MODEL REFERENCE ADAPTIVE CONTROL OF A SERVO SYSTEM

Onogu, M. I. and Omogbemeh, C. I.
School of Engineering and Engineering Technology
Abubakar Tafawa Balewa University, Bauchi.

Abstract

Spacecraft servo system optimisation is adversely affected by uncertainties in operating parameters, wears and tears. The paper presents an algorithm using model reference adaptive control (MRAC) for elegant and robust control of a servo under stochastic disturbances. The adaptive law developed using discrete time system with direct parameter adaptive algorithm (PAA), is based on minimisation of the error signal between the response of the actual system and a reference model.
Key words: Adaptive control, model reference servo system.

1 Introduction

The control system design with adaptive viewpoint became a challenge in the 1950s on the need for high performance air craft with varying flight conditions. Industrial process optimisation under slowly varying operating parameters, wears and tears further made adaptive control a necessity. The control systems design have some or all of the coefficients automatically adjusted in response to on-line measurements of process variables. It is the intent of this paper to present an adaptive control algorithm using the model refernce approach and to investigate a microprocessor implementation for servo system.

An earlier recorded work on MRAC was in 1958 when the analogue hardware was suggested as a solution to aerospace problems (1). However, the advancements in digital technology has led to tremendous progress in adaptive control techniques. Hwang(2) proposed MRAC without any assumption on unstable plant zeros. The published works (2 - 6) on MRAC confirm the potential viability of adaptive algorithm over linear controllers for both non-linear and linear stochastic system. The algorithm presented here are in two phases viz:

i On-line identification of the servo system
ii Minimising identified model output error from refernce model output while simultaneously adjusting the controller parameters.

2.0 SYSTEM IDENTIFICATION

A linear discrete time dynamic system with single input single output (SISO) employed is characterised by the relations;

$$Ay_t = z^{-1}B_t \quad \text{.....1}$$

where y_t and u_t are the discrete system output and input respectively at sample instant t. Polynomials A and B are in the backward shift operator z^{-1} and defined by

$$A = 1 + a_1z^{-1} + a_2z^{-2} + \ldots + a_{na}z^{-na}$$

$$B = b_0 + b_1z^{-1} + b_2z^{-2} + \ldots + b_{nb}z^{-nb} \quad \text{.....2}$$

where na and nb are the order of the system assigned a priori as part of the assumption of the dynamic structure to be identified.

Equation 1 can be rewritten as

$$y_t = \varphi^T_t \Theta + e_t$$

where Θ is the vector of unknown parameters defined by

$$\Theta^T = (-a_1, -a_2, \ldots, -a_{na}, b_o, b_1, \ldots b_{nb})$$

and φ^T is a regression vector consisting of measured

output and input variable defined as

$$\varphi^T_t = \{ y_{t-1}, y_{t-2}, \ldots y_{t-na}; u_{t-1}, u_{t-2}, \ldots u_{t-nb-1} \}$$

e_t is the aggregate of random distrubances corrupting the measurable output

The identification of system model given by Eqn. 3 is performed using recursive least square (RLS) estimation and performed using recursive least square (RLS) estimation and shown in Fig. 4 while the identified parameters are shown in Fig.2 The model based on past information is used to obtain an estimate y of the current output . The algorithm of RLS is given in Appendix.

3 Design of MRAC Parameters

The desirable properties of an adaptive controller algorithm are that it should stabilise any system in the sense of yielding bounded deviation of the output from the set point and should converge eventually to a control correspond ing to the underlying design method. The MRAC structure is shown in Fig. 3.

The general linear regulator is of the form

$$T_{Wt} = Gy_t + Fu_t \qquad \text{.....4}$$

where T, F and G are polynomials defined by

$$T = t_o = t_1 z^{-1} + t_2 z^{-2} + \ldots + t_{nt} z^{-nt}$$
$$F = f_o + f_1 z^{-1} + f_2 z^{-2} + \ldots + f_{nf} z^{-nf}$$
$$G = g_o + g_1 z^{-1} + g_2 z^{-2} + \ldots + g_{ng} z^{-ng}$$

The servo system model considered in the design is

$$G(z) = \frac{Y(z)}{U(z)} \qquad \text{.....5}$$

$$= z^{-k} \frac{b_o B}{A}$$

while the reference model is

$$G(z)_m = \frac{X(z)}{W(z)}$$

$$= z^{-k} \frac{b_m B}{A_m} \qquad \text{.....6}$$

k is the discrete dead time of the system. The closed loop output is

$$y_t = \frac{z^{-k} TB}{AF + z^{-k} BG} \qquad \text{.....7}$$

where T/F is a precompensator and G/F is a feed back element.

