DESIGN OF COMPOSITE DIES BY THE DIE EXPANSION METHOD

A. O. A. IBHADODE
Production Engineering Department,
University of Benin, Benin City.

ABSTRACT

An analytical method based on die expansion is used to derive a general expression for the optimum interface diameters in composite dies such that minimum die expansion and minimum resultant maximum circumferential die stress are obtained. It enables the specified forging tolerance and allowable maximum die stress to be maintained and may also give rise to savings in die material.

INTRODUCTION

In completely closed die forging, as in other applications where the cylindrical die is subjected to very high pressures, the dies are usually compounded by shrinking one cylinder into the other, increase the strength. The elasto-plastic process of autofrettage is also used to achieve this. These operation are exemplified by the works of Blazynski (1989), Riley (1985), Farmer (1976), Lengyel et al (1966), Franklin (1960) and Horne (1955).

In addition to increasing die strength, die compounding also increases the die stiffness which reduces die expansion. Ibhadode (1997) has shown that a method based on die expansion may be used to select the external diameter of a die with respect to the specified forging tolerance.

In this paper, the die expansion method is used to select the optimum interface diameters in composite dies in order to have (i) minimum die expansion and (ii) equal and minimum resultant maximum circumferential stresses at the different cylinders in the die. Based on the method, a design procedure is outlined for selecting an appropriate composite die.

THEORETICAL ANALYSIS

By considering the residual stresses acting on a cylindrical workpiece in monoplex (1-plex) die container (Figure 1 [a]) Ibhadode (1988,1997) has shown that the internal pressure in the die container is a function of the die elastic properties, die geometry, workpiece shear flow stress, and die workpiece interface friction.

By extending this concept to a multiplex (n-plex) die (figure 1 [e]), of internal die radius a, external die radius b, and interface radii R_{n2} R_{n3},.. R_{ni},... R_{nn}, it can be shown as derived in the Appendix, that at the end of forging and before workpiece ejection, the die expansion is given by

$$\Delta a_n = 2k\beta_n (4\pi a^3 + M_w V_o) / (4\pi a^2) \quad \text{..... (1)}$$

where

$$\beta_n = [1 + \nu B + 2\nu\alpha_n / (b^2 - a^2)] / E \quad \text{..... (2a)}$$

and

$$B = [(b/a)^2 + 1] / [(b/a)^2 - 1] \quad \text{..... (2b)}$$

is the die elastic constant, α_n is the die geometric constant which is a function of the die interface radii, E is the modulus of elasticity, ν is the Poission's ratio, k is the shear flow stress of workpiece, M_w is the workpiece-container wall friction factor, and V_o is the effective workpiece volume; and the maximum circumferential stress in the die is given by

$$\sigma_{\theta mn} = 2kA [B + 2\alpha_n / (b^2 - a^2)] \quad \text{..... (3a)}$$

where

$$A = 1 + M_w L / (2a) \quad \text{..... (3b)}$$

and 2L is the effective workpiece-container contact length.

It can also be shown that i-th interface radius $R_{n,i}$ (excluding $R_{n,1}$ and $R_{n,i}$ and $R_{n,n+1}$) that gives the minimum die expansion and also the minimum resultant maximum circumferential die stress is given by

$$R_{n,i} = \sqrt{R_{n,i-1} R_{n,i+1}} = (a^{n+1-i} b^{i-1})^{1/n} \quad \text{..... (4)}$$

where i = 1,2,3,....., n + 1; n = 1,2,3, ...;

$R_{n,1} = a$ and $R_{n,n+1} = b$.

The derivations are given in the Appendix.

3. EXPERIMENTS

Experiments were carried out to verify the theoretical method by using a model die. Figure 2 shows the set-up for the experiments. The punch and counterpunch were made of low-carbon steel. The die container was made from polypropylene with an elastic modulus E =1400 N/mm², Poission's ratio, v = 0.4. Three different die blocks were used whose dimensions are given below by applying equation (4)

Monoplex - internal die radius, a = 12.85mm external die radius, b = 43.15mm;

Duplex - a = 12.85mm, s = 30mm, b = 43.15mm

Triplex: a = 12.85, s = 19.24mm, t = 28.82mm, b = 43.15mm

The height of each die was 35mm. Figure 3 shows a photograph of the dies and forged workpieces.

Lead workpieces measuring 20mm diameter by 20mm high were forged in the die containers. The lead workpieces had a flow stress of about $\sigma = 2k = 7N/mm^2$ determined from a pure compression test.

For each forging test, the dimensions of the lead workpiece were taken, the workpiece was degreased all over with alcohol, and placed in the die set. A 60-tonne Avery hydraulic testing machine was used to forge the workpiece at a load of 6kN. This load ensured that the workpiece filled out at its corners without fin extrusion, and that the bore of the die container was not deformed plastically. The workpiece was ejected from the die container and its diameter was measured at two perpendicular planes on a toolmaker's microscope.

This test procedure was carried out for the monoplex, duplex and triplex dies.

4. RESULTS AND DISCUSSION

4.1 CASES OTHER THAN NO-LOAD CONDITION

From equations (1 and 3), which were obtained for the no-load condition, we find that

$$\Delta a_n = C^* [D\alpha_n (R_{n,}) / E \text{ and } \sigma_{\theta mn} = F [D\alpha_n (R_{n,})] / a \quad \text{.....(5)}$$

where

$$C = 2kaA (1+vB) / E,\ D = 2kaA [2v/(b^2-a^2)],\ F=2kAB \quad \text{.....(5a)}$$

Equations (5) show that Δa_n and $\sigma_{\theta mn}$ are functions of $\alpha_n(R_{n,i})$ only, while C, D and F are load constants. $\alpha_n (R_{n,i})$ is a geometric quantity which is a function of the interface radii of the n-plex die. By partial differentiation of Δa_n and $\sigma_{\theta mn}$ with respect to the different $R_{n,i}$, the load constants do not appear in the results for optimum values of $R_{n,i}$ that minimise Δa_n and $\sigma_{\theta mn}$ as shown in equation (4). Therefore the result obtained in equation (4) is independent of the value of load acting on the die. This implies that equation (4) is vaild for all pressures acting the die including that related to maximum forging load.

4.2 PHYSICAL INTERPRETATION OF THE RESULT IN EQUATION (4)

Consider Figure 1(e) which shows an n-plex die consisting of n cylinders each having elastic constants $B_{n,1}$, $B_{n,2}$,...' $B_{n,j}$,... '$B_{n,n}$.

From the Appendix (equation [36], the die elastic constant for the i-th cyclinder is given by

$$\beta_{n,i} = [1+v[(R_{n,i+1}/R_{n,i})^2+1] / [(R_{n,i+1}/R_{n,i})^2-1]] / E \quad \text{.....(6)}$$

For minimum die expansion and minimum resultant maximum die stress, the outer radius of the i-th cylinder is given by

$$R_{n,i+1} = (a^{n-i}\, b^i)^{1/n} \quad \text{.....(7)}$$

from equation (4). The inner radius $R_{n,i}$ of the i-th cylinder is given by equation (4).

Therefore from equations (4 and 7), equation (6) becomes

$$B_{n,i} = [1+v[(b/a)^{2/n}+1][(b/a)^{2/n}-1]] / E \quad \text{.....(8)}$$

Similarly, the die elastic constant for the (i+1)-th cylinder is given by

$$B_{n,i+1} = [1+v[R_{n,i+2}/R_{n,i+1})^2+1]/[(R_{n,i+2}/R_{n,i+1})2-1]]/E \quad \text{.....(9)}$$

From equation (4) the outer radius for the (i+1)-th cyclinder is given by

$$R_{n,i+2} = (a^{n-i-1}\, b^{i+1})^{1/n} \quad \text{.....(10)}$$

Therefore from equations (4 and 10), equation (9) becomes equation (8).

