\right) \quad \text{(9)}$$

Geometric flame configurations generated from equations (8) and (9) are shown in Figure 3. These configurations can be used to determine the gross physical characteristics of the jet flame, for a given set of initial conditions and burner geometry.

6. DERIVATION OF FLOW VELOCITIES

6.1 INITIAL VELOCITY DISTRIBUTION

From fluid mechanics considerations, according to poiseuille flow, the theoretical initial fuel and air velocity distributions through the inner and outer circular pipes respectively are:

$$U_i = 2\,U_{fuel,\,av}\left[1-\left(\frac{r}{kR}\right)^2\right];\ 0 \le r \le kR;\ x = 0 \quad \text{(10)}$$

$$U_i = \frac{2U_{air,av}}{\left(1+k^2-\frac{1-k^2}{In\frac{1}{k}}\right)}\left[1-\left(\frac{r}{R}\right)^2+\frac{1-k^2}{In\frac{1}{k}}In\frac{r}{R}\right];\ kR \le r \le R;\ x=0 \quad \text{(11)}$$

In equation (10) and (11), k is essentially the ratio of burner to outer pipe radii $\left(\frac{a}{R}\right)$

In practice, a flat velocity profile is assumed over the width of the burner mouth and the outer shield. Since zero velocity is not allowed in the assumed profile, it is therefore chosen such that over the burner pipe wall thickness where there is no flow initially, being a thick wall, the initial velocity over this pipe width is consistent with obtaining the correct mass and enthalpy balances through the system. Thus, on this basis, the initial velocity over the burner wall thickness is taken to be equal to the flat velocity through the burner mouth.

6.2 DERIVATION OF EXPRESSION FOR THE VELOCITY FIELD

The velocity function U is obtained by transformation of the diffusion equation (1) using the generalised correlation of equation (5). The results is:

$$U = \frac{4\beta LD}{\alpha a^2 G_1}\left(1-\beta\left(\frac{r}{a}\right)^2(1+B)\right) \quad \text{(12)}$$

Where: $B = \frac{\partial In\left(\frac{\mu}{Sc}\right)}{\partial InG}$

μ is the laminar kinematic viscosity coefficient

Sc is the Schmidt Number (Sc = 0.7)

Equation (12) is not readily amenable for the evaluation of the velocity function (U) owing to insufficient information on the behaviour of the factor B. Since B is a complex logarithmic function of both the viscosity coefficient and the mixing property, its evaluation from purely laminar considerations is not likely to yield representative result especially in the mixing region between the fuel and air flows which are characterised by intensive viscous dissipation.

However, examination of equation (12) shows that the velocity on the axis of symmetry is readily obtained by setting $\frac{r}{a}=0$ and this yields:

$$U(0, x) = \frac{4\beta LD}{\alpha a^2 G_1} \quad \text{(13)}$$

From this result, certain fundamental characteristics of the flame can be evaluated as follows:

The flow time t_F is given by:

$$t_F = \int_0^x \frac{dx}{U(o,x)} = \frac{-a^2 InG_1}{4\beta D} \quad \text{(14)}$$

The Acceleration of the flow on the axis of symmetry is obtained as:

$$a_o = U\frac{\partial U}{\partial x} = \frac{16\beta^3 D^2 L}{\alpha a^4 G_1} \quad \text{(15)}$$

The Local Damkohler No. respecting to the flow and diffusion time is evaluated form the expression:

$$\text{Damkohler No.} = \frac{\text{Flow time } (t_F)}{\text{Diffusion time } (t_d)}$$

$$\text{i.e. Damkohler No.} = \frac{a^2 (\nabla G)^2 InG_1}{4\beta} \quad \text{(16)}$$

With reference to the initial flow conditions for the entering fuel and air streams, and allowing for buoyancy, the flame gases start with initial velocity (U_i) and continue with upward acceleration (a_o) through the flame. The axial velocity (U) attained at a distance (x) from the burner mouth is given by the equation:

$$U = \left(U_i^2 + 2a_o x\right)^{\frac{1}{2}} \quad \text{(17)}$$

A binomial expansion of equation (17) yields the first two terms as:

$$U = U_i\left(1+\frac{a_o x}{U_i^2}\right) - - - \quad (18)$$

Using equation (15), the axial velocity is obtained in terms of the initial velocity distribution as:

$$\frac{U}{U_i} = 1+\frac{16\beta^2 D^2 L^2 \bar{x}}{\alpha a^4 G_1 U_i^2} \quad \ldots\ldots(19)$$

