LOAD CHARACTERISTICS OF SINGLE-PHASE STATOR-EXCITED INDUCTION AND RELUCTANCE GENERATORS

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ABSTRACT

The performance characteristics of a single-phase stator excited generator are presented. Some salient points inherent in the generators ability to produce a voltage under different conditions are highlighted. The generator is shown to be capable of stand-alone activity and providing easily controllable output. The model is developed with a general R-L load to represent a common application of these generators as stand-alone power sources. The same stator is used with two rotors, one round and the other with salient poles, yielding an induction and synchronous reluctance generators respectively. It is shown that in both cases, the frequency of the output load current is twice that of the rotor current and that the load characteristics are superior when a salient pole rotor is used instead of a round rotor.

Keywords: ***stator-excited, salient-pole, single-phase generator, induction generator, synchronous reluctance generator***

1.0 INTRODUCTION

Small capacity (0.5 – 3kVA) single-phase synchronous generators are utilized in locations where the grid is either inaccessible or where power supply is unreliable. Such generators are driven by small petrol engines and have been in existence for over seven decades. Unlike its motor counterpart, its demand and manufacture in a particular region tends to reduce as the grid power spreads. Following population growth in some developing countries (e.g. Nigeria) which are not correspondingly matched by growth in electrical energy supply, grid system supply inefficiencies increased and therefore these generators are rapidly becoming a greater part of electricity supply in both rural and urban homes and small commercial outfits. Apart from conventional single-phase synchronous generators with brush-type exciters [1], there have evolved many configurations of truly single-phase generators - and self-exited induction [2, 3] and synchronous reluctance generators [4]. Both of these generator types are brushless and therefore have robust rotor structure capable of operating for a long time without needing maintenance.

Single-phase induction [2, 3] and synchronous reluctance [4] generators have been shown to be very robust and do not require rotating rectifiers, self excitation being achieved by connecting capacitors to both stator windings in the presence of some remanent magnetism. Loads are connected in parallel with the main winding capacitor. Load regulation in these generators is however poor and output voltage control in respect of load changes requires advanced technology and add to the cost of the machine. Brushless generators with [5] or without [6] rotating exciters reported by Nonaka consist of a field winding short-circuited with a diode plus an unbalanced two-phase stator winding, one connected directly to load and other capacitor excited for load regulation. Reverse revolving fields induced on the field winding due to armature reaction is half-wave rectified and is responsible for self-excitation.

The generator that forms the subject of this paper was invented by Fujii [7, 8], has no rotating diodes and therefore has a robust rotor structure. The rotor is of salient pole form and has a damper cage in the q-axis but in this study, a round rotor is also considered. Owing to rotor saliency that results in difference in d- and q-axis reactances, it is possible to obtain an output power even in the absence of rotor conductors. The two stator windings (named power and dc windings) are 90° displaced from each other, one supplying the load and the other connected to a d-c supply. The advantage of this generator is that output voltage can be kept constant under varying load and speed by adjusting the stator dc current; and the high frequency output voltage can produce smooth dc sources when rectified. The focus of this paper is to develop the load characteristics of this machine using the phase-variable approach, as against the conventional d-q theory and to show the load behaviour when different rotor-types and loads are impressed on the generator.

2.0 Energy conversion principle

A general feature of all ac generators is that output voltage production is possible if there is initial excitation either in the rotor or stator in the form of a dc source, a residual (or permanent) magnetism in the core or a pre-charged capacitor. If none of these initial conditions is present, excitation and subsequently, output voltage production will not be possible no matter the torque and speed of the prime mover. This implies that the concept 'self-excitation' is actually an adopted nomenclature to distinguish such generators from those that have a separate excitation standing alone, since voltage production must originate from somewhere (no matter how small) as discussed above.

Self-excitation using capacitors are undesirable since it can cause severe overheating, torque and machine speed fluctuations, and only a fraction of the current in the stator winding is utilized by the load as part of the current charges the capacitor. Of all the papers cited, only [9] used a stator excitation with dc voltage for energy conversion. The rest relied on either self-excitation and curtailed the stated disadvantages with circuits of varying complexities or used the well known rotating diodes embedded in the rotor assembly, each adding to the overall cost of the system. In [9] an already existing three-phase stator winding was employed for a single-phase power generation. Two of the phases were connected in series to make the power winding and the third phase was excited with a dc voltage. For voltage production in the main winding, there had to be some form of asymmetry in the rotor. Two possibilities of creating this asymmetry were considered in [9]: (a) rotor saliency, resulting in a reluctance generator and (b) rotor asymmetrical windings without saliency resulting in induction generator. The voltage regulation of the machine was not as good as that of the ordinary single-phase synchronous generators and the authors had to utilize a PI controlled voltage stabilizer relying on solid-state devices – another addition to cost.