The condition for perfect model is satisfied in the equation:

$$\frac{A_m}{b_m} = \frac{1}{b_o}[AF = z^{-k}BG] \quad \text{.....8}$$

The vector Eqn. 3 is modified for RLS controller design as

$$\theta^T = [f_o f_1 \ldots f_{nf}; g_o g_1 \ldots g_{ng}] \quad \text{.....9}$$

$$\varphi = [u_t u_{t-1} \ldots u_{t-nf}; y_t y_{t-1} \ldots y_{t-ng}]$$

to generate

$$W_t = \varphi^T_t \theta_{t-1}$$

It is shown in Onogbemeh [18] that for the convergence of the error equation

$$\lim_{t \to \infty} e_t = \lim_{t \to \infty} (X_t - y_t) \quad \text{.....10}$$

$$= 0$$

The time varying equations to be satisfied are

$$A_m e_t = b_m z^{-k} B(w_t - \varphi^T \theta)$$

$$U_t = \frac{1}{f_o}[w_t - f_1 u_{t-1} - f_2 u_{t-2} \ldots - f_{nf} u_{t-nf}; -g_o y_t - g_1 y_{t-1} - g_{ng} y_{t-ng}]$$

$$A_m e_t = b_m z^{-k} B \varphi^T_t [\hat{e}_{t-1} - \hat{e}_t]$$

The on-line controller parameter vector θ is updated as

(new estimate) = (old estimate) + Gain x (Prediction error)

θ_t θ_{t-1} e_t

The flow chart diagrams for identification and MRAC algorithm are shown in Fig. 4 and Fig 5 respectively.

4.1 Simulation

The servo system considered has the difference equation

$$(1 + 0.5z^{-1} + 0.7z^{-2})y_t = (z^{-1} + 0.2z^{-2})u_t$$

The estimated controller parameters are shown in Fig. 6 and Fig. 7 is the square-wave response.

The servo system has found significant applications in aerospace and proc ess industry. However their operations are affected by frictions, wear, tear, ageing, ambient temperation and other environmental conditions. It is conceived that adap tive control should provide a robust control algorithm. A hypothetical arrangement is shown in Fig 8. The feature of the microprocessor is:

Processor	Intel	80386
Base memory		640KB
External memory		3072KB
Serial ports		3F8
Parallel ports		378

5.0 Conclusion

The results obtained have shown that satisfactory model following is achieved with MRAC algorithm. It has the potential to solve the stochastic distur bances of precision drives in aerospace and industrial process. Friction, wears and tears which are major shortcomings of DC servo are effectively compensated by online controller law synthesis. The rapid and revolutionary progress in microelec tronics and computers have contributed immensely to the practical feasibility of MRAC.

6 Acknowledgement

The authors wish to express their gratitude to the Abubakar tafawa Balewa University for sponsoring the research work.

7 References

1 Jacobs O. L. R. (1981) "Introduction to Adaptive Control" In: Self-Turning and Adaptive Control: Theory and Applications (2nd Edition) -IEE Control Engineering series: 15, Peter Peregrinus Ltd. London. pp. 1-35.

2 Hwang C. and Chen B. (1989) "Model Reference Adaptive Control via the Minimisation of Output Error and Weighting Control Input" In: IEE Pro-ceedings Vol. 136. Pt. D. Do. pp 231 - 237.

3 Pajunen G. A. (1987) "Comparison of Linear and Nonlinear Adaptive Control of a pH-Process" In: IEEE Control systems Magazine Vol. 7 No. 1 pp. 39 - 44.

4 Krause J. M. and Khargonekar P. P. (1987) "Parameter Information Con tent of Measurable signals in Direct Adaptive Control" In: IEEE Transaction on Automatic Control, Vol. AC 32, No. 9, pp 802-810.s

5 Edge K. A. and Figueredo K. R. A. (1987) "An Adaptively controlled Electrohydraulic Servo-mechanism part 1: Adaptive Controler Design" In: Proceedings of the Institution of Mechanical Engineer Part B - Management and Engineering Manufacture Vol. 201 No. B3, pp 175-180

6 Canudas C., Astrom, K. J. and Braun K. (1987) "Adaptive friction Compensa-tion in DC Motor Drives" In: IEEE Journal of Robotics and Automation Vol. RA-3, No 6, pp 681 - 685.

7 Wellstead P. E. and Zarrop M. B. (1991) "Self-Tuning Systems Control and signal processing, Wiley and Sons Ltd.

8 Parks P. C. (1981) "Stability and Convergence of Adaptive Controllers - continuous systems" In: self-Tuning and Adaptive Control: Theory and Applications (2nd Edition) - ELL Control Engineers series: 15, Peter Peregrinus Ltd. pp. 93 -100.

Edge K. A. and Figueredo K. R. A. (1987) "Adaptively Controlled Electrohydraulic servo-mechanism Part 2: Implementation" In: Proceedings of the Institution of Mechanical Engineer Part B-Management and Engineering Manufacture vol. 201 No. pp 181-189.

10 Astrom K. J. and Wittenmark B. (1991) "Adaptive Control" Addsin - Wesley Publishing Co. U. K.

11 Petros A. I. and Aniruddha D. (1991) "Robust Adaptive Control: A Unified Approach" In: Proceedings of the IEEE, Vol. 79. No. 12, pp. 1736-1767.s

12 Miyamoto S. ; Shin S. ; Okubo S. and Kitamori T. (1987)"Continuous-Time Model Reference Adaptive Control Based on Weighting Functions" In: IEEE Transaction on Automatic Control, Vol. AC-32 No. 5 pp. 442-444.

13 Janecki D (1987) "Direct Adaptive Pole Placement for Plants Having Purely Deterministic Disturbances" In IEEE Transactions on Automatic Control, Vol. AC-32 No. 2, pp. 187-189.

