NON-LINEAR STABILITY ANALYSIS OF COOLING TOWER SHELLS

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ABSRACT

A new method for analysing the bucking failure of cooling tower shells of any geometric surface of revolution is proposed. The method is based on decomposing the stress-strain conditions into fourier series along the circumferential spatial co-ordinate .The obtained system of ordinary differential equations is then discretized using the basic trigonometric function. The application of the approach is demonstrated on the geometric nonlinear stability analysis of a reinforced concrete hyperbolic cooling tower of height 150m, base diameter 120m and wall thickness of 20cm. Results show that the tower lost stability asymmetrically with the formation of circular wave along the cicumferential coordinate. The response of the tower under its own self weight is also studied.

1.0 INTRODUCTION

The development of natural draught cooling towers has been very rapid during the last 25 years. Until 1970, cooling towers were built for a plant capacity of about 350 mega watts (MW). These wet type cooling towers had maximum height of 120m and base diameter of about 90m. However, beginning from 1975, very large cooling towers up to 165m height and 150m base diameter for Nuclear power plant capacities of over 1000MW have been designed and constructed in Britain, China, France, Germany, Iran, Russia and U. S. A.

Cooling towers are traditionally built in reinforced concrete in the shape of hyperboloids of revolution. Such towers are extremely difficult to build, because the geometry and depth of the shell vary continually over the height of the tower. Deviation from the designed geometric configurations of the tower may increase the danger inherent in imperfections, which if accumulating, may cause the tower to collapse as a result of loss of stability. The collapse in Britain of the Ferrybridge towers in 1965 and the Arder tower in 1973 are two telling examples. So, as far as the structural safety of cooling tower shells is concerned, it is important to consider the effect of geometric non-linearity, since linear analysis more often than not, overestimates the critical loads of the shell.

Many scientists and engineers have been involved in this area for some time now. Mungan I. (1985) experimentally investigated the buckling failure of a large number of relatively perfect cooling tower models. He considered both geometric and material non-linearities. His tests showed that buckling started always locally, even under axisymmetric loading conditions due to unavoidable even very slight local imperfections. He also proved that tensile strength required against tensile failure of concrete is an important criterion in determining the shell thickness of reinforced concrete cooling towers.

Mang and Trappel (1984) used finite element method (FEM) to compare the cracks and ultimate loads of two different reinforced concrete cooling towers, namely, a 93m high cooling tower built in Austria and 150m high Port-Gibson cooling tower in the U.S.A Both towers were subjected to dead load and quasi-static wind loads. On the basis of their numerical results obtained, the authors concluded that the ultimate load is considerably smaller than the corresponding initial buckling load; and that the crack load indicating initial cracking is a lower bound to the ultimate load.

In another paper, Gioncu V. et. al. (1985) proposed a method for analysing prestressed cable cooling towers, taking into account the interaction between the reinforced concrete central mast and the prestressed

cable shell of the tower. The method considered the effect of concrete creep and shrinkage of the mast under axisymmetric and wind loadings.

When a cooling tower is subjected to vigorous wind or earthquake perturbations, up-lift of the base footing may occur and the resistance of the soil will decrease due to decrease in contact area. Shiro Kato et. al. (1991) examined the responses in a cooling tower shell and the soil foundation using N0 - Tension method. The tower has ring footings supported on V-type columns. He showed that the stresses on the base soil are largely affected by the up-lift of the ring footing.

Although for reinforced concrete cooling towers, surfaces of hyperboloids of revolution type are commonly being used, because there are straight generatrix lines on the surface which makes it easier to handle the form-work, however, recent researches have shown the possibilities of steel as an alternative material for cooling towers with other forms of geometric surfaces. Cholnoky and Kollar (1989) designed a 160m cooling tower made of combined conical and cylindrical steel recticulated space trusses. Though this promises to be an interesting alternative to reinforced concrete shell type, the authors did not report on the draught efficiency of the cylindrical form and on the protection of the steel bars against corrosion in long term.

N. Saheed (1989) analysed a dry cooling tower also constructed of steel for a power plant in Iran. The behaviour of the tower was studied by a general purpose structural analysis program based on finite element method. The author analysed the stability of the tower under different combination of loading such as self-weight, ice, wind, earthquake and thermal effects, taking geometric non-linearity of the shell into account.

The effects of stiffening rings on the structural response of cooling towers also attracted theoretical and experimental interests of researchers. Peters (1984) reported on the application of rings at the cooling towers of Doel III Nuclear power plant in Belgium. He emphasized that ring-stiffened cooling towers are not only a structural alternative but also an economical one for cooling towers of larger dimensions. In a symposium on Natural Draught Cooling Towers in Paris, Bosman P. B. (1989), presented a report on the application of stiffening rings on the repairs of two cooling towers of the Kelvins Power station in South Africa, which developed vertical cracks shortly after their commissioning in 1960. The propagation of the cracks was closely monitored and at that time a 50mm thick reinforced gunite skin was applied to the outside of the tower. On inspection in 1983, additional severe cracks were observed, so that it was decide to repair the cooling towers by arranging seven external rings.

The dynamic properties of cooling towers and their responses under dynamic loading have also been researched by several authors (Gould et. al., 1974; Gran and Yang, 1978; Hashish and Abusitta, 1971; Lu et. al.1984; Sen and Gould, 1974; Obanishola M. Sadiq (1993) and many others.