3.0 Machine description

The structural arrangement of the machine under study is shown in Fig. 1. The stator power and dc exciting windings are evenly distributed in all the slots and laid 90^{o} out of phase resulting in a pure single-phase arrangement. Since there is no excitation on the rotor conductors, an induction generator results if there is no rotor saliency (rotor is round) and a reluctance generator if rotor saliency is present. The initial condition for possible excitation as discussed above is the dc excitation supply E_E connected to the exciting winding of the stator. A free-wheeling diode is connected across the dc winding as shown in Fig. 1 to reduce ripples of dc control current and enhance self-excitation. The presence of E initiates a standstill magnetic flux Φ_E which induces a current I_R on the rotor damper winding. The motion of the rotor at speed ω enables I_R to produce an ac flux Φ_R. This ac flux is made up of positive-sequence flux Φ_{RP} rotating at 2ω in space with the rotor and the negative-sequence flux Φ_{RN} which links the power winding and induces a voltage E_M of frequency 2ω on it. The direction of Φ_{RN} is against the direction of the rotor.

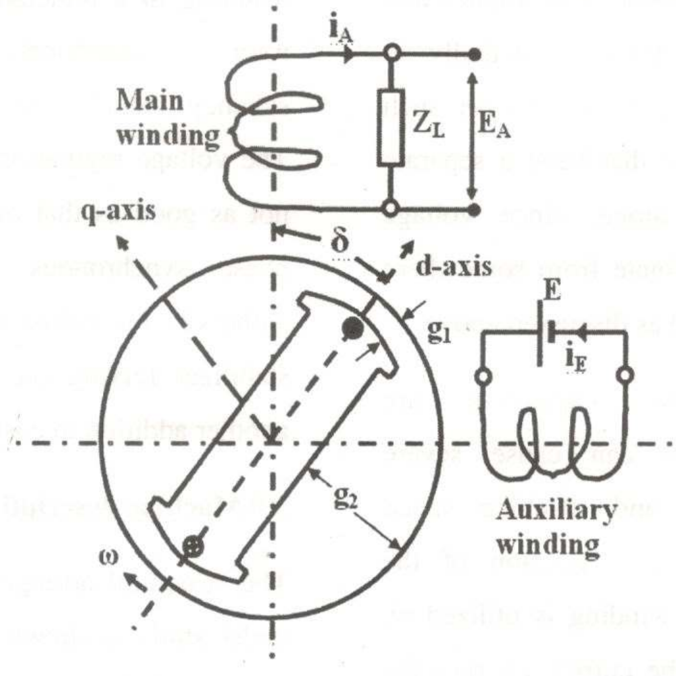


Figure 1: Machine connection diagram

4.0 The mathematical model

The main and auxiliary windings are not magnetically coupled since they are displaced by 90 degrees in space. However, each of the stator windings are coupled to the rotor winding. In the analysis that follows, the rotor shown in Fig. 1 is used. This is a more general approach since the equations can easily be adapted to that of a round rotor machine by setting $g_1 = g_2$. Neglecting saturation and core loss, the voltage induced in the main winding by a flux in the rotor is given by:

$$0=(R_A+R_L)i_A+\frac{d}{dt}((L_{AA}+L_L+L_{RA})i_A-L_{RA}i_R) \quad (1)$$

Similarly, the voltage across the rotor winding (which is zero) is given by:

$$E_R=0=R_Ri_R+\frac{d}{dt}((L_{RR}+L_{RE}+L_{AR})i_R-L_{RE}i_E-L_{RA}i_A) \quad (2)$$

$$E_E=R_Ei_E+\frac{d}{dt}((L_{EE}+L_{RE})i_E-L_{RE}i_R) \quad (3)$$

In the above equations, E_E is the excitation voltage, E_R the rotor voltage, i_E, the excitation current, i_A the load current and i_R the rotor current. L_{RR} represents the self-inductance of the rotor winding while L_{RA} and L_{RE} are the mutual inductance between the main winding and the rotor winding and between the exciting winding and the rotor winding respectively. Equations (1) to (3) suggest the equivalent circuit shown in Figure 2. Since a dc voltage is impressed upon the auxiliary stator winding, only the rotor and main winding equations will be of interest.