Appendix

Recursive Least Square Algorithm (RLS)

The loss function is defined as

$$J = \sum_{t=0}^{t} e_t^2$$

$$= \mathcal{E}_t^T \mathcal{E}_t$$

$$= (Y_t - X_t\Theta)^T (Y_t - X_t\Theta) \qquad \text{..... A1}$$

where $y_t = \begin{bmatrix} y_1 \\ y_2 \\ . \\ . \\ . \\ y_t \end{bmatrix}$; $X_t = \begin{bmatrix} X_{1-k} \\ X_{2-k} \\ . \\ . \\ . \\ X_{t-k} \end{bmatrix}$; $\mathcal{E}_t \begin{bmatrix} e1 \\ e2 \\ . \\ . \\ . \\ e_t \end{bmatrix}$

The objective function $\frac{\partial J}{\partial \theta} = o$ implies

$$-2(Y^{\prime}_{t}X_{t} - \theta^{\prime}X^{\prime}_{t}X_{t}) = o \qquad \text{.....A. 2}$$

and the estimate of θ is

$$\hat{\theta} = (X^{\prime}_{t}X_{t})^{-1}X^{\prime}_{t}Y_{t} \qquad \text{.....A. 3}$$

The information matrix otherwise known as covariance matrix is given by

$$P^{-1}_{t} = X^{\prime}_{t}X_{t}$$

$$= \begin{bmatrix} X_{t-1} \\ \cdots \\ x^{\prime}_{t-k} \end{bmatrix}^{\prime} \begin{bmatrix} X_{t-k} \\ \cdots \\ x^{\prime}_{t-k} \end{bmatrix}$$

$$= X^{\prime}_{t-1}X_{t-1} + X_{t-k}X^{\prime}_{t-k} \qquad \text{.....A. 4}$$

$$= P^{-1}_{t-1} + x_{t-k}x^{\prime}_{t-k}$$

Equation A. 4 is inverted by matrix inversion lemma and becomes

$$P_{t} = [P^{-1}_{t-1} + x_{t-k}x_{t-k}]^{-1} \qquad \text{... A. 5}$$

$$= (I - K_{t}x^{\prime}_{t-k})\, p_{t-1}$$

where the Kalman gain K_t is given by

$$K_{t} = \frac{p_{t-1}x_{t-k}}{1 + x^{\prime}_{t-k}p_{t-1}x_{t-k}}$$

if

$$X_{t}Y_{t} = \begin{bmatrix} X_{t-l} \\ \cdots \\ X^{l}_{t-k} \end{bmatrix}^{\prime} \begin{bmatrix} Y_{t-l} \\ \cdots \\ y^{l}_{t} \end{bmatrix}$$

$$= X^{l}_{t-1}Y_{t-l} + x_{t-k}Y_{t}$$

then

$$\hat{\theta}_t = \hat{\theta}_{t-1} + K_t Y_t - K_t x^{l}_{t-k} \hat{\theta}_{t-1}$$
$$= \hat{\theta}_{t-l} + K_t (Y_t - x^{l}_{t-k} \theta_{t-l}) \quad \text{.....A. 6}$$

The recursive least square algorithm is defined by equation A. 6. It is necessary that the initial covariance matrix p_0 is defined as aI where $0 < a < 1$ for a good initial estimate $\hat{\theta}_o$

Otherwise α is chosen as $100 < a < 10{,}000$.