If follows, therefore, that for an n-plex die to have a minimum die expansion and minimum resultant maximum stress, the die elastic constants $B_{n,1}$, $B_{n,2}$,...$B_{n,i}$,..., $B_{n,n}$, of the n-cyclinders making up the die must all be equal.

The die elastic constant β_n is a measure of the stiffness of the die. From equation (1), the lower the value of β_n, the lower the die expansion Δa_n.

Therefore, equation (4) implies that for a multiplex die to have minimum die expansion, and also minimum maximum stress, the stiffness of each cyclinder that makes up the die must all be equal.

4.3 FEATURES OF A DUPLEX (2-PLEX) DIE

Figure 4 shows the plots of die expansion Δa_n, maximum circumferential stresses $\sigma_{\theta n2I}$, and $\sigma_{\theta n2II}$, die geometric constant α_2 and shrinkage P_{s2} against the interface radius s for a duplex die.

The figure shows that as s increases from the internal die radius a to the external die radius b, α_2 decreases in an unsymmetric parabolic manner from zero to a minimum value of -2757mm^2 at about $s = 79$mm, and thereafter increases to zero again at $s = b$. This same behaviour is exhibited by the plots of Δa_2, and $\sigma_{\theta n2I}$, and $\sigma_{\theta n2II}$. This is due to the fact that these variables are functions of α_2 as given by equations (5).

Equation (4) predicts that the minima of α_2, Δa_2, and $\sigma_{\theta m2I}$ and $\sigma_{\theta m2II}$ will occur at the optimum value of s_{opt} = 79.06mm which agrees with Figure 4.

The unsymmetric parabolic plots of α_2, Δa_2, and $\sigma_{\theta m2I}$ and $\sigma_{\theta m2II}$ imply that these variables attain their minimum values at distances in the die nearer the die bore than its external surface.

The figure shows that the plots of $\sigma_{\theta m2I}$ and $\sigma_{\theta m2II}$ are coincident, as given by equation (28).

The plot of P_{s2} in Figure 4 also exhibits a similar behaviour to the above plots except that its minimum value occurs later at about s = 89mm than for the other plots. The value of s at which P_{s2} attains its minimum value does not have a simple expression but may be determined numerically. The value is however not important as far as minimum die expansion and minimum die stress are concerned, as the value of P_{s2} that makes these parameters minimum may be obtained by substituting $s = s_{opt}$ in equation (29).

4.4 EFFECT OF MULTIPLEX DIES

Figure 5 shows the plots of minimum values of Δa_n, $\sigma_{\theta mn}$, β_n and α_n against the number of cyclinders making up a die, *n*-plex, for constant die material volume. The figures were obtained by using equations (1-4, 30, 34 and 35). For this figure and the following ones, Δa_n and $\sigma_{\theta mn}$ represent minimum values, obtained for optimally designed dies.

The figure shows that the above variables decrease as the number of cylinders making up the die increases. Particularly, it shows that Δa is reduced by 8.92%, 11.29% and 12.37% by the use of duplex, triplex and quadruplex dies respectively when compared with the monoplex die. Similarly $\sigma_{\theta mn}$ is reduced by 30.42%, 38.54% and 42.20% when the duplex, triplex and quadruplex dies are used respectively.

The figure shows that the effect of using increasing number of cyclinders to compound a die, in reducing die expansion and maximum die stress, decreases as the number of cyclinders increases. For example, the additional reduction in Δa when a quadruplex die is used in place of a triplex die is only 1.08% compared to an additional reduction of 2.37% when a triplex die is used instead of a duplex die. This goes to show that very little may be gained by using an increasing number of cyclinders to compound a die beyond a certain value.

4.5 EFFECT OF EXTERNAL DIE RADIUS IN MULTIPLEX DIES

Figure 6 shows the plots of relative multiplex die expansion $\Delta a_n/\Delta a_1$ and relative maximum die stress $\sigma_{\theta mn}/\sigma_{\theta m1}$ against n-plex die for various values of b, Δa_n and $\sigma_{\theta mn}$ are the die expansion and maximum die stress respectively for each *n*-plex die where *n*=1,2,3 and 4; and Δa_1 and $\sigma_{\theta m1}$ are the respective symbols for the 1-plex die, for optimally designed dies.

The figure shows that as *b* increases the relative multiplex die expansion and relative maximum dies stress decrease for each *n*-plex die. For example, for the 2-plex die, the relative multiplex die expansion and relative maximum die stress are 0.91 and 0.70, 0.90 and 0.61, and 0.89 and 0.55 for *b* = 125mm, 170mm and 215mm respectively. These values respresent 8.87% and 30.42%, 10.41% and 39.46%, and 11.20% and 45.04% reductions in relative multiplex die expansion and relative maximum die stress with respect to the monoplex die for *b* = 125mm, 170mm and 215mm respectively.

The figure also shows that as b increases, its effectiveness in reducing die expansion and maximum die stress reduces. For example, for the 4-plex die, for a 36% increase in *b* from 125mm to 170mm, the relative multiplex die expansion and relative maximum die stress are reduced by 2.15% and 20.38% respectively while for a 72% increase in b from 125mm to 215mm, the relative multiplex die expansion and relative maximum die stress are reduced by only 3.35% and 32.94% respectively, which are less than their respective two-fold increases in the former case compared to the two-fold increases in the former case compared to the two-fold increase in *b*. This result shows that not much may be gained by using dies of large external die radius beyond a certain size, depending on prevailing conditions.

4.6 DESIGN METHOD

The following steps may be taken in designing by this method:

1.0 **Determination of external die diameter, 2*b* (Ibhadode, 1997)**

1.1 Calculate effective workpiece volume V_0

1.2 Specify die wall friction factor, M_w

1.3 Determine minimum tolerarice on forging diameter from

$$\Delta D_{min} = kN(4\pi a^3 + M_w V_0)/(\pi a^2) \quad \text{(11a)}$$

where $N = (1+v)/E$ (11b)

1.4 Select $\Delta D > \Delta D_{min}$

1.5 Determine the die elastic constant for the monoplex die from $\beta_1 = \pi a^2 \Delta D/(k(4\pi a^3 + M_w V_0))$ (12)

1.6 Determine minimum external die radius from

$$b_{min} = a\sqrt{(E\beta_1 + V\text{-}1)/(E\beta_1 - V\text{-}1)} \quad \text{(13)}$$

1.7 Select the eternal die radius for the monoplex die $b_1 > b_{min}$

Note that equations (11-13) are good approximations for the respective equations for $\Delta a/a << 1$ (Ibhadode, 1997)

2.0 Selection of multiplex die

2.1 Determine the die expansion Δa_1' (2. $\Delta a_1 = \Delta D$) , (equation (1) and maximum die stress $\sigma_{\theta m1}$ (equation (3)) for the monoplex die.

2.2 Using b_1 above, determine Δa_{nb1} and $\sigma_{\theta mnb1}$ for *n*-plex dies, where $n = 2,3,4,...$, Δa_{nb1} and $\sigma_{\theta mnb1}$ are the die expansion and maximum die stress respectively for each *n*-plex die at the external die radius $b_n = b_1$.

2.3 By trial-and-error determine an appropriate b_n such that $\Delta a_{nb1} < \Delta a_{nbn} < \Delta a_1$ and $\sigma_{\theta mnb1} < \sigma_{\theta mnbn} \leq \sigma_{\theta m1}$ ..... (14a and b) if it is required to reduce b_1. Δa_{nbn} and $\sigma_{\theta mnbn}$ are the die expansion and maximum die stress respectively for each *n*-plex die at the external die radius b*n*.

2.4 Select an appropriate *n*-plex die by considering the economics of manufacture with respect to additional improvements in die expansion and maximum die stress reductions over the duplex (2-plex) die.