Despite the above mentioned research efforts of scientist, international research activities are still being intensified aimed at improving the methods of analysis and design of these largest reinforced concrete shell structures of the present day. In this paper an efficient method of analysing the stability of cooling tower shells is presented. The method is capable of analysing the stress-strain conditions and non-linear stability of cooling tower shells of hyprbolic, conical, cylindrical or combined surfaces of revolution. The allowable correlations describing the geometric and mechanical characteristics of the towers are formulated using tensor analysis which considerably increase the possibility of reducing the volume of computer program entry informations.

2.0 EQUATIONS OF EQUILIBRIUM

Stability analysis of shells of revolution on the basis of geometric nonlinearity is connected with the necessity of considering the stress-strain components and local basic vectors during the process of deformation.

Differential equations of equilibrium of a shell element in the local orthogonal curvilinear coordinate system, satisfying thin shell hypothesis can be expressed as

$$\frac{\partial \sqrt{a}\,\vec{T}^{\alpha}}{\partial x^{\alpha}} + \sqrt{a}\,\vec{q} = 0 \quad \ldots\ldots \quad 1(a)$$

$$\frac{\partial \sqrt{a}\,\vec{M}^{\alpha}}{\partial x^{\alpha}} + \left[\vec{e}_{\alpha} \cdot \vec{T}^{\alpha}\right]\sqrt{a} = 0 \quad \ldots\ldots \quad 1(b)$$

where a is the fundamental determinant of metric tensor of shell's middle surface which characterizes it's geometry; $\vec{T}^{\alpha}$ and $\vec{M}^{\alpha}$ are contravariant tensor respectively of internal forces and moments; $\vec{e}$ local basis vectors; $\vec{q}$ internal or external load vector.

$$\vec{T}^{\alpha} = T^{\alpha\beta}\vec{e}_{\beta} + T^{\alpha 3}\vec{e}_{3} \quad \ldots\ldots\ldots \quad (2)$$

$$\vec{M}^{\alpha} = c_{\beta\gamma}\, M^{\alpha\beta}\,\vec{e}^{\gamma} \quad \ldots\ldots\ldots \quad (3)$$

$$(\alpha, \beta, \gamma = 1,2)$$

Here T^{11}, T^{22} are membrane forces; $T^{\alpha 3}$ are shear forces ($T^{12} = T^{21}$); M^{11}, M^{22} are bending moments, while M^{12}, M^{21} are twisting moments; $C\beta\gamma$ is the discriminant tensor of curved surfaces. For shells of revolution, $C_{11} = C_{22} = 0$; $C_{12} = \sqrt{a}$; $C_{21} = -\sqrt{a}$

These contravariant tensor components of membrane $T^{\alpha\beta}$ and bending $M^{\alpha\beta}$ forces are expressed through their convariant tensor components of membrane $\varepsilon^{\alpha\beta}$ and bending $\mu^{\alpha\beta}$ deformations, following the laws of theory of elasticity:

$$T^{\alpha\beta} = \frac{Eh}{(1-\nu^2)}\varepsilon_{\gamma\omega}\left[\nu a^{\alpha\beta}\, a^{\gamma\omega} + (1-\nu)a^{\alpha\gamma}\, a^{\beta\omega}\right] \quad (4)$$

$$M^{\alpha\beta} = \frac{Eh^3}{12(1-\nu^2)}\mu_{\gamma\omega}\left[\nu a^{\alpha\beta}\, a^{\gamma\omega} + (1-\nu)a^{\alpha\gamma}\, a^{\beta\omega}\right] \quad (5)$$

where E is Young's modulus of elasticity, ν is Poisson coefficient, h is shell's thickness.

The deformation components $\varepsilon^{\alpha\beta}$ and $\mu^{\alpha\beta}$ are expressed as follows:

$$\varepsilon_{\alpha\beta} = \frac{1}{2}\left[\frac{\partial\vec{u}}{\partial x^{\alpha}}\vec{e}_{\beta} + \frac{\partial\vec{u}}{\partial x^{\beta}}\vec{e}_{\alpha}\right] + \frac{1}{2}\vartheta_{\alpha}\vartheta_{\beta} \quad \ldots\ldots \quad (6)$$

$$\mu_{\alpha\beta} = \frac{1}{2}\left[\frac{1}{c^{\alpha\beta}}\frac{\partial\vec{\pi}}{\partial x^{\beta}}\vec{e}_{\gamma} + \frac{1}{c^{\beta\omega}}\frac{\partial\vec{\pi}}{\partial x^{\alpha}}\vec{e}^{\omega}\right] \quad \ldots\ldots \quad (7)$$

$$(\alpha, \beta, \gamma, \omega = 1,2)$$

Here $\vec{\pi}$ is twisting angle vector, $\vec{u}$ - displacement vector; ε_{11}, ε_{22} are Normal deformations respectively in the direction of $\vec{e}_1$, $\vec{e}_2$ local basis vectors; $\varepsilon_{12} = \varepsilon_{21}$ - angular deformation between e_1 and e_2; μ_{11}, μ_{22} - bending deformations; μ_{12} - characterizes the twisting of the surface, ϑ - twisting angle normal to the shell's middle surface.