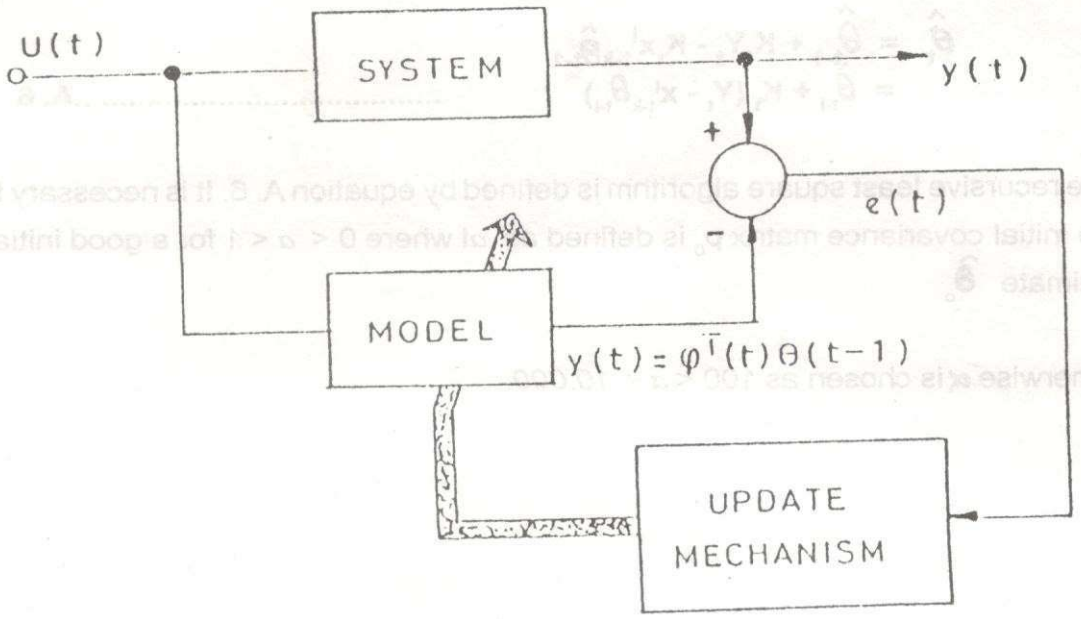
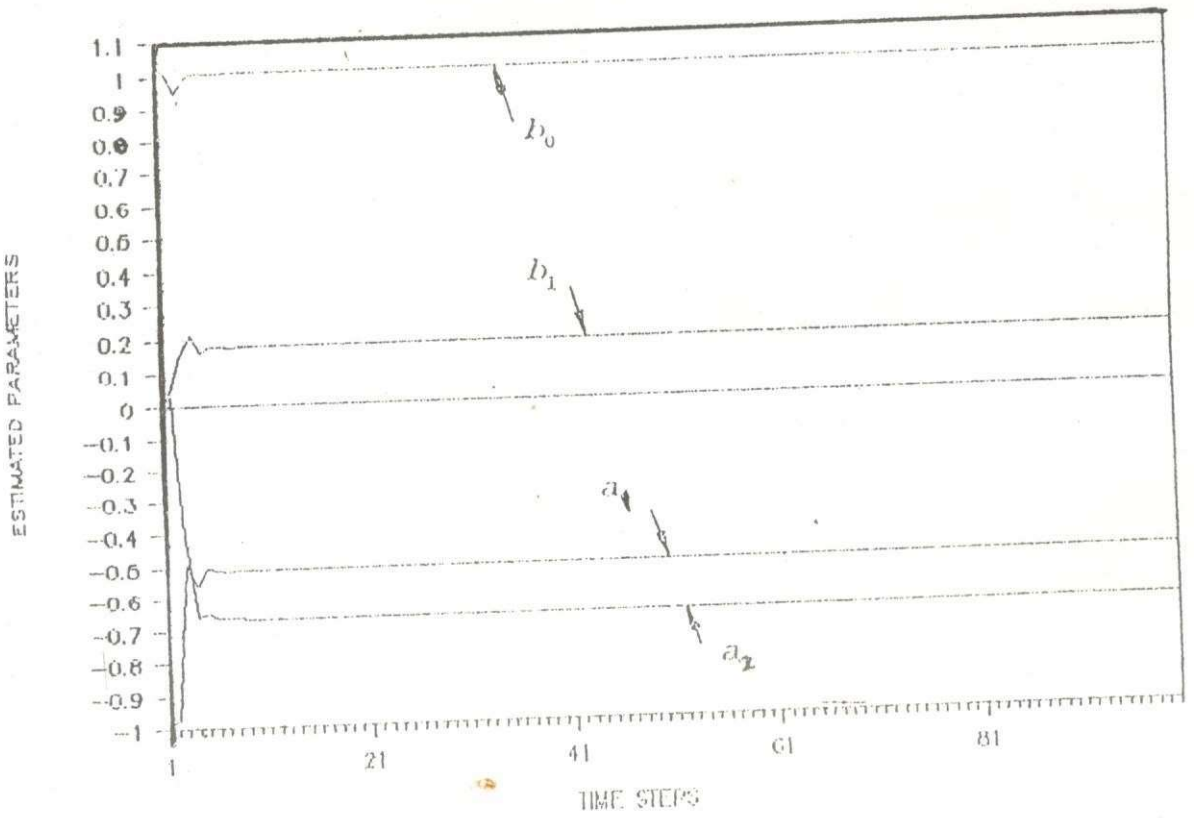


Fig. 1 On-line system modelling



'ig. 2 Estimated system parameters with $P_o = 100I$

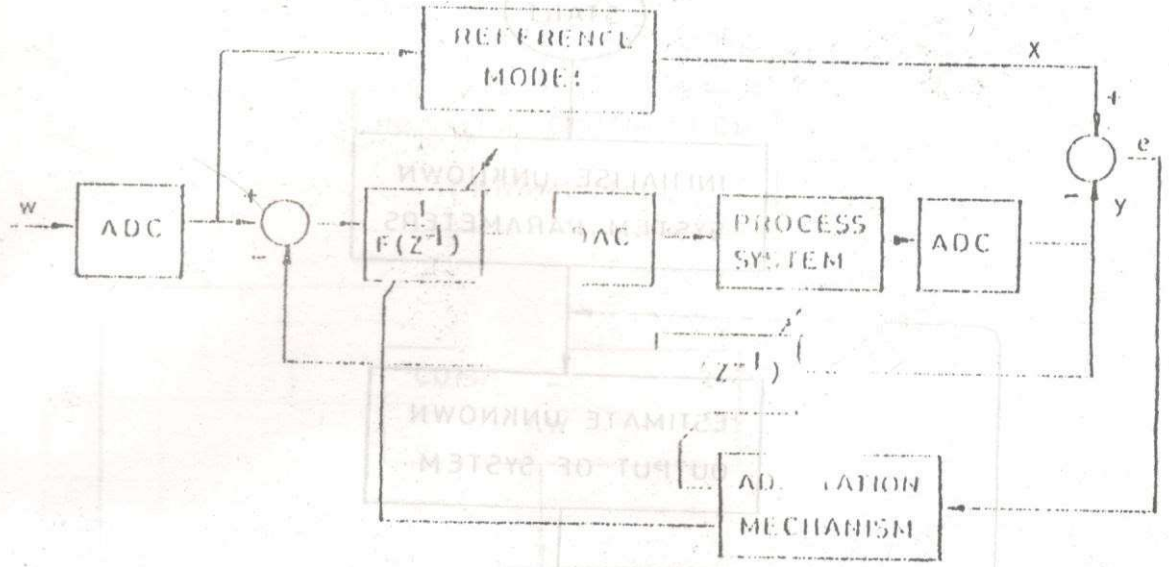


Fig. 3a Parallel model reference adaptive controller.