4.7 CONSTANT DIE EXPANSION IN MULTIPLEX DIES

Figure 7 shows the plots of relative external die radius b_n/b_1, relative multiplex die expansion $\Delta a_n / \Delta a_1$ and relative maximum die stress $\sigma_{\theta mn} / \sigma_{\theta m1}$ against *n*-plex die for constant external die radius b_1 and constant die expansion Δa_1 obtained by following the steps outlined in Section 4.6.

The figure shows that by requiring a constant die expansion in the multiplex dies equal to that for the monoplex die, the external die radius *b* is reduced by 38.24%, 42.35%, and 44.12% for the duplex, triplex and quadruplex dies respectively compared to the monoplex die. However, the maxiumm die stress is slighly increased by 0.62%, 2.7% and 4.4% for the 2-plex, 3-plex and 4-plex dies respectively compared to the 1-plex die. The reductions in external die radius by this requirement gives rise to savings in die material of 67.71%, 73.09% and 75.28% for the 2-plex, 3-plex and 4-plex dies respectively compared to the 1-plex die.

Equal stress levels in multiplex dies as for the monoplex die may be maintained by requiring that die expansions in the multiplex dies are slightly less than for the monoplex die. This gives rise to substantial savings in die material while at the same time maintaining the required forging tolerance. As *n* increases for the *n*-plex die, the difference between its die expansion and that of the 1-plex die should be increased in order to obtain equal maximum die stress at the die bore.

For the case where the multiplex external die radius b_n is equal to the monoplex external die radius b_1, figure 5 shows that die expansion and maximum die stress are reduced by 11.59% and 39.45%, 13.18% and 49.55%, and 14.35% and 54.00% for the 2-plex, 3-plex, and 4-plex dies respectively compared to the 1-plex die. In this case, a high factor of safety is imposed, which may lead to over-designing.

4.8 THEORETICAL AND EXPERIMENTAL RESULTS

Figure 8 shows the comparison of theoretical and experimental results. A die wall-workpiece friction factor $M_w = 0.5$ was used for the theoretical evaluations. Both results indicate that as *n*-plex increases, the relative die expanison $\Delta a/a$ decreases. However, the theoretical results over-estimate the die expansion and as *n*-plex increases the divergence in results increases. The over-estimates are 12.8%, 28.8% and 46% for the 1-plex, 2-plex and 3-plex dies respectively. The results show that the theory is able to predict die expansion qualitatively.

More tests are required to determine the extent of agreement between the theory and practice quantitatively with steel dies.

5. CONCLUSIONS

The results above show that

(a) The interface diameters of a multiplex die can be determined optimally such that the die will have minimum die expansion and minimum resultant maximum circumferential stress.

(b) An appropriate composite die may be selected for a given forging operation such that any or all of the following requirements may be met:

(i) Maintenance of the specified forging tolerance,
(ii) Maintenance of specified maximum die stress and
(iii) Savings in die material.

(c) Agreement between theoretical and experimental results is generally good especially in qualitative terms.

6. REFERENCES

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APPENDIX

A.1 DIE EXPANSION AND DIE STRESS IN QUADRUPLEX (4-PLEX) DIE

Consider the quadruplex die made up of four cylinders I, II, III and IV shown in figure 1(d) with internal radius *a*, external radius *b* and interface radii *s, t* and *u*, The shrinkage stresses are P_s' P_t' and P_u between cylinders I and II, II and III, and III and IV respectively.

At the end of forging and before workpiece ejection it can be shown (Ibhadode 1997, Ibhadode 1988) that the internal pressure in the die container is given by

$$P = -2k\,[1+M_w L/[2(a+\Delta a)]] \sim -2kA = \sigma_{ra} \quad \text{.....(15)}$$

where σ_{ra} is the radial stress at the bore, and $\Delta a/a << 1$
From thick hollow cylinder theory, the circumferential stress at the bore is

$$\sigma_{\theta a} = 2kAB \quad \text{.....(16)}$$

The maximum circumferential stresses $\sigma_{\theta msI}$, $\sigma_{\theta msII}$, $\sigma_{\theta msIII}$ and $\sigma_{\theta msIV}$ in cylinders I, II, III and IV respectively due to shrinkage are

$$\sigma_{\theta msI} = 2s^2P_s/(s^2-a^2) \quad \text{.....(17)}$$

$$\sigma_{\theta msII} = P_s + 2t^2(P_t-P_s)/(t^2-s^2) \quad \text{.....(18)}$$

$$\sigma_{\theta msIII} = P_t + 2u^2(P_u-P_t)/(u^2-t^2) \quad \text{.....(19)}$$

and

$$\sigma_{\theta msIV} = (u^2+b^2)\,P_u/(u^2-b^2) \quad \text{.....(20)}$$

The resultant stresses at the bore (r=a) due to the applied internal pressure and shrinkage stresses are

$$\sigma_{rm4I} = \sigma_{ra} \quad \text{.....(21)}$$

$$\sigma_{\theta m4I} = \sigma_{\theta a} + \sigma_{\theta msI} = 2kAB + 2s^2P_s/(s^2-a^2) \quad \text{.....(22)}$$

and σ_y, the axial stress, is zero.

The magnitude of the radial strain at the bore is given by

$$|\varepsilon_r| = |\,[\Delta a/(a+\Delta a)\,]\,| \sim |\,(\Delta a/a)\,| = |\,[\,(\sigma_{rm4I} - V\sigma_{\theta m4I})/E\,]\,| \quad \text{.....(23)}$$

From equations (21, 22 and 23)

$$(\Delta a/a) = [2kA(1+vB) + 2vs^2P_s/(S^2-a^2)]/E \quad \text{.....(24)}$$

P_s can be determined by requiring that the resultant maximum circumferential stresses σ_{ms4I}, σ_{ms4II} $\sigma_{\theta ms4III}$, and $\sigma_{\theta ms4IV}$ in the four cylinders are equal.

At $r = s$,

$$\sigma_{\theta m4II} = \sigma_{\theta s} + \sigma_{\theta msII} = 2kA[(b/s)^2+1]/[(b/a)^2-1] + P_s + 2s^2(P_t - P_s)(t^2 - s^2) \quad \text{.....(25)}$$

At $r = t$,

$$\sigma_{\theta m4III} = \sigma_{\theta t} + \sigma_{\theta msIII} = 2kA[(b/t)^2+1]/[(b/a)^2-1] + P_t + 2u^2(P_u - P_t)(u^2 - t^2) \quad \text{.....(26)}$$

At $r = u$,

$$\sigma_{\theta m4IV} = \sigma_{\theta u} + \sigma_{\theta msIV} = 2kA[(b/u)^2+1]/[(b/a)^2-1] + (u_2 + b^2)P_u/(u^2 - b^2) \quad \text{.....(27)}$$

For equal maximum circumferential stresses in the four cylinders,

$$\sigma_{\theta m4I} = \sigma_{\theta m4II} = \sigma_{\theta m4III} = \sigma_{\theta m4IV} \quad \text{.....(28)}$$

From equations (22, 25 - 28), the shrinkage stress between cylinder I and cylinder II in the 4-plex die is given by

$$P_{s4} = 2kA\,(s^2 - a^2)\,\alpha_4\,[s^2(b^2 - a^2)] \quad \text{.....(29)}$$

Where α_4 is the die geometric constant, given by

$$\alpha_4 = (c_1 + c_2 + c_3 - c_4 - c_5 - c_6)/(d_1 + d_2 + d_3 - d_4 - d_5) \quad \text{.....(30)}$$

Where

$$c_1 = a^2(b^2+s^2)(u^2+t^2)(u^2+b^2)(t^2-s^2) \quad \text{.....(30b)}$$

$$c_2 = 2a^2s^2(b^2+t^2)(u^2+b^2)(u^2-t^2) \quad \text{.....(30c)}$$

$$c_3 = 4s^2t^2u^2(a^2+b^2)(u^2-b^2) \quad \text{.....(30d)}$$

$C_4 = 4a^2s^2t^2 (u^2 + b^2) (u^2 - b^2)$ ..... (30e)