From equation (19), the effects of buoyancy are evident. The above expression also shows that buoyancy will have a profound effect on the flow over the flame height; if the modified Froude No. Given by $Fr = \frac{U_i^2}{a_o x}$ is of the Order Fr << 1, that is, if

$$\frac{16\beta^2 D^2 L^2 \bar{x}}{\alpha a^4 G_1 U_i^2} >> 1.$$

Using data from Flame AAX:

$$\beta = 2.08;\ D = 0.2 \times 10^{-4}\left(\frac{T}{273}\right)^{1.69} m^2/s;$$

$L = 250 \times 10^{-3}$ m; $x = 1$; $\alpha = \frac{941}{59}$, G1 = 1; Ui=0.1m/s; T = 2000 K and a= 8.73×10^{-3}m, the buoyancy factor is of the order of 10^3.

Equation (19) is the basis for the evaluation of the velocity function at any point in the flow, since it is also consistent with the initial condition of flow corresponding to x = 0

The velocity profiles computed by means of equation (19) are presented in Figure 4 and 5, for velocities on the axis of symmetry and for the radial distribution respectively. The computational data are taken from the flame condition AAX in Table 1.

It is evident from Figure 4 that the flow undergoes rapid acceleration through the flame. Figure 4 also shows that the axial velocities peak in the neighbourhood of the maximum temperature. This observation gives further credence to the dominant effect of buoyancy on the flame.

Comparison of equations (12) and (13) shows that both equations take identical form when the factor B assumes the value B = -1, irrespective of the value of $\frac{r}{a}$. In real situation, it is observed that the real and theoretical values of B agree only in the fuel core region (G ≥ 0.3). It is also observed that the value B = -1 is obtained when equation (12) is matched with the known initial air flow conditions.

DISCUSSION

A complete description of the velocity field in laminar jet flow is obtained from the momentum conservation equation in the form:

$$\frac{\partial U}{\partial x} - \frac{1}{\rho U r}\frac{\partial}{\partial r}\left(r\frac{\mu}{Sc}\frac{\partial U}{\partial r}\right) = \frac{1}{\rho U}\left(\frac{\partial p}{\partial x}+\rho g\right) \quad (17)$$

In order to explicitly solve the diffusion equation (1), it is necessary to decouple it from equation (17), as both equations are coupled by the density function. This is not analytically feasible because of the complexity of equation (17) and considering that both pressure gradient and buoyancy force terms are non-linear functions of velocity, and are not usually specified. On the other hand, the diffusion equation (1) readily lends itself to both analytic and numerical solution once the velocity field is known.

The method described in this paper uses known properties of the flame and intial flow conditions to determine the velocity field. A transformation of the diffusion equation using the generalised correlations reveals very significant features of the mixing and flow fields.

In the first place, the gross physical characteristics of the flame, namely the length and size of the flame an other geometrical attributes are determined from the mixing parameter which have well defined values at the stoichiometric and maximum temperature boundaries, for given initial conditions and burner configuration.

In the second place, examination of the velocity profiles on the axis of symmetry and the radial distribution reveals that the flow undergoes rapid acceleration from its initial outlet from the burner month, through the flame. This means that buoyancy exerts very significant effect on the flow velocity over the flame height. Consequently, the modified Froude Number defined by $\frac{U_i^2}{aL}$ is expected to be of an order much less than unity, i.e. Fr << 0(1), throughout the flame.

The observed discrepancy between the actual values of the factor

$B = \frac{\partial \ln(\rho_\infty)}{\partial \bar{x}}$ and the theoretical values

determined from laminar properties, especially in the overlapping air-fuel mixing regions is indicative of intensive mixing in this region, a clear departure from laminar flow. As the flame boundary moves higher up towards the axis of symmetry, so also does this region move towards the axis until the entire flow is encompassed beyond the flame.