Equations 4 - 7 are expressed in a more general form as follows:

$$T^{11} = \frac{Eh}{(1-\nu^2)}\left(a^{11}a^{22}\varepsilon_{11} + \nu a^{11}a^{22}\varepsilon_{22}\right)$$

$$T^{22} = \frac{Eh}{(1-\nu^2)}\left(a^{22}a^{22}\varepsilon_{22} + \nu a^{11}a^{22}\varepsilon_{11}\right)$$

$$T^{12} = T^{21} = \frac{Eh}{(1-\nu^2)}\, a^{22}a^{11}\varepsilon_{12}$$

$$T^{13} = \frac{\partial M^{11}}{\partial x^1} + 2\Gamma^{1}_{12}M^{22} + \frac{\partial M^{12}}{\partial x^2} + \Gamma^{1}_{22}M^{22}$$

$$T^{23} = \frac{\partial M^{12}}{\partial x^1} + \Gamma^{1}_{12}M^{22} + \Gamma^{2}_{11}M^{11} + \frac{\partial M^{22}}{\partial x^2} + 2\Gamma^{2}_{22}M^{22}$$

$$M^{11} = \frac{Eh^3}{12(1-\nu^2)}\left(a^{11}a^{11}\mu_{11} + \nu a^{11}a^{22}\mu_{22}\right) \quad (8)$$

$$M^{22} = \frac{Eh^3}{12(1-\nu^2)}\left(a^{22}a^{22}\mu_{22} + \nu a^{11}a^{22}\mu_{11}\right)$$

$$M^{12} = M^{21} = \frac{Eh^3}{12(1-\nu^2)}(1-\nu)a^{11}a^{22}\mu_{12}$$

$$\varepsilon_{11} = \frac{\partial u_1}{\partial x^1} - u_2 - b_{11}u_3 + \frac{1}{2}(\vartheta_1)^2$$

$$\varepsilon_{22} = \frac{\partial u_2}{\partial x^2} - \Gamma^{2}_{22}u_2 - b_{22}u_3 + \frac{1}{2}(\vartheta_2)^2$$

$$\varepsilon_{12} = \varepsilon_{21} = \frac{1}{2}\left[\frac{\partial u_2}{\partial x^1} + \frac{\partial u_1}{\partial x^2} - u_1\Gamma^{1}_{21} - \Gamma^{1}_{12}\right) + \vartheta_1\vartheta_2\Big] \quad (9)$$

$$\mu_{11} = \frac{\partial\vartheta_1}{\partial x^1} - \vartheta_2\Gamma^{2}_{11}\,; \quad \mu_{22} = \frac{\partial\vartheta_2}{\partial x^2} - \vartheta_1\Gamma^{1}_{22}$$

$$\mu_{12} = -\sqrt{a}\frac{\partial}{\partial x^2}\left(\frac{1}{\sqrt{a}}\vartheta_1\vec{e}_2 - \frac{1}{\sqrt{a}}\vartheta_2\vec{e}_1\right)$$

$$\vartheta_1 = \frac{\partial u}{\partial x^1} - u_1 b^{1}_{1}\,; \quad \vartheta_2 = -\frac{\partial u}{\partial x^2} - u_2 b^{2}_{2}$$

Here, b is the second quadratic forms of curved surfaces, Γ- Christoffel's symbol. For shells of revolution $b_{12} = b_{21} = b_2^1 = b^2{}_1 = 0; \Gamma^1_{11} = \Gamma^2_{21} = \Gamma^3_{31} = \Gamma^3_{32} = 0$; u_1, u_2, u_3 = (u,v,w) - displacement components respectively along the x^1, x^2 and x^3 curvilinear coordinate system.

3.0 STABILITY ANALYSIS TECHNIQUE

In stability analysis of shell structural systems, it is usually assumed that their buckling failure occurs either axisymmetrically or asymmetrically, with the formation of a number of circular waves (harmonic number) in the circumferential direction. Therefore, it is more convenient to express the unknown displacement vector $u(x^1)$ in Fourier series. The technique is as follows:

Assuming an operatorial equation in the form

$$Au = 0 \quad \ldots\ldots \quad \ldots\ldots \quad (10)$$

where A is a matrix operator, Eq. (10) has a system of solutions which can be represented in the form

$$u(x^1) = n = 1\ u_n\ n(x^1) \ldots \quad (11)$$

where u_n are defined by, the orthogonality of Au to $\psi_1(x^1), \psi_2(x^1), \ldots\ldots, \psi_\infty(x^1)$ elements.

i.e. $$\left\{ A \left(\sum_{n=1}^{\infty} u_n \varphi_n \right) \psi_m \right\} = 0 \ldots (12)$$

$$(m = 1, 2, \ldots\ldots \infty)$$

Here $\varphi_n(x^1)$; $\psi_m(x^1)$ are systems of linearly independent coordinate functions.

Stability equation components can be divided into symmetric $(u_2, u_3, q^2, q^3, \varepsilon_{11}, \varepsilon_{22}, \mu_{11}, \mu_{22}, \nu_2, T_1'', T^{22}, T^{23}, M^{11}, M^{22})$ and non-symmetric $(u_1, q^1, \varepsilon_{12}, \mu_{12}, \nu_1, T^{12}, T^{13}, M^{12})$ components relative to the plane $x^1 = 0$. In that case, the symmetric components can be represented in form of Fourier cosine series, while the non-symmetric components in the form of sine series as follows:

$$C(x^1) = \sum_{n=0}^{\infty} {}^n C \cos nx^1 \quad \ldots\ldots \quad \ldots\ldots \quad (13)$$

$$S(x^1) = \sum_{n=1}^{\infty} {}^n S \sin nx^1 \quad \ldots\ldots \quad \ldots\ldots \quad (14)$$

where

$${}^n C = \frac{1}{\pi} \int_0^{2\pi} \cos nx^1 dx^1 \quad \ldots\ldots \quad \ldots\ldots \quad (15)$$

$${}^n S = \frac{1}{\pi} \int_0^{\pi} \sin nx^1 dx^1 \quad \ldots\ldots \quad \ldots\ldots \quad (16)$$