$C_5 = 2s^2t^2 (a^2 + b^2) (u^2 + b^2) (u^2 - t^2)$ ..... (30f)

$C_6 = s^2 (a^2 + b^2) (u^2 + b^2) (u^2 + t^2) (t^2 - s^2)$..... (30g)

$d_1 = 4s^2t^2 (u^2 + b^2) (u^2 - t^2)$..... (30h)

$d_2 = 2t^2 (u^2 + b^2) (u^2 + t^2) (s^2 - a^2)$..... (30i)

$d_3 = 2s^2 (u^2 + b^2) (u^2 + t^2) (t^2 - s^2)$..... (30j)

$d_4 = (u^2 + b^2) (u^2 + t^2) (s^2 - a^2) (t^2 - s^2)$..... (30k)

$d_5 = 8s^2 t^2u^2 (u^2 - b^2)$ ..... (30l)

From equations (24, 29 and 30, the die expansion Δa_4 is given by

$\Delta a_4 = 2kA [1 + \nu B+2\nu\alpha_4/(b^2-a^2)]/E$ .....(31)

or

$\Delta a_4 = 2k\beta 4 (4\pi a^3 + M_w V_o)/(4\pi a^2)$ ..... (32a)

where

$V_o = \pi(a + \Delta a)^2(2L)$ ..... (32b)

and β_4 is given by equation (2) for $n = 4$.

By partial differentiation of Δa_4, equation (31 or 32), with respect to *s,t* and *u* separately, the optimum interface radii are

$s = \sqrt{at} = (a^3b)^{1/4}$, $t = \sqrt{su} = \sqrt{ab}$, $u = \sqrt{tb} = ab^3)^{1/4}$ .....(3)

A.2 TRIPLEX (3-PLEX) DIE

For the triplex die, figure 1(c), *u=t*. Substituting for *u* in equations (30),

$\alpha_3 = (c_1+c_3-c_4-c_5)/(d_2+d_3-d_4-d_5) = (e_1+e_2-e_3)/(f_1+f_2-f_3-f_4)$

where

$e_1 = a^2(b^2+s^2) (b^2+t^2) (t^2-s^2)$ ..... (34b)

$e_2 = 2b^2s^2 (t^2-a^2) (t^2-b^2)$ ..... (34c)

$e_3 = s^2(a^2+b^2) (b^2+t^2)(t^2-s^2)$ .....(34d)

$f_1 = 2t^2(b^2+t^2)(s^2-a^2)$ .....(34e)

$f_2 = 2s^2 (b^2+t^2)(t^2-s^2)$ ..... (34f)

$f_3 = (b^2+t^2)(s^2-a^2)(t^2-s^2)$ .....(34g)

$f_4 = 4s^2t^2(t^2-b^2)$ .....(34h)

A.3 DUPLEX (2-PLEX) DIE

For the duplex die, figure 1(b), *u=t=s.* Substituting for *u* and *t* in equations (30) or for t in equations (34),

where

$\alpha 2 = (c_3-c_4)/(d_2-d_5) = e_2/(f_1-f_4) = g_1/(h_1-h_2)$ ..... (35a)

where

$g_1 = b_2(s_2-a_2) (s^2-b^2)$, $h_1 = (s^2+b^2)(s^2-a^2)$, $h_2 = 2s^2(s^2-b^2)$ .....(35b)

A.4 MONOPLEX (1-PLEX) DIE

For the monoplex die, u=t=s=a or b. Substituting for u, t and s in equations (30), $\alpha_1 = 0$, such that from equation (2),

$\beta_1 = (1+B)/E$ .....(36)

A.5 n-plex Die

Equations (34) and (35) for the 3-plex, and **2-plex** dies respectively were deduced from equations **(30)** for the 4-plex die. Equations (34) and (35) can also be derived independently from first principles. Similar expressions may also be derived for higher orders of compounding. Therefore from equation (32), the die expansion Δa_n for an *n*-plex die is given by equation (1). The solution for α_n has been given for n= 1, 2, 3, and 4.

By partial differentiation of Δa_3 and Δa_2 with respect to *s* and *t*, and s respectively, a pattern of results for the optimum interface radii similar to those in equation (33) is derived.

It can therefore be deduced that for an n-plex die, the i-th interface radius $R_{n,i}$ (excluding $R_{n,1}$ and $R_{n,n+1}$), that gives the minimum die expansion is given by equation (4).

A.4 MAXIMUM CIRCUMFERENTIAL STRESS

From equations (22 and 29), the maximum circumferential stress at the bore for an n-plex die is given by equation (3). By partial differentiation of $\alpha_{\theta mn}$ with respect to the different interface radii $R_{n,i}$' equation (4) is also obtained for minimum maximum circumferential stress.

FIGURE CAPTIONS

FIGURE 1. Monoplex and multiplex dies.

FIGURE 2. Experimental set-up.

FIGURE 3. Die containers and forged workpieces used in the tests.

FIGURE 4. Plots of die expansion, maximum circumferential stress, die geometric constant and shrinkage stress against intermediate radius for a duplex die.

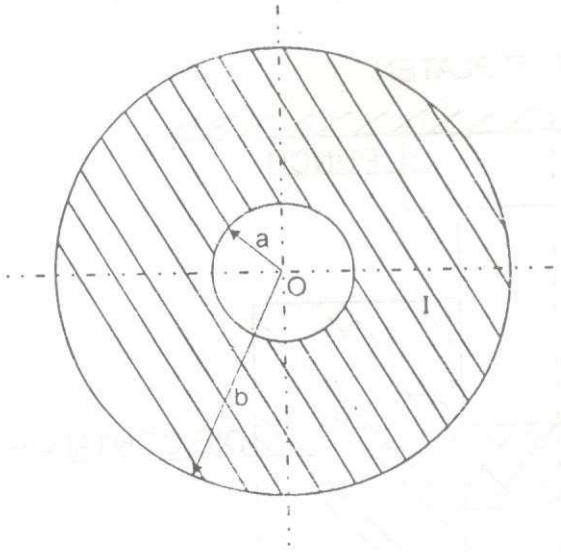
FIGURE 5. Plots of minimum values of die expansion, maximum die stress, die elastic constant and die geometric constant

against the numbers of cylinders making up a die.

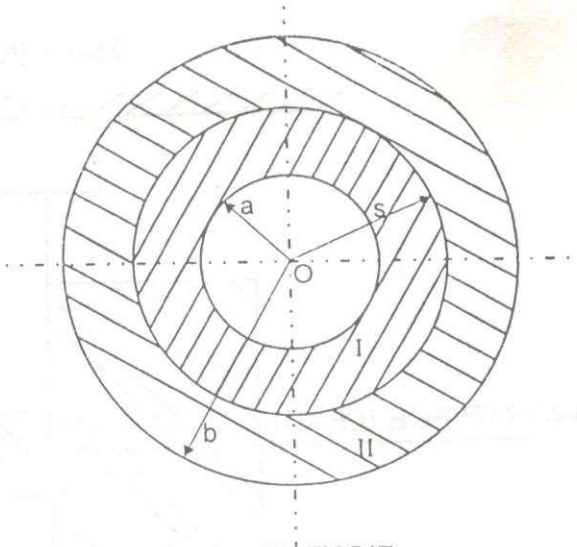
FIGURE 6. Plots of relative die expansion and relative maximum die stress against the number of cylinders making up a die.

FIGURE 7. Plots of relative external die radius, relative multiplex die expansion and relative maximum die stress against the number of cylinders making up a die.