CONCLUSION

This paper describes a method for indirectly computing flow velocities and mixing characteristics in jet diffusion flames. The results of the computation reveal that buoyancy and viscous dissipation forces play vital roles in properly locating the boundaries of the flame and locating the velocity field. Inside the fuel core, buoyancy forces are predominant resulting in the acceleration of the flow through the flame by two orders of magnitude from the starting condition. Outside the fuel core, viscous dissipation is adverse as a result of departures of the temperature dependent transport coefficients from laminar conditions.

The knowledge of the velocity field provides strong measure of the aerodynamic structure of the flame and hence the characteristics of the flame properties which relate to its aerodynamics.

NOMENCLATURE

- L - Flame height (m)
- a - Burner port radius (m)
- a_o - Acceleration (m/S^2)
- U_i - Initial fuel and air flow velocities (m/s)
- U - Axial velocity (m/s)
- R - Outer Shield radius (m)
- a, β - Correlation coefficients
- B - Ratio of incremental change in viscosity coefficient to incremental change in mixing property ($B = \frac{\partial In \mu_{eff}}{\partial In G}$)
- x, r - Axial and radial coordinates (m)
- G - Mixing property otherwise called Mixture Fraction
- G_1, G_2 - Functions of axial and radial coordinates respectively
- $\bar{x}$ - Scaled axial coordinate ($\bar{x} = \frac{x}{L}$)
- D - Diffusion coefficient (m^2/s)
- ρ - Density (kg/m^3)
- m_i - mass fraction of species (i)
- f_i - mole fraction of species (i)
- $\bar{M}$ - Mean Molecular weight of the mixture at any point in the flame (kg)
- EQ - Equivalent ratio
- T - Temperature (Kelvin)
- Cp - Specific heat at constant pressure
- Hi - Enthalpy of formation of species (i) under standard conditions of temperature and pressure
- k - Radial coefficient ($k = a/_a$)
- r_{stoic} - Stoichiometric Flame Boundary
- r_{Tmax} - Maximum Temperature Flame Boundary

TABLE 1

DETAILS OF FLAMES INVESTIGATED

Investigator	Burner I. D. (a) (mm)	Outer Pipe dia (cm)	Fuel inlet velocity (m/s)	Air inlet velocity (m/s)	Flame height (L) (m)	$\frac{L}{a^2}$ $(\frac{1}{m})$
The Author (5)						
AAX	17.46	29.52	.1	1.43	.25	820.07
ABX	17.46	29.52	.043	.87	.10	328.03
Tuteja (6)						
TAX	16.00	4.3	.22	.405	.413	1613.28
TBX	16.00	4.3	.22	.542	.362	1414.06
TCX	16.00	4.3	.22	.671	.349	1363.28
Mitchell (7)						
MX	12.7	5.08	0.45	.0988	.058	359.60

REFERENCES

1. **Burke. S. P.** and **Schumann, T. E. W.** Diffusion Flames. Ind. and Engng Chem. Vol. 20, No. 10, 1928

2. **Shvab, V. A.** Acta Physicochimica, URSS, 21, 577, (1946).

3. **Patankar, S. V.** and **Spalding, D. B.** Heat and Mass Transfer in Boundary Layers, Intertext Books, 1970.

4. **Moore, J. G.** and **Moore, J.** Sixteenth Symposium (International) on Combustion, The Combustion Institute (1976).

5. **Abam, D. P. S.** Computation of Aerothermodynamic Field of Laminar Diffusion Flames. Proceedings of the Institution of Mechanical Engineers, London, Journal of Mechanical Engineering Science, Part C, Vol. 198 C, No. 12 (1984).

6. **Tuteja, A. D.** and Nitric Oxide Formation in Laminar Diffusion Flames. Newhall, H. K. Symp. on Emmissions from Continuous Combustion Systems 1971, Plenum Press, New York.

7. **Mitchell, R. E.** Sc. D. Thesis, Massachusetts Institute of Technology, Cambridge, Mass (1975).

8. **Jellings, W.** Unpublished Research, University Engineering Laboratory, Trumpington Street, Cambridge (1975).

9. **Bilger, R. W.** The Structure of Diffusion Flames, Combustion Science and Technology Vol. 13, pp 155 - 170 (1976); Progress in Energy and Combustion Science 16, 87 (1976).