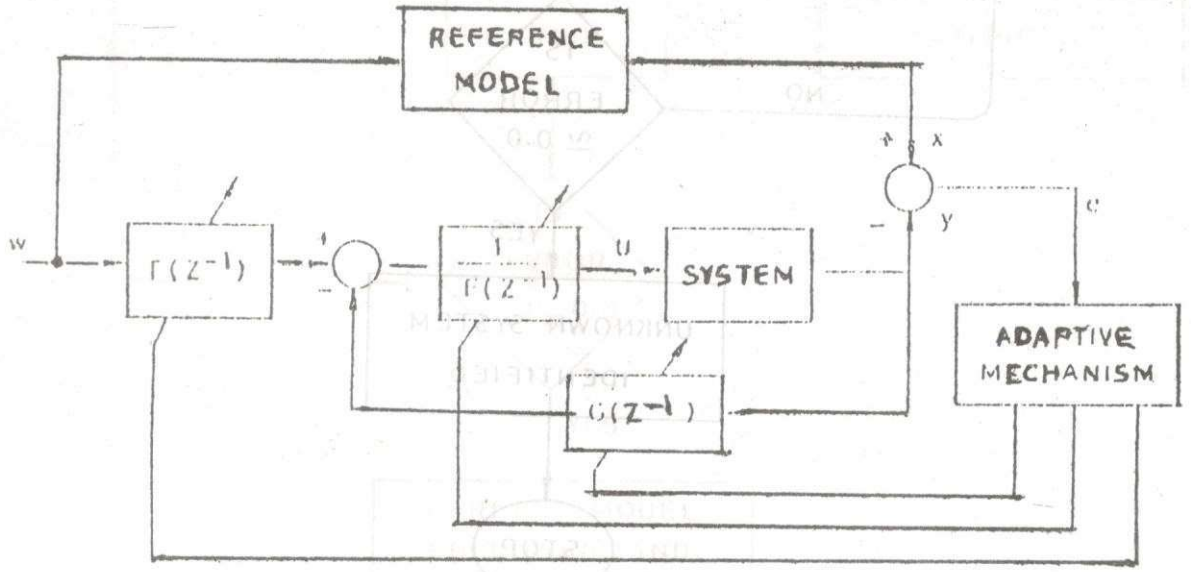


Fig. 3b Generalized parallel model reference adaptive Controller.