Now substituting eqs. (15) and (16) respectively into eqs. (13)and (14), and then multiplying the symmetric components by $\frac{1}{\pi}\cos nx^1$ and the nonsymmetric components by $\frac{1}{\pi}\sin nx^1$ and intergrating them accordingly, we obtain a system of nonlinear algebraic equations of equilibrium, internal forces, moments and deformation of the shell of revolution. The solutions of the system of nonlinear algebraic equations are built using the method of continuation of solutions by loading parameter in combination with Newton's iterative procedures (Ortega J.M.et. al., 1970). Thus a system of linearized equations is obtained. In the process of building the solution curves in the space of states, critical points (limit and bifurcation) which characterize loss of stability are defined. Criterion for loss of stability of the shell is the change in sign of the determinant of the matrix of the system of linearized equations.

4.0 EXAMPLES AND DISCUSSIONS

The stability analysis of a Reinforced concrete hyperbolic cooling tower shell (Fig.1) is taken as a demonstration problem. The equations of the middle curved surface of the hyperbola are taken in parametric forms as:

$$X = \left(\frac{a}{b}\sqrt{(x^2 - Z_b)^2 + b^2 + c}\right) \cos x^1 \quad \ldots\ldots \quad (17a)$$

$$Y = \left(\frac{a}{b}\sqrt{(x^2 - Z_b)^2 + b^2 + c}\right) \sin x^1 \quad \ldots\ldots \quad (17b)$$

$$Z = x^2 \quad \ldots\ldots \quad (17c)$$

when the X, Y, Z. are the cartesian coordinate system; x^1, x^2 are the curvilinear coordinate system orientated respectively along the circumferential direction and the hyperbola; a = 32m; b = 57.23m; c = 8m; Z_b = 40m.

The tower has the following geometrical characteristics: Base diameter, 2Ra = 120m; Height, H = 150m, throat diameter, $2R_T$ = 64m. The thickness of the shell is the same throughout and is given as h = 20cm. In some hyperbolic cooling towers, the axis of the hyperbola may not coincide with the axis of the tower. So is the case here. The aixs of the hyperbola stands 8m from the central axis of the tower. This is shown in Fig. 1 as C = 8m. The mechanical properties of the tower are as follows:

Young's modulus, E = 3.10^7 kN/m^2; Position coefficient,

ν = 0.166; material density, γ = 2500kg/m^3.

4.1 BOUNDARY CONDITIONS

For this hyperbolic cooling tower, the following boundary conditions must be satisfied:

a) At the free edge: $T^{22} = T^{23} = M\,22 = 0$

$$M^{11} = 0;\ M^{22} = 0$$

b) At the fixed edge: $u_1 = u_2 = u_3 = 0$;

$$\nu_2 = \nu_1;\ \frac{\partial u_3}{\partial x^1} = 0$$

4.2 RESPONSE OF THE TOWER UNDER EVENLY DISTRIBUTED EXTERNAL PRESSURE

Fig.2 shows the pressure-displacement curves of the tower under evenly distributed external pressure. We show the behaviour of the curves at three distinct points (A, B, C) on the hyperbola. Denoted in brackets on the curves are the harmonic number (n) at loss of stability. The limit critical point is denoted as (O) on the curves. At this point the critical pressure was found to be q^*_o = 665.369kN/m^2. Bifurcation analysis however showed that the tower lost stability asymmetrically with the formation of seven circular waves along the circumferential coordinate. This is denoted on the curves as (7).

At this point the critical pressure was found to be $q^*_{(7)}$ = 24.95kN/m^2.

Fig. 3 shows the diagrams of the membrane forces T^{11}, T^{22} and bending moments M^{11}, M^{22}.

4.3 STABILITY ANALYSIS OF THE TOWER UNDER SELF-WEIGHT

The self-weight of a shell element can be expressed in the form:

$$\vec{P} = \sqrt{a}h\,\gamma\vec{g} \qquad \ldots\ldots \quad (18)$$

where g is the acceleration due to gravity vector. Projecting eq. (18) on the local basis vectors (e_1, e_2, e_3) of an arbitrary point on the middle surface of the hyperbola, we obtain:

$$P_1 = 0;\ P_2 = P\sin\alpha\ ;\ P_3 = -P\cos\alpha \qquad \ldots\ (19)$$

where α is the angle between the tangent to the hyperbola at the arbitrary point and the OX - axis. Indeed, α = arctan $\frac{dz}{dx}$. Then eq. (19) can be rewritten in the form:

$$P_1 = O$$

$$P2 = \frac{\sqrt{a}h\,\gamma g\ \frac{dz}{dx}}{\sqrt{1 + \left(\frac{dz}{dx}\right)^2}} \qquad \ldots\ldots \quad (20)$$

$$P3 = \frac{-\ ah\,\gamma g}{\sqrt{1+\left(\frac{dz}{dx}\right)^2}}$$

Now the relationship between the membrane forces

T^{22} and the self-weight, P, of a hyperbolic shell element can be expressed as (Sadiq, 1990):

$$2\pi R_T T^{22} = \lambda P \quad \ldots \quad \ldots \quad (21)$$

where R_T is the throat radius and λ is the self-weight loading parameter.