RADIAL DISTRIBUTION OF MIXTURE FRACTION (G)

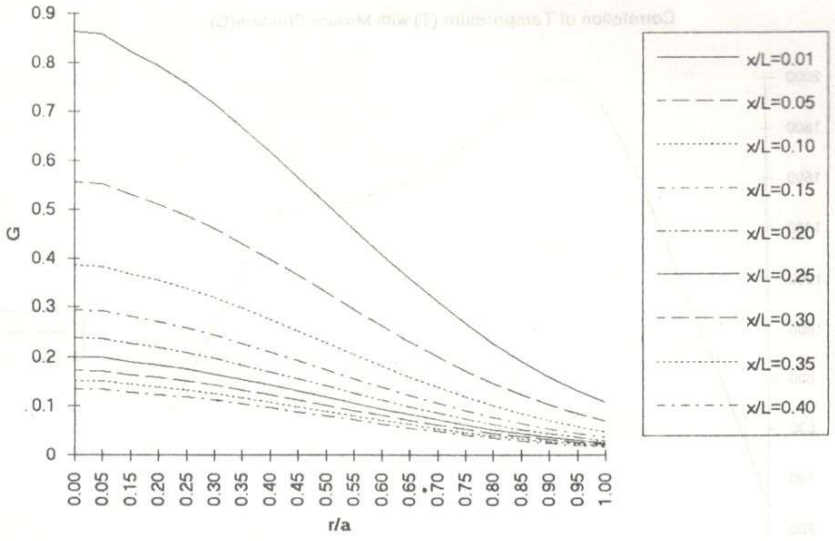


FIGURE 1.1

RADIAL DISTRIBUTION OF MIXTURE FRACTION (G)

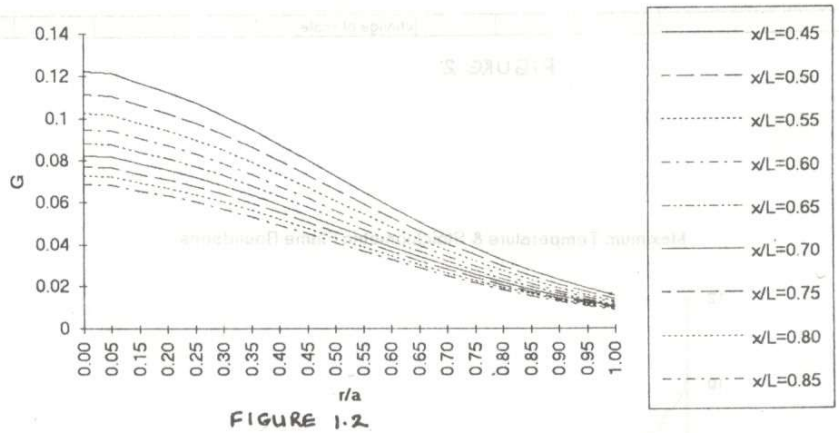


FIGURE 1.2

RADIAL DISTRIBUTION OF MIXTURE FRACTION (G)