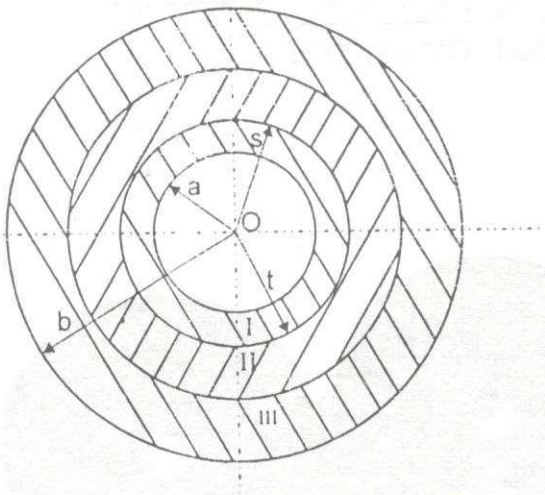
FIGURE 8. Comparison of experimental and theoretical results for the variation of relative die expansion with degree of compounding of die container.



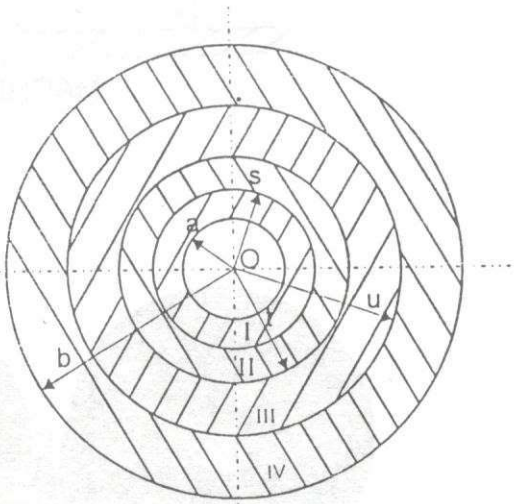
(a) 1 - PLEX DIE



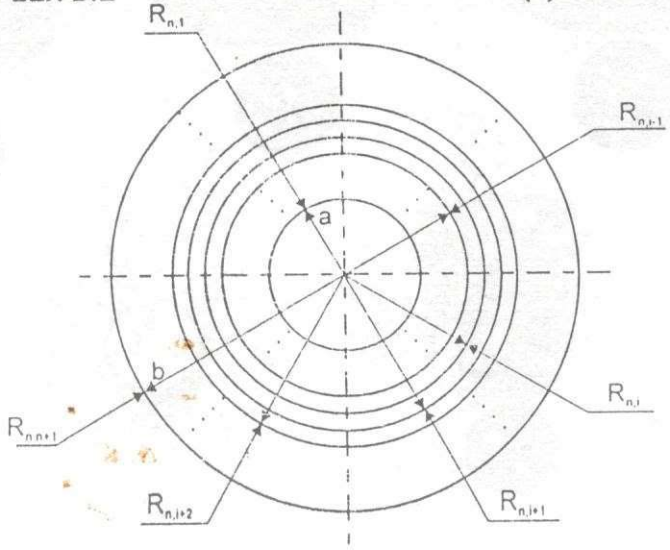
(b) 2 - PLEX DIE



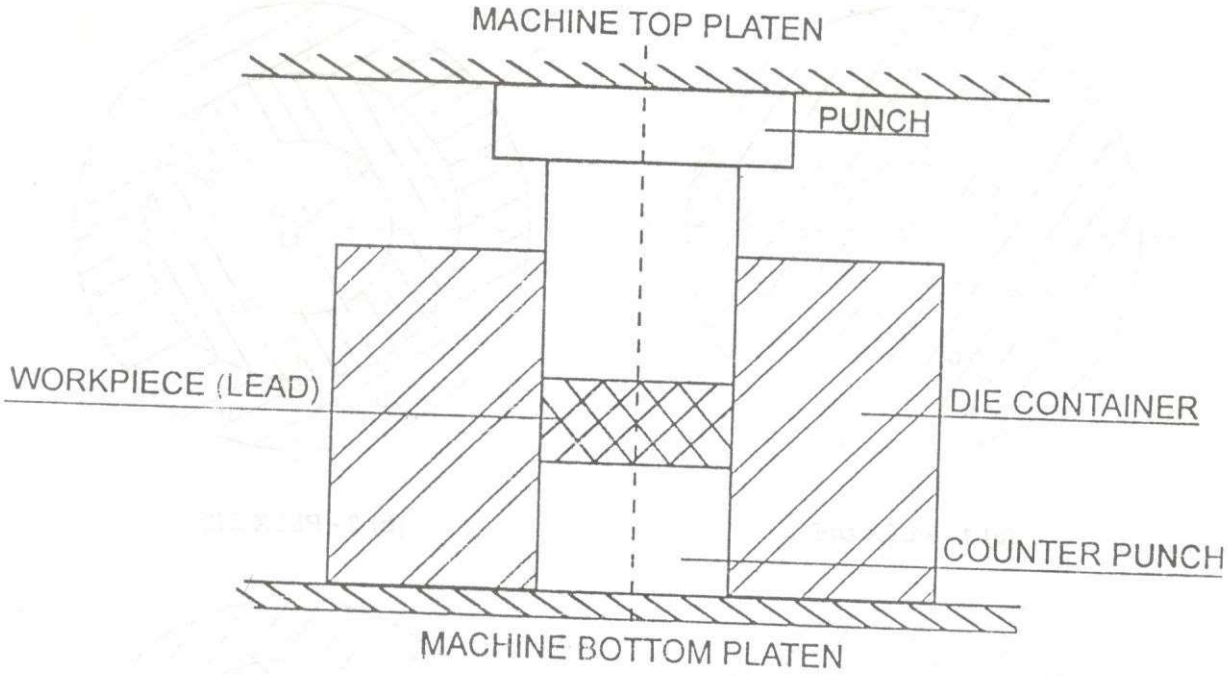
(c) 3 - PLEX DIE

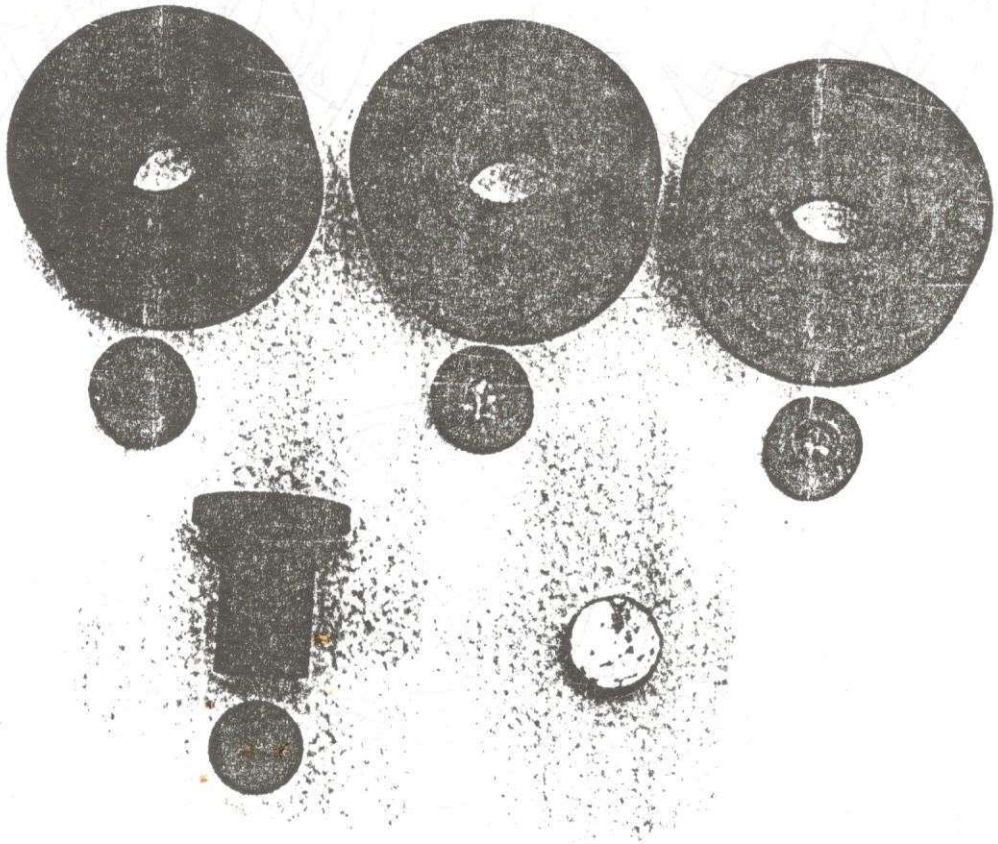


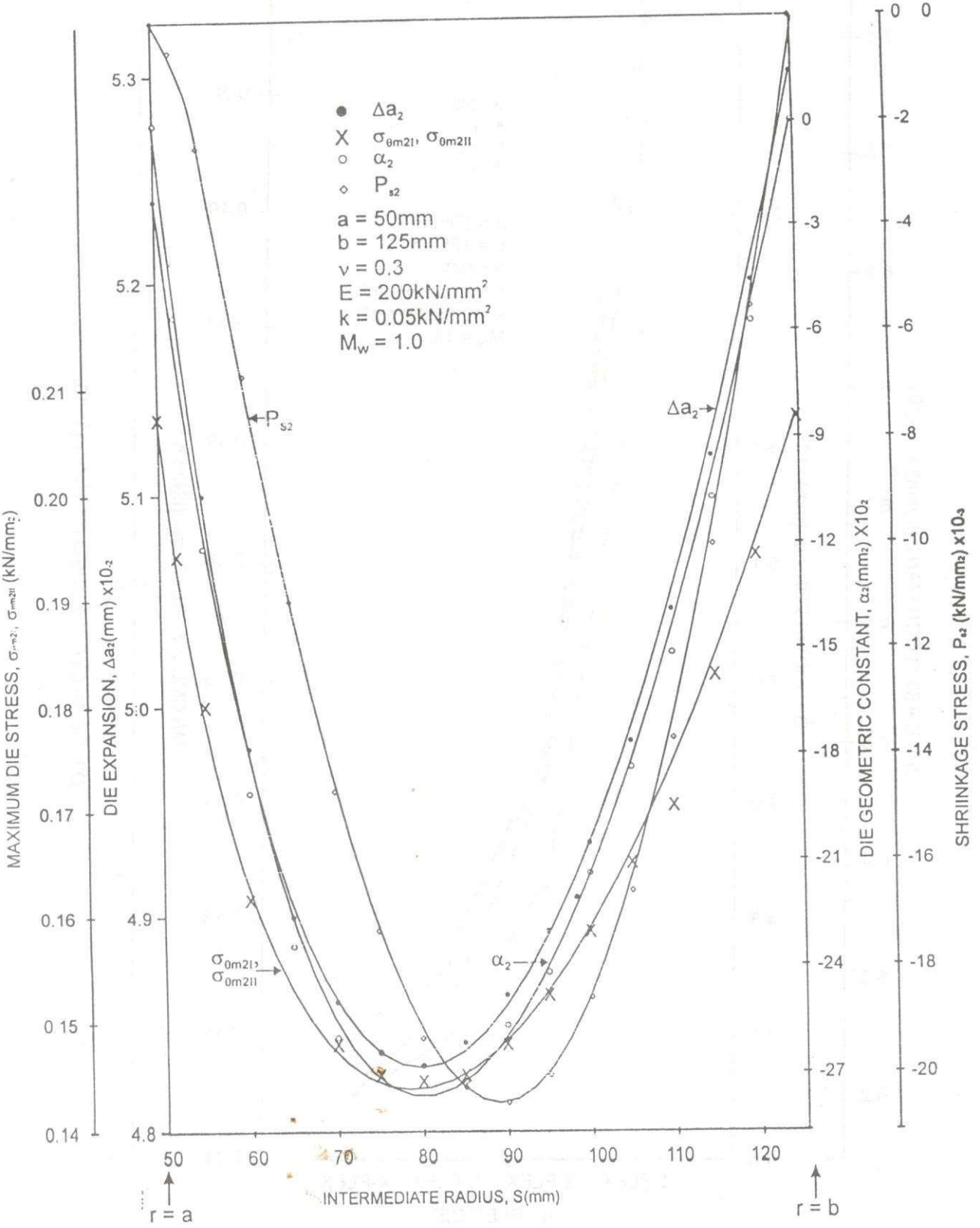
(d) 4 - PLEX DIE

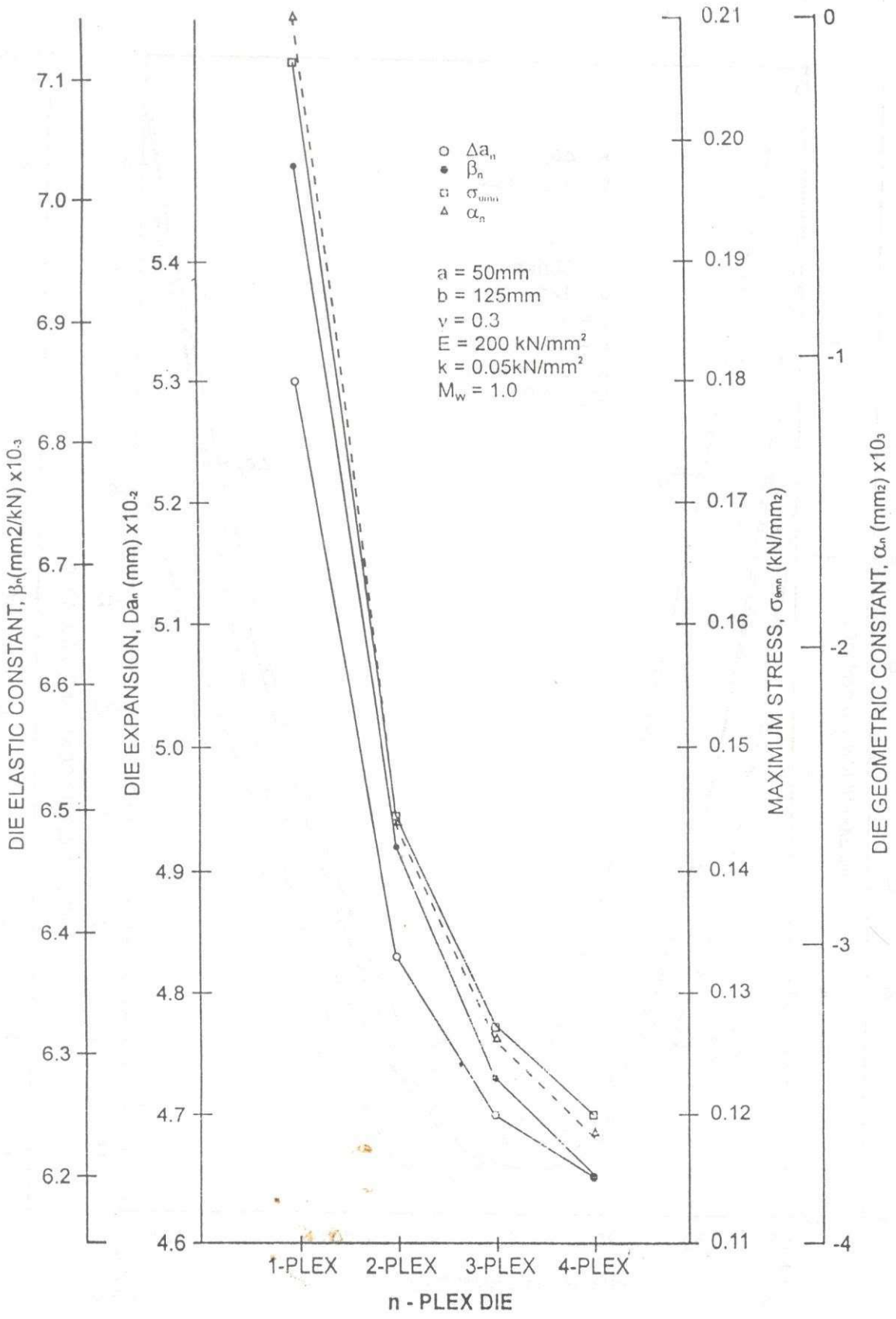


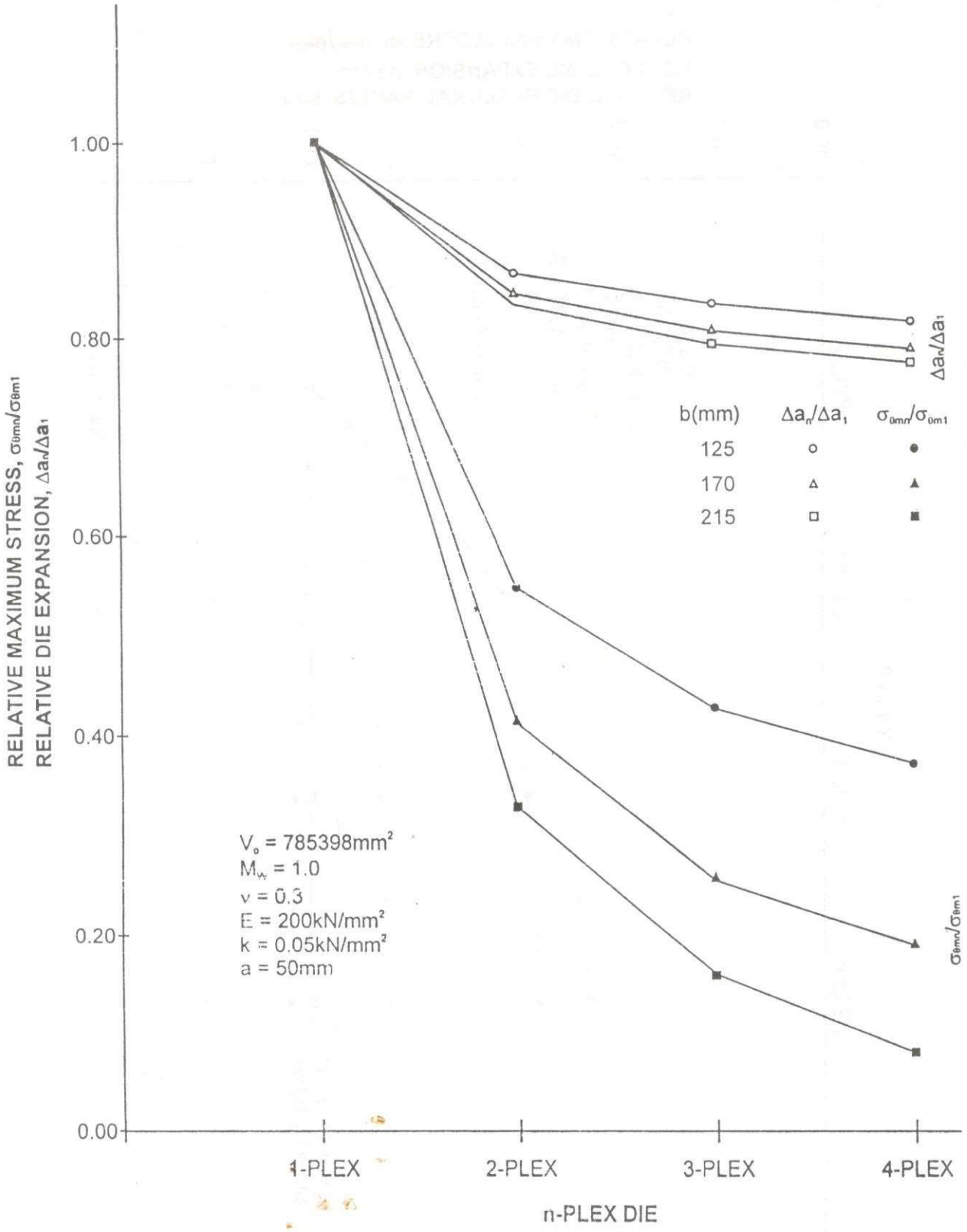
(e) n - PLEX DIE

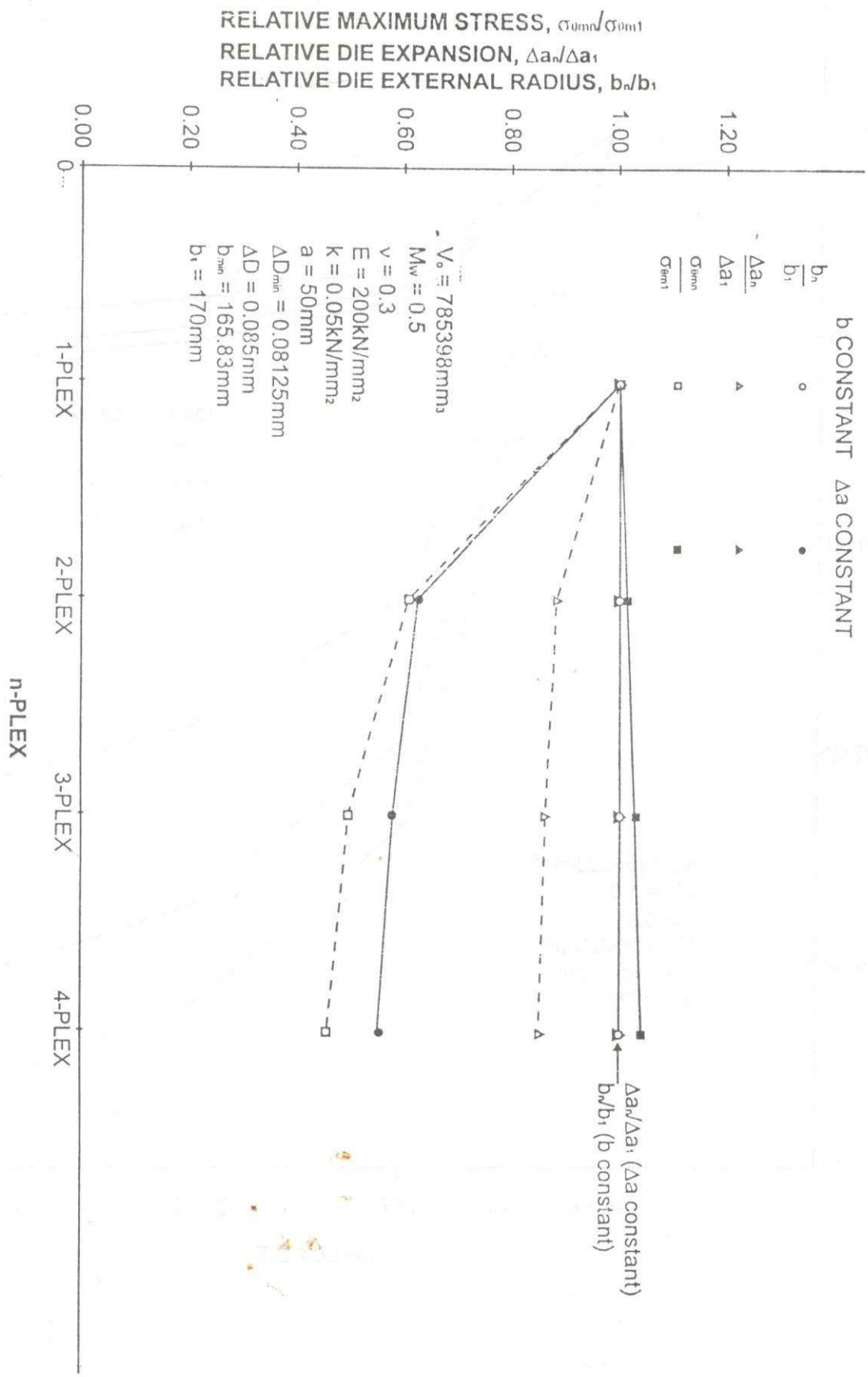

MACHINE TOP PLATEN