Table 1 shows the computed values of T^{22} with respect to various corresponding values of λ. Fig. 4 shows the graph of T^{22} against λ. When $\lambda = 1$, $T^{22} = 214.48$kN/m and the self-weight of the tower was found to be 43125.17kN.

Stability analysis shows that the tower lost stability when n = 8. At this point $P^*_{(8)}$ was found to be 361911.47kN. This implies that the tower will fail under a dead load about eight times it's own self weight. Fig. 5 shows the diagrams of the membrane forces and the bending moments under self weight.

5. CONCLUSION

A new method of analysing the stability of cooling tower shells of any geometric curved surface of revolution, taking geometric non-linearity into account is presented. The method is then used to study the response of a real life reinforced concrete hyperbolic cooling tower of height 150m, base diameter of 120m and shell thickness of 20cm. Our analysis showed that under evenly distributed external pressure, the tower lost stability asymmetrically with the formation of seven circular waves along the circumferential direction.

We also discovered that the tower will fail if subjected to a dead load of about eight times it's own self weight.

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Table 1: Hyperbolic Cooling Towers Shell under Self-weight.

Height - 150m; Base Diameter - 120m; Thickness - 20cm.

λ	T^{22} kN/m
0.4	85.8
1.0	214.48
1.56	334.59
2.16	463-28
2.76	591-97
3.16	677.76
3.76	806.44
4.16	892.24
4.76	1020.92
5.16	1106.72
5.76	1235.40
6.76	1449.88
7.16	1535.67
7.76	1664.36
8.16	1750.15
8.76	1878.84
9.16	1964.63

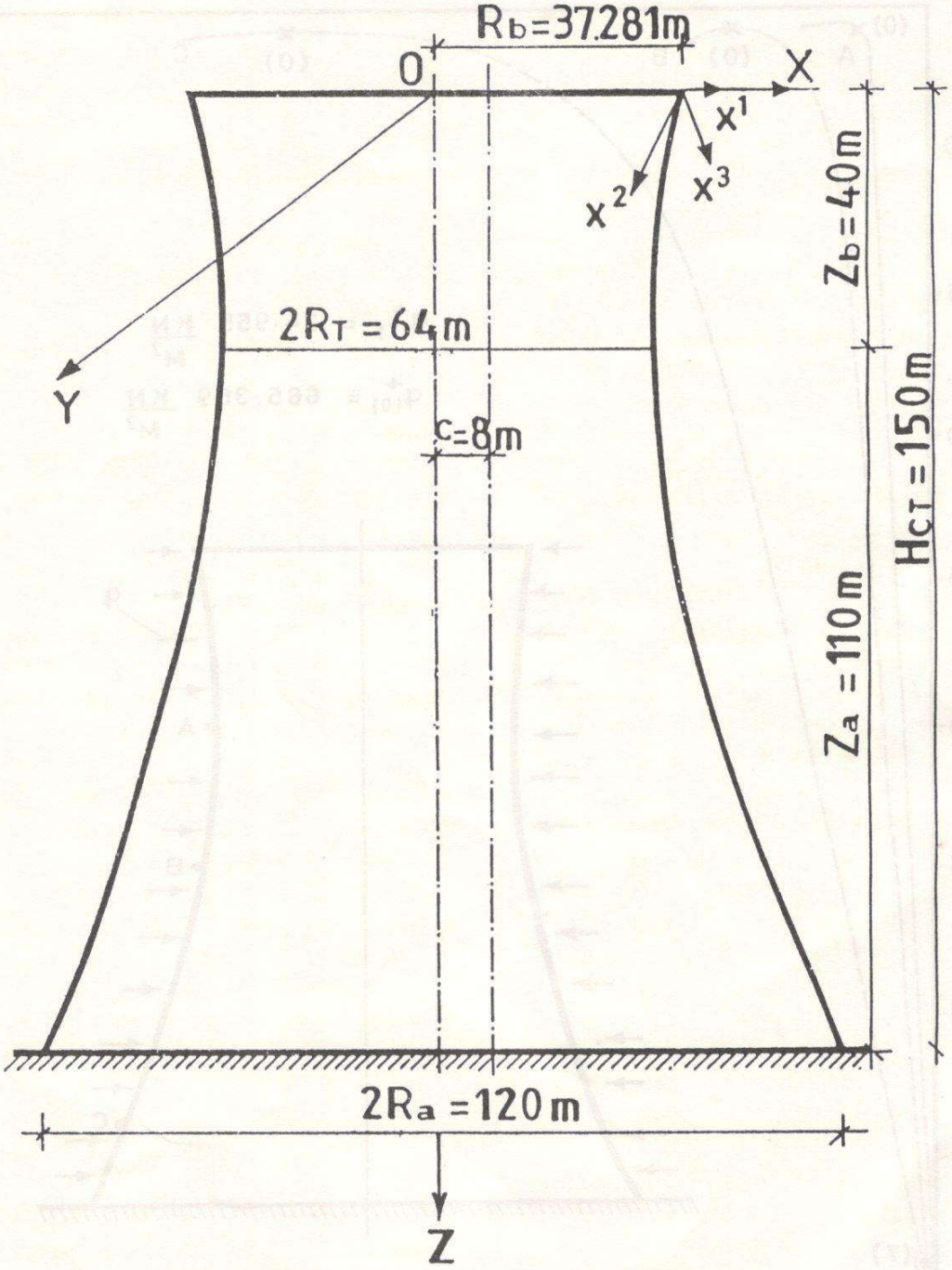


Fig. 1 Hyperbolic Reinforced Concrete Cooling Tower.