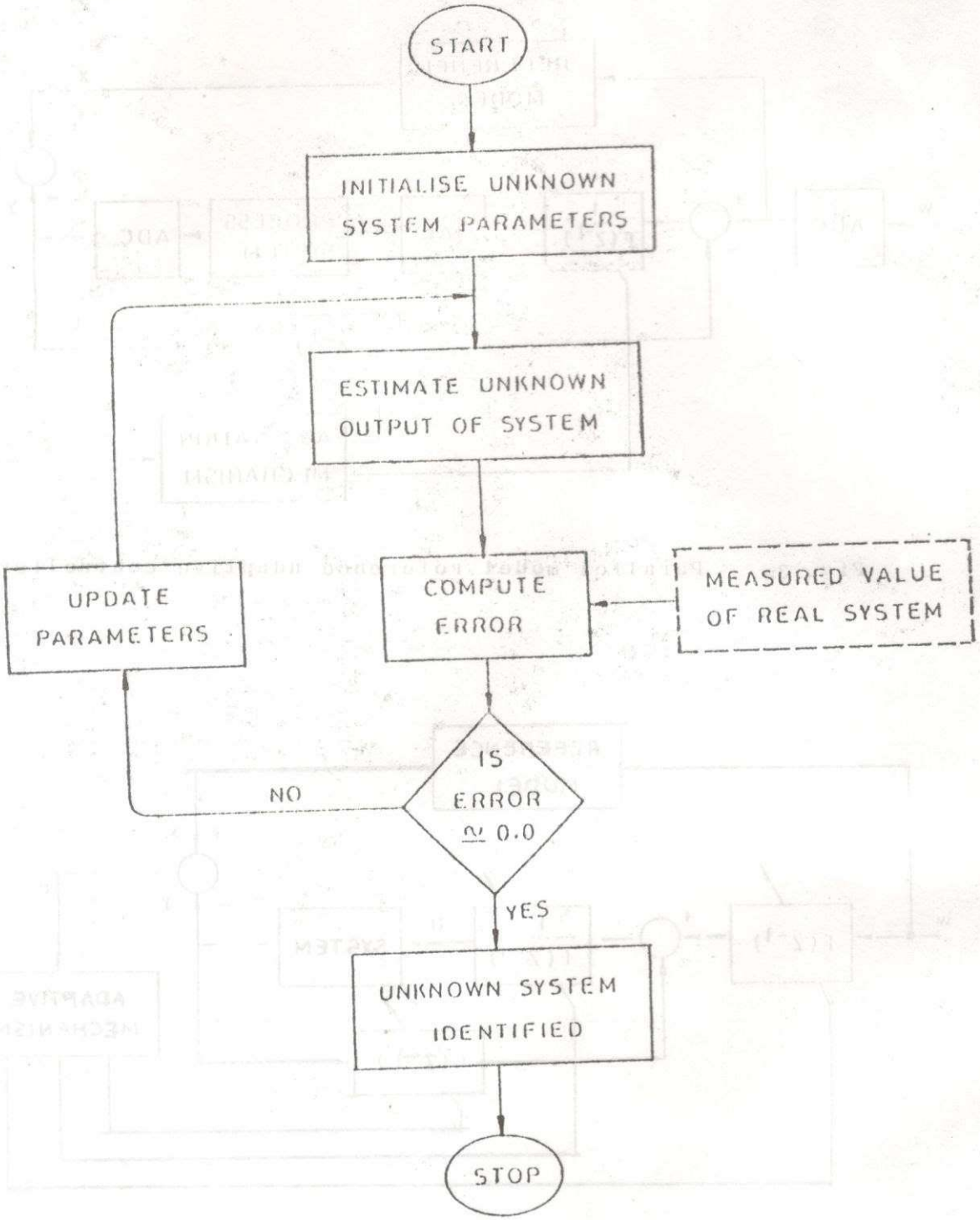


Fig. 4 Flow diagram for the computer implementation of system identification.

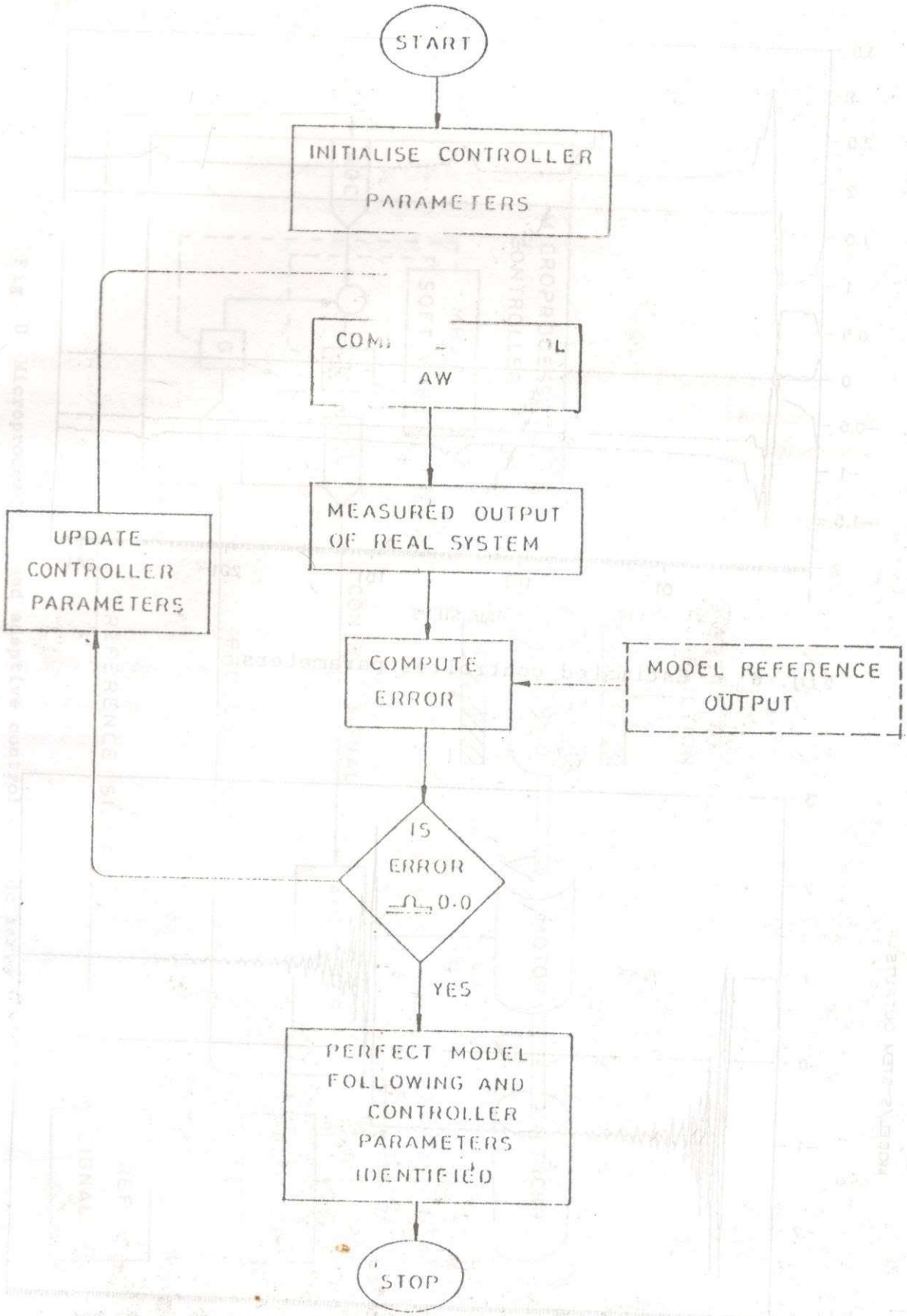


Fig. 5 Flow diagram for the computer implementation of MRAC algorithm.