PUNCH
WORKPIECE (LEAD)
DIE CONTAINER
COUNTER PUNCH
MACHINE BOTTOM PLATEN

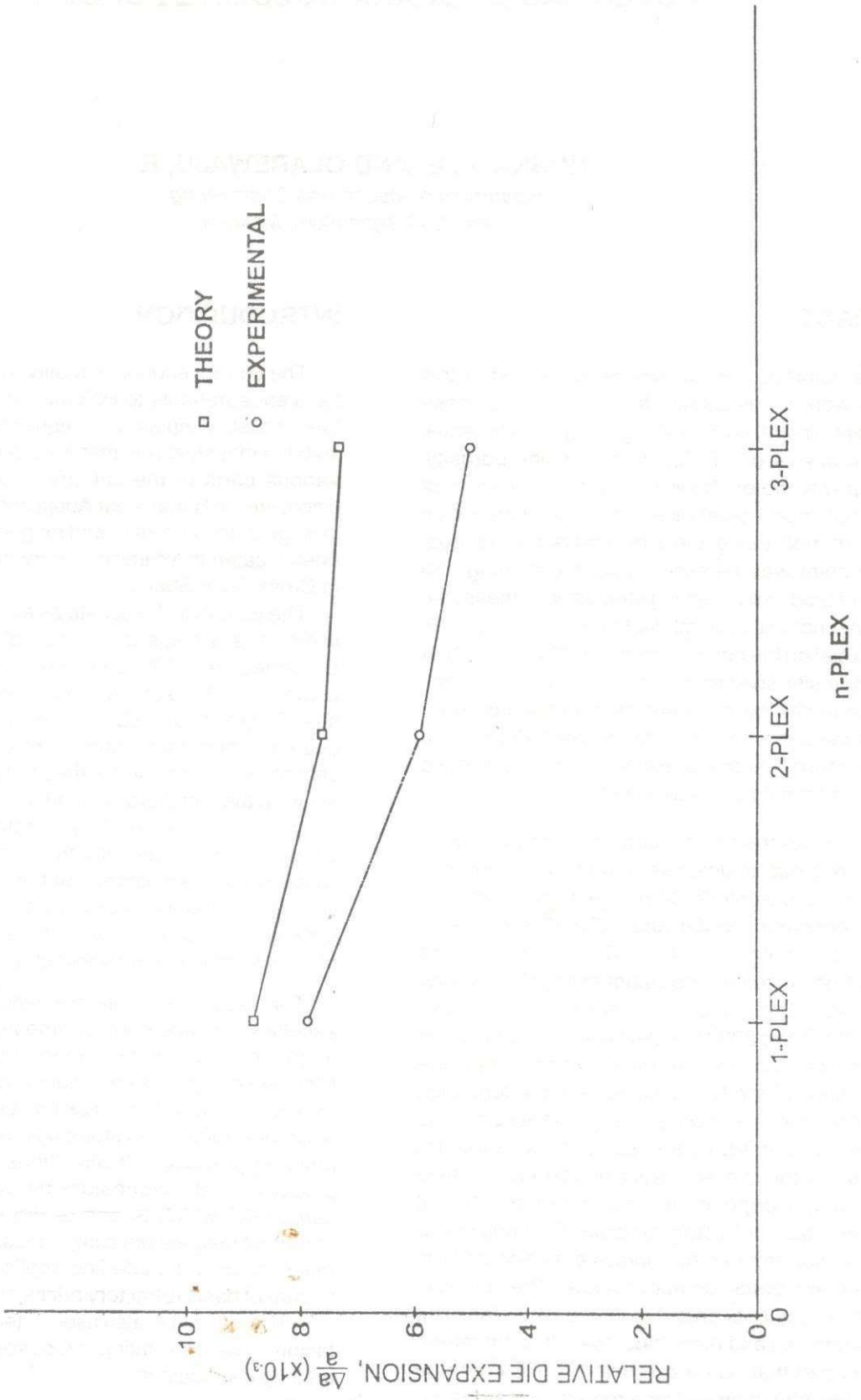



Δa2
σθm2I, σθm2II
α2
Ps2
a = 50mm
b = 125mm
ν = 0.3
E = 200kN/mm²
k = 0.05kN/mm²
Mw = 1.0
Ps2
Δa2
σθm2I, σθm2II
α2
MAXIMUM DIE STRESS, σm2I, σm2II (kN/mm2)
DIE EXPANSION, Δa2(mm) x10-2
DIE GEOMETRIC CONSTANT, α2(mm2) X10-2
SHRIINKAGE STRESS, Ps2 (kN/mm2) x10-3
INTERMEDIATE RADIUS, S(mm)
r = a
r = b


o Δa_n
• β_n
□ σ_{mm}
△ α_n
a = 50mm
b = 125mm
ν = 0.3
E = 200 kN/mm²
k = 0.05kN/mm²
M_w = 1.0
DIE ELASTIC CONSTANT, β_n(mm2/kN) x10₋₃
DIE EXPANSION, Da_n (mm) x10₋₂
MAXIMUM STRESS, σ_{mn} (kN/mm₂)