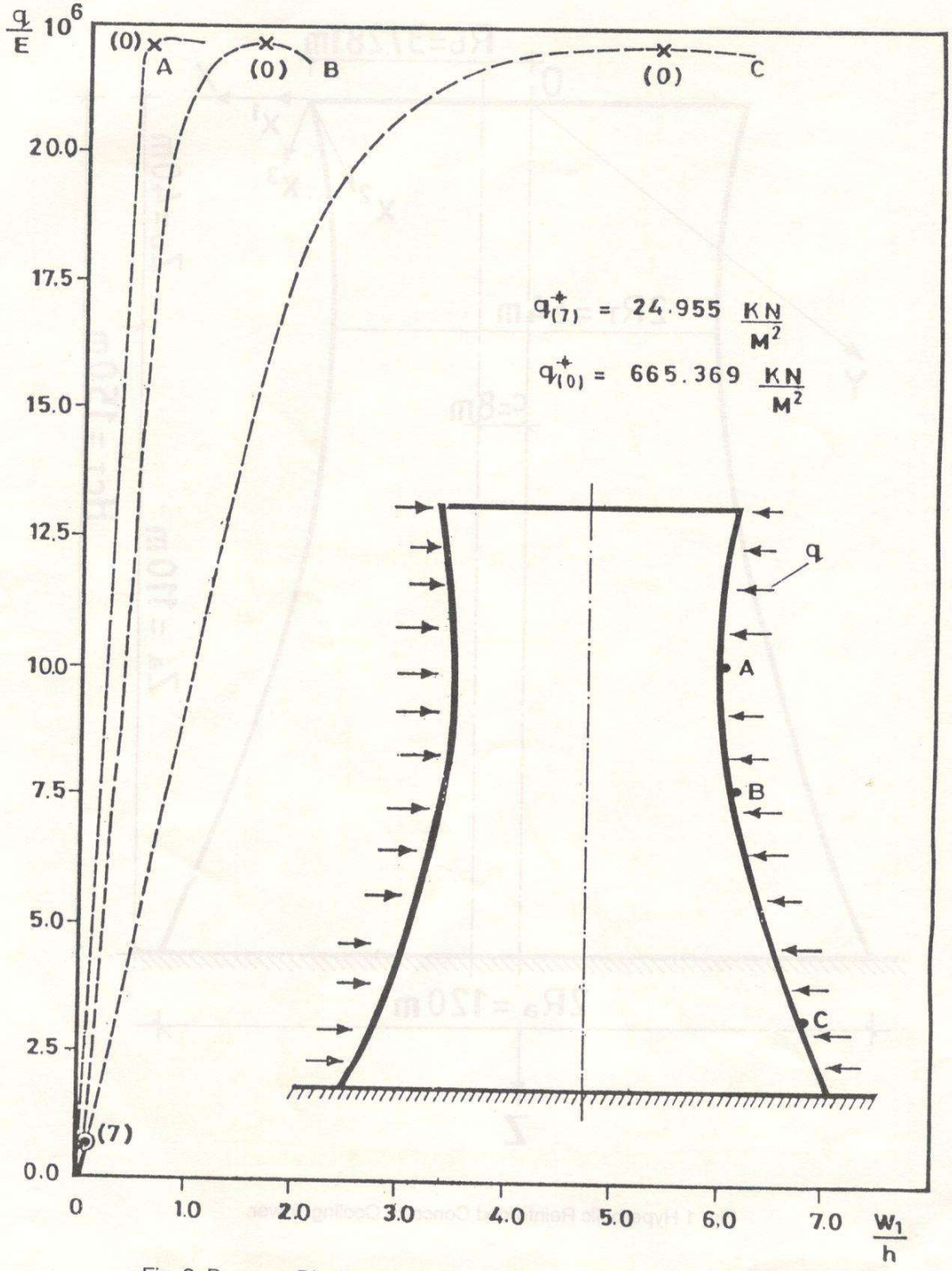


Fig. 2: Pressure-Displacement Curves of the Cooling Tower Under Evenly Distributed External Pressure.