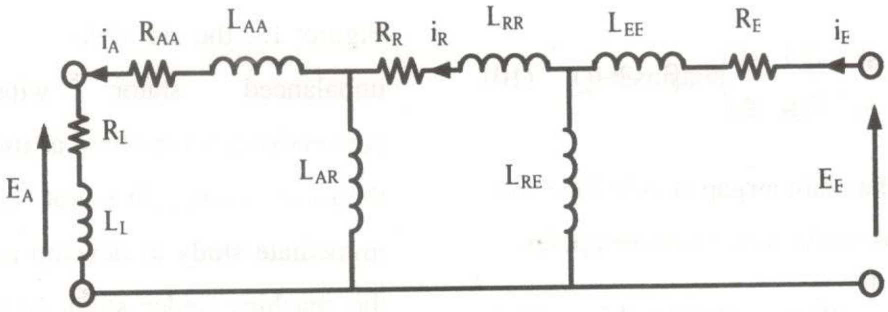


Figure 2: Equivalent circuit of the stator-excited generator

The inductances L_{AA}, L_{RA} and L_{RE} can be readily deduced with knowledge of the machine geometry and winding functions. By winding function theory [10], the inductances can be calculated from (4) to (6) below.

$$L_{AA}=\mu_oRL\int_0^{2\pi}g^{-1}(\phi,\theta_r)N_A(\phi)N_A(\phi)d\phi \quad (4)$$

$$L_{RA}=\mu_oRL\int_0^{2\pi}g^{-1}(\phi,\theta_r)N_R(\phi,\theta_r)N_A(\phi)d\phi \quad (5)$$

$$L_{RE}=\mu_oRL\int_0^{2\pi}g^{-1}(\phi,\theta_r)N_R(\phi,\theta_r)N_E(\phi)d\phi \quad (6)$$

where R is the radius of the airgap, L is the machine stack length, $N_A(\varphi,\theta_r)$, $N_R(\varphi)$, $N_E(\varphi)$ are the winding functions of the stator main windings, rotor winding, and the stator exciting windings, respectively. Their fundamental terms are defined here as:

$$N_A(\phi)=\frac{4N_a}{\pi}\cos\phi \quad (7)$$

$$N_E(\phi) = \frac{4N_e}{\pi}\cos\left(\phi - \frac{\pi}{2}\right) \quad (8)$$

and

$$N_R(\phi, \theta_r) = \frac{4N_r}{\pi}\cos 2(\phi - \theta_r) \quad (9)$$

where N_a, N_e and N_r are the number of turns of stator main, stator exciting and rotor windings respectively.

The expressions $g^{-1}(\varphi, \theta_r)$ appearing in (4) to (6) is the inverse airgap expressions representing the airgap geometry. It is given by:

$$g^{-1}(\phi,\theta_r) = \frac{1}{2}\left(\frac{1}{g_1} + \frac{1}{g_2}\right) - \frac{2}{\pi}\left(\frac{1}{g_1} - \frac{1}{g_2}\right)\sin\pi\beta\cos 2(\phi - \theta_r) \quad (10)$$

where g_1 is the main airgap at pole-face and g_2 is the depth of the space between poles.

Now if (7) to (10) are substituted into (4) to (6) and the integrations performed, we will obtain:

$$L_{AA} = L_{ls} + \frac{L_d + L_q}{2} + \frac{L_d - L_q}{2}\cos 2\theta_r, \quad (11)$$

where $L_d = L_q$ for round rotor machine, L_{ls} is the leakage inductance of the main winding.

$$L_{RA} = L_{ra}\cos\theta_r \quad (12)$$

$$L_{RE} = L_{re}\sin\theta_r \quad (13)$$

where

$$\theta_r = \omega t + \delta \quad (14)$$

$L_{AR} = L_{RA}$ and L_{rr}, L_{ra}, L_d, L_q and L_{re} are obviously constants that depend on number of turns and machine dimensions and depends on rotor-type. Their values for the machine under study are given in section 8.

Even though the inductances expressions L_{AA}, L_{AR} and L_{RE} have been derived, direct solution of the differential equations (1) and (2) can be challenging due to the dependence of the inductances on rotor position, θ_r. Additionally, three asymmetric conditions are identified in the machine of Figure 1:- the dc field in the stator, the unbalanced stator windings (not conventional three-phase as used by [9]) and the rotor saliency. It is not envisaged in the immediate study to develop a d-q model of the machine under study because of these asymmetries. Nevertheless, this does not preclude the study of the load characteristics of the generator in the steady-state. To obtain the steady-state equations for currents i_A (main winding) and i_R(rotor), equations (1) and (2) must be re-written as:

$$0 = (R_A + R_L)i_A + (L_{AA} + L_L + L_{RA})\frac{di_A}{dt} + i_A\frac{d(L_{AA} + L_L + L_{RA})}{dt} + i