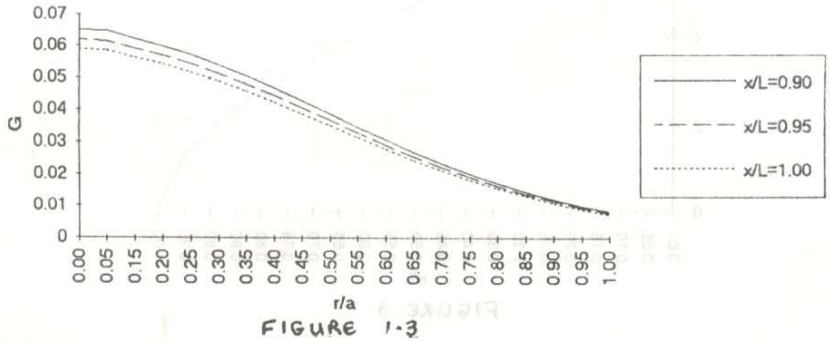


FIGURE 1.3

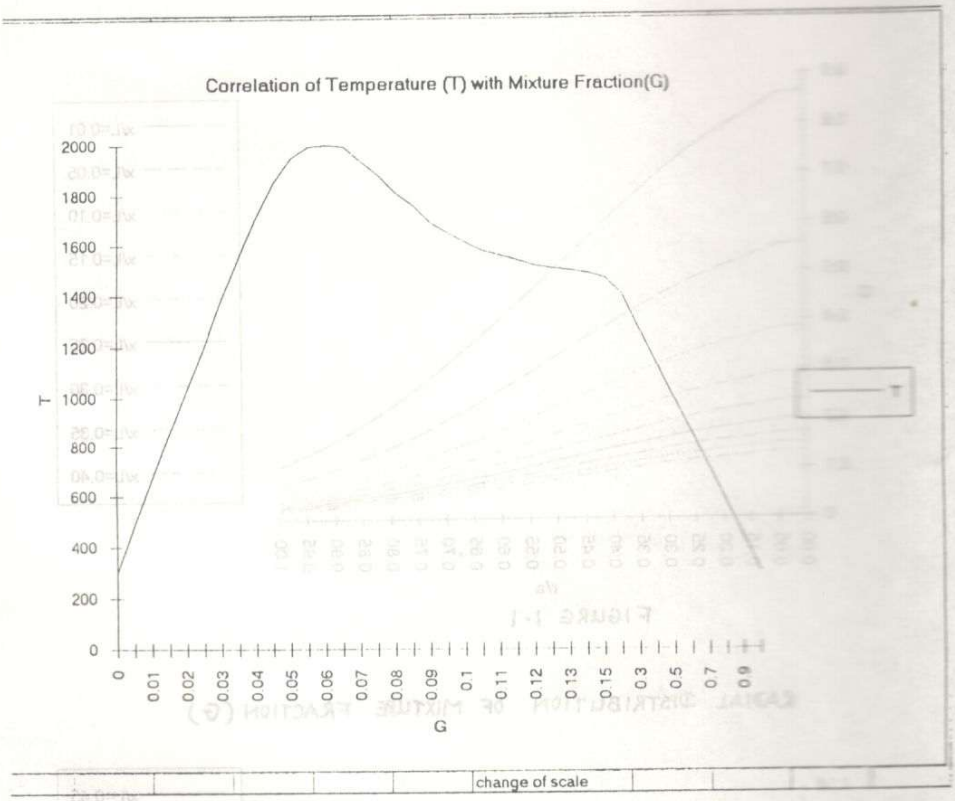


FIGURE 2

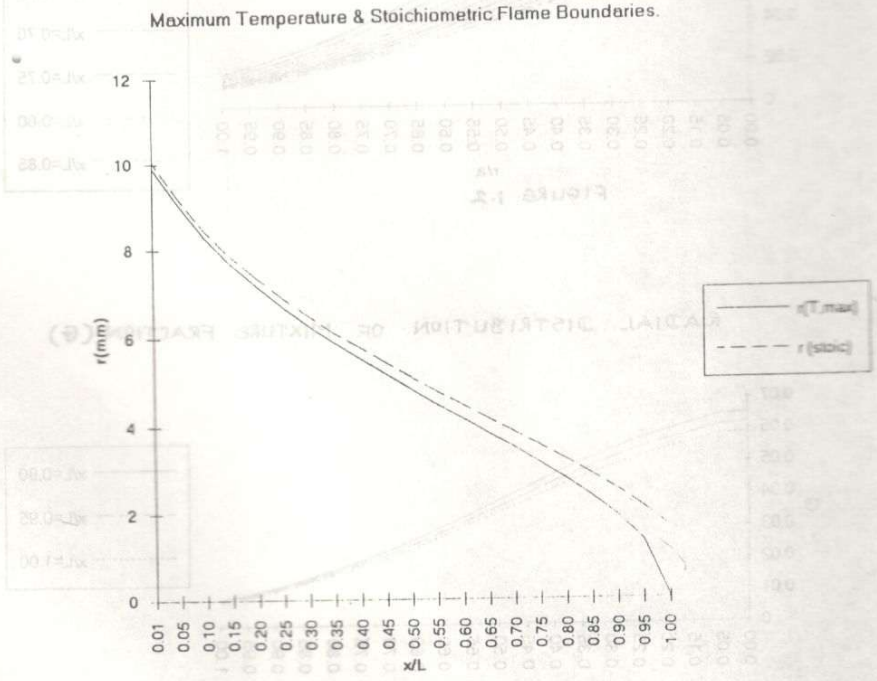


FIGURE 3

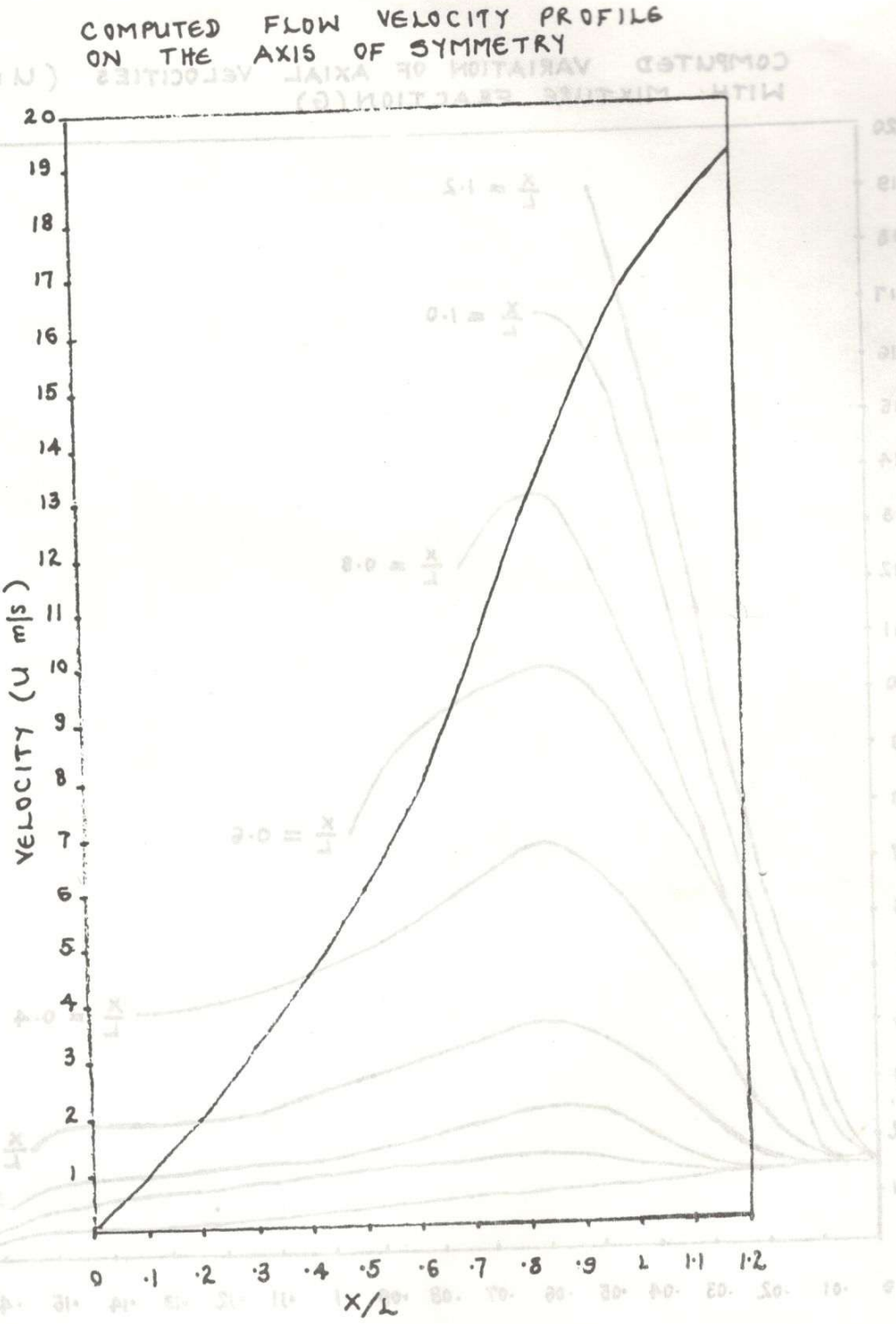

COMPUTED FLOW VELOCITY PROFILE
ON THE AXIS OF SYMMETRY
VELOCITY (U m/s)
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0 ·1 ·2 ·3 ·4 ·5 ·6 ·7 ·8 ·9 1 1·1 1·2
X/L