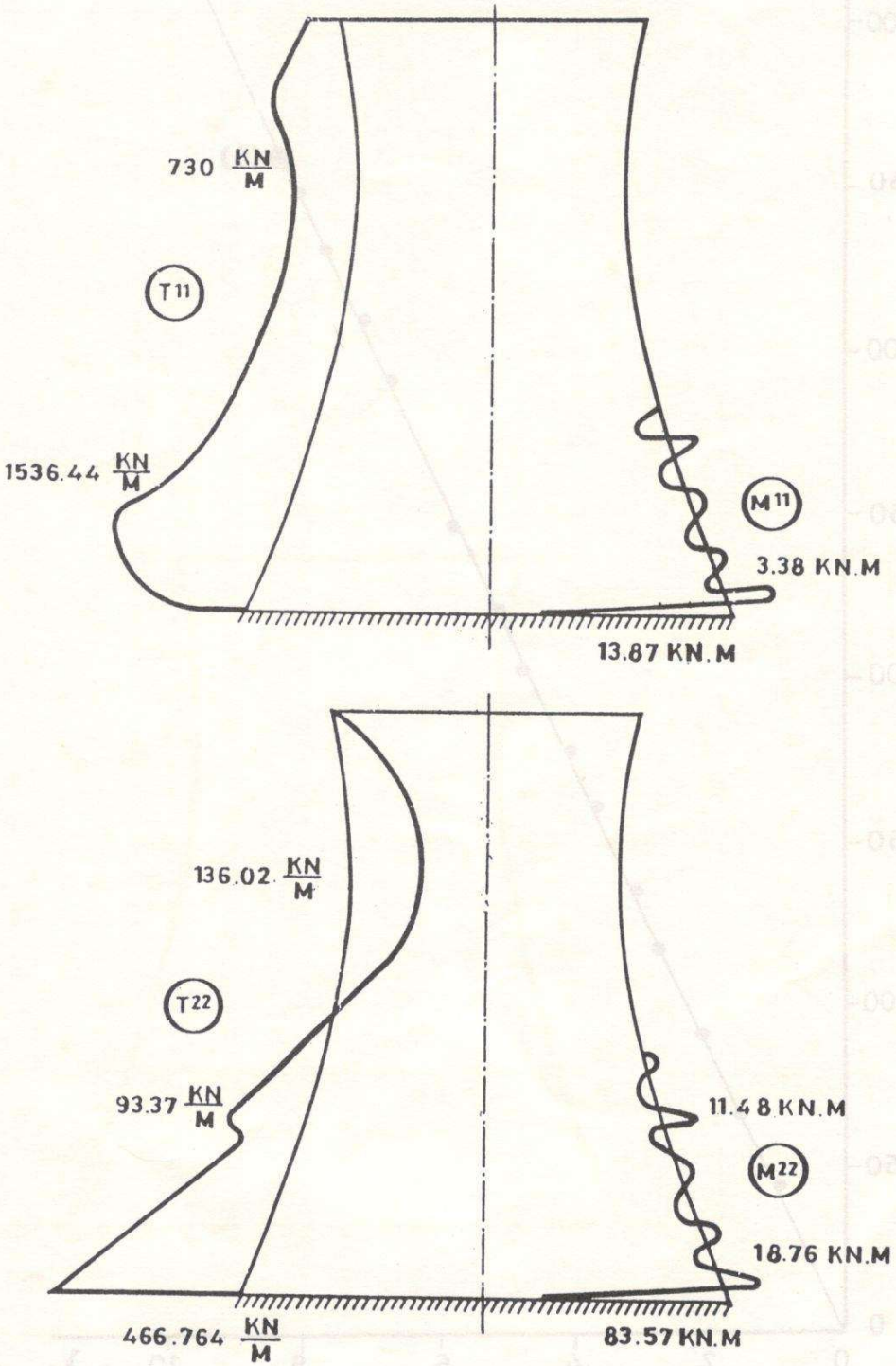


Fig. 3: Membrane Forces and Bending Moments Diagrams of the Tower under evenly distributed external pressure.