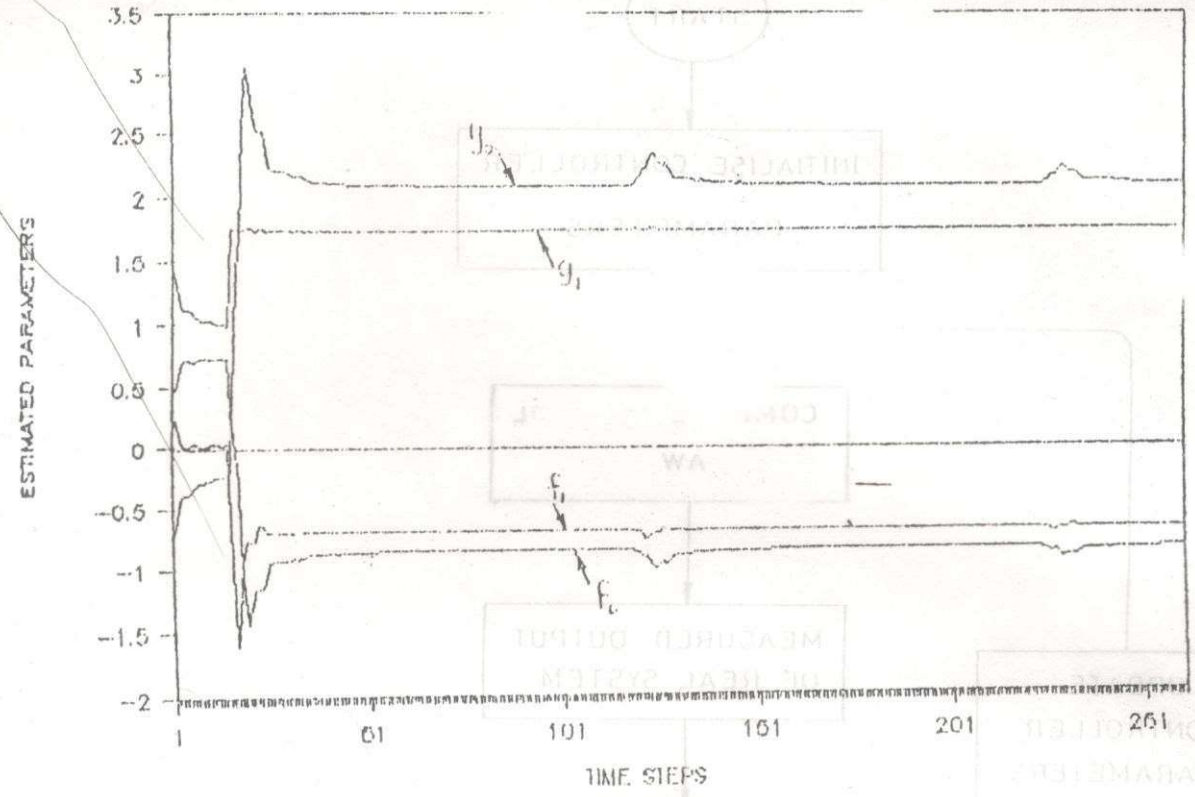


Fig. 6 Estimated controller parameters

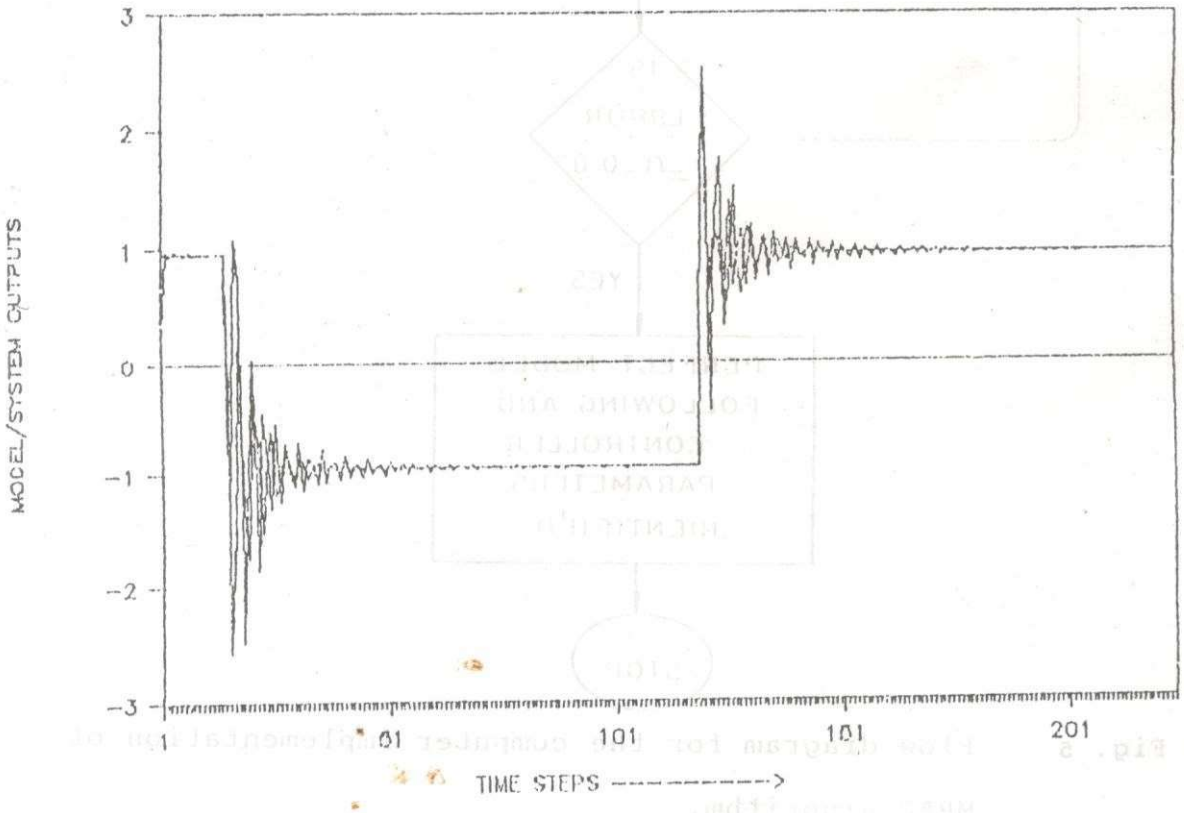