DIE GEOMETRIC CONSTANT, α_n (mm₂) x10₃
7.1
7.0
6.9
6.8
6.7
6.6
6.5
6.4
6.3
6.2
5.4
5.3
5.2
5.1
5.0
4.9
4.8
4.7
4.6
0.21
0.20
0.19
0.18
0.17
0.16
0.15
0.14
0.13
0.12
0.11
0
-1
-2
-3
-4
1-PLEX
2-PLEX
3-PLEX
4-PLEX
n - PLEX DIE


RELATIVE MAXIMUM STRESS, $\sigma_{\theta mn}/\sigma_{\theta m1}$
RELATIVE DIE EXPANSION, $\Delta a_n/\Delta a_1$
1.00
0.80
0.60
0.40
0.20
0.00
b(mm) $\Delta a_n/\Delta a_1$ $\sigma_{\theta mn}/\sigma_{\theta m1}$
125
170
215
$\Delta a_n/\Delta a_1$
$\sigma_{\theta mn}/\sigma_{\theta m1}$
V_o = 785398mm²
M_w = 1.0
ν = 0.3
E = 200kN/mm²
k = 0.05kN/mm²
a = 50mm
1-PLEX
2-PLEX
3-PLEX
4-PLEX
n-PLEX DIE


RELATIVE MAXIMUM STRESS, $\sigma_{\theta mn}/\sigma_{\theta m1}$
RELATIVE DIE EXPANSION, $\Delta a_n/\Delta a_1$
RELATIVE DIE EXTERNAL RADIUS, b_n/b_1
0.00
0.20
0.40
0.60
0.80
1.00
1.20
1-PLEX
2-PLEX
3-PLEX
4-PLEX
n-PLEX
b CONSTANT Δa CONSTANT
$\frac{b_n}{b_1}$
$\frac{\Delta a_n}{\Delta a_1}$
$\frac{\sigma_{\theta mn}}{\sigma_{\theta m1}}$
V_o = 785398mm3
M_w = 0.5
v = 0.3
E = 200kN/mm2
k = 0.05kN/mm2
a = 50mm
ΔD_{min} = 0.08125mm
ΔD = 0.085mm
b_{min} = 165.83mm
b_1 = 170mm
$\Delta a_n/\Delta a_1$ (Δa constant)
b_n/b_1 (b constant)


THEORY
EXPERIMENTAL
RELATIVE DIE EXPANSION, $\frac{\Delta a}{a}$ ($\times 10^{-3}$)
0
2
4
6
8
10
1-PLEX
2-PLEX
3-PLEX
n-PLEX