FIGURE 4

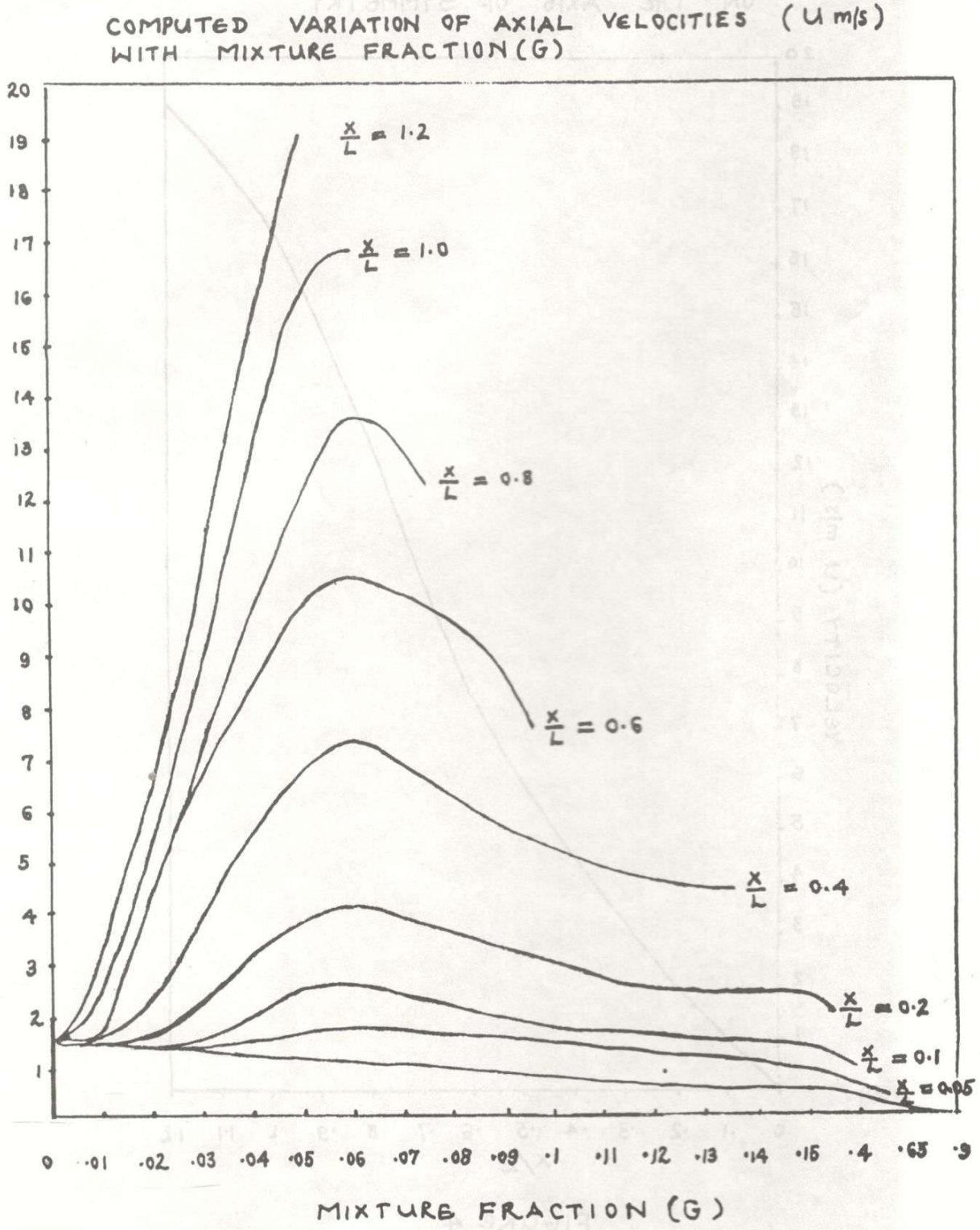

COMPUTED VARIATION OF AXIAL VELOCITIES (U m/s)
WITH MIXTURE FRACTION (G)
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
X/L = 1.2
X/L = 1.0
X/L = 0.8
X/L = 0.6
X/L = 0.4
X/L = 0.2
X/L = 0.1
X/L = 0.05
0 .01 .02 .03 .04 .05 .06 .07 .08 .09 .1 .11 .12 .13 .14 .15 .4 .65 .9
MIXTURE FRACTION (G)

FIGURE 5