Fig. 7 Square wave response of the adaptive servo

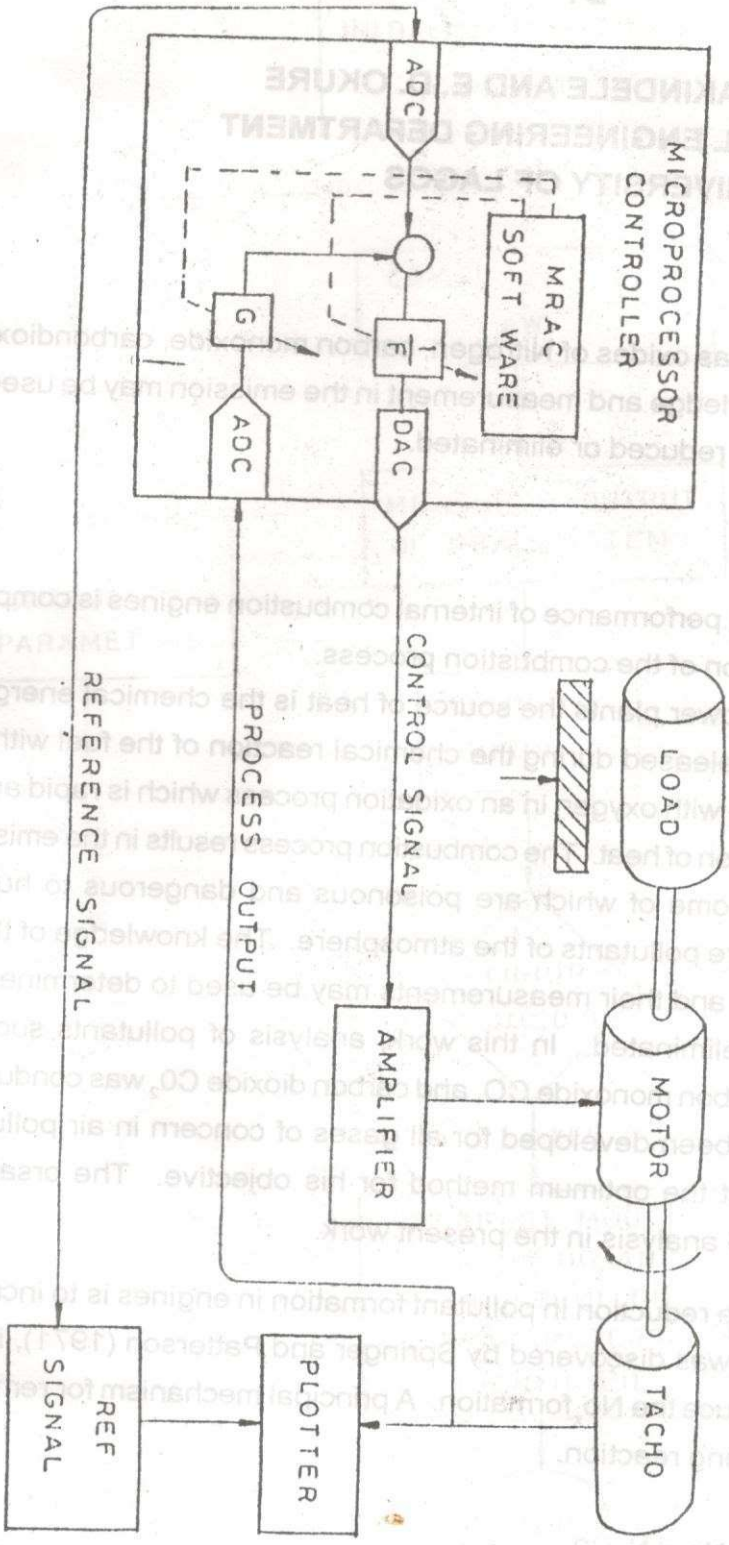


Fig. 8 Microprocessor based adaptive control of a dc servo motor