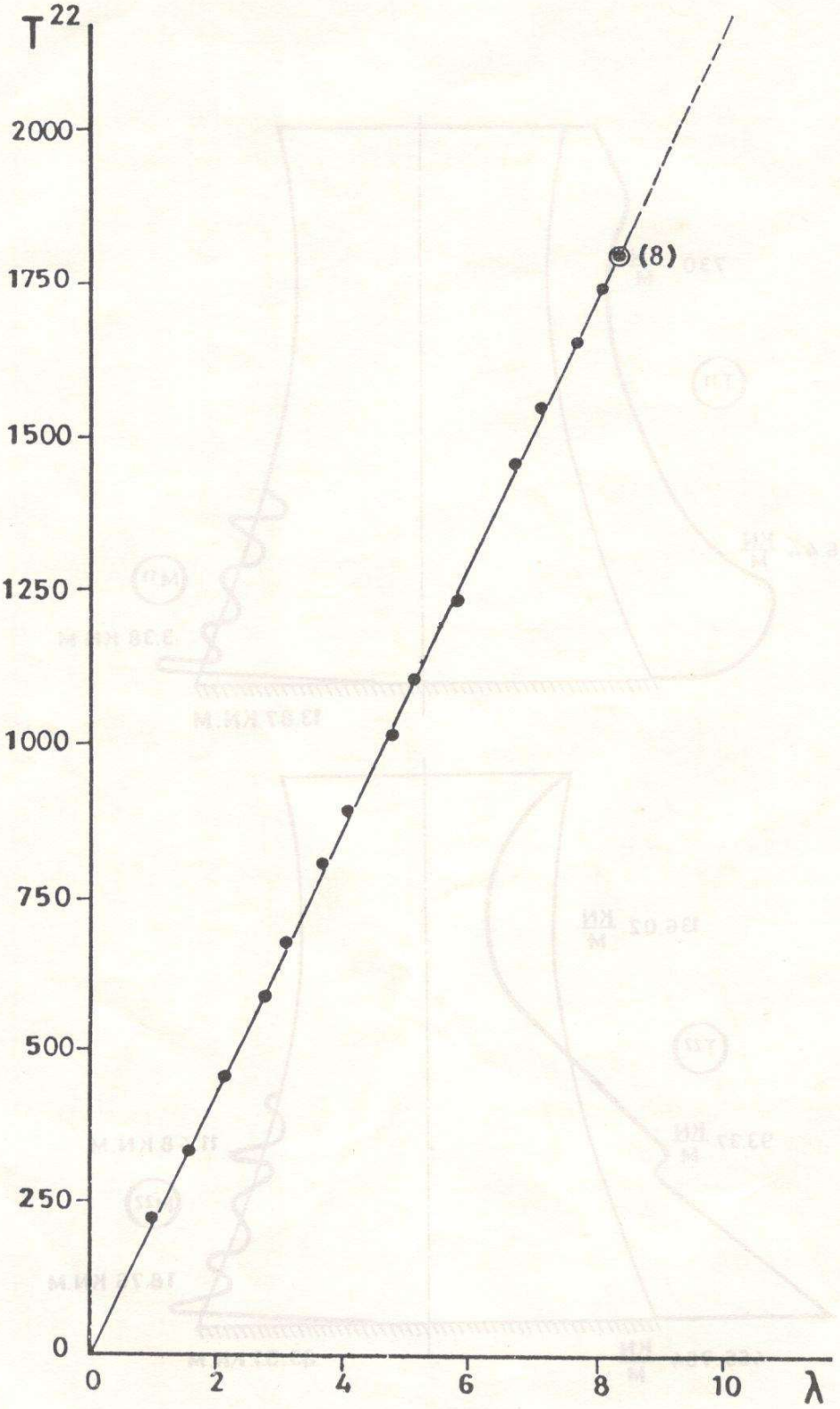


Fig. 4: T^{22} Vs λgraphy of the Tower under Self-weight.

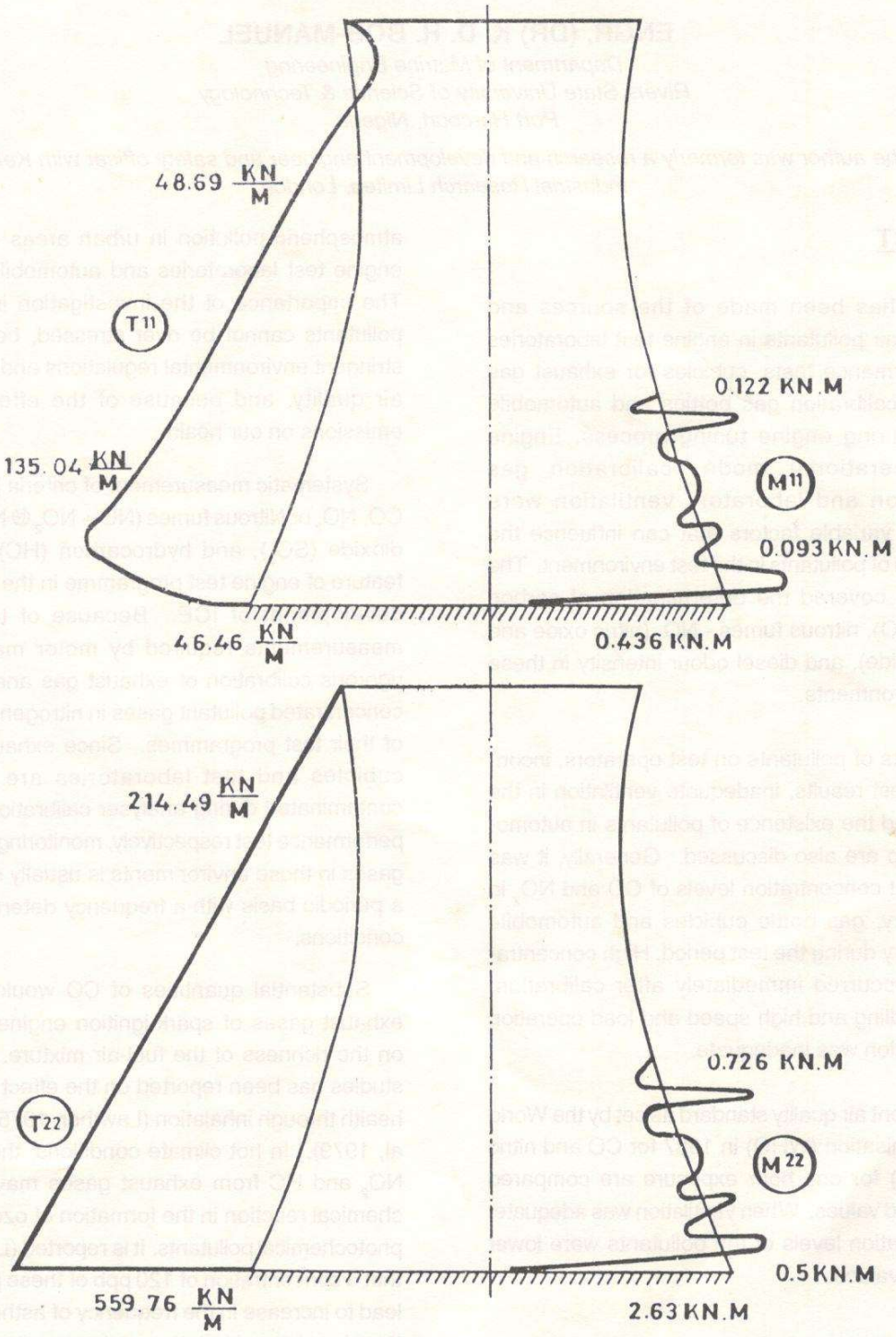


Fig. 5 Membrane Forces and Bending Moments Diagrams of the Tower under Self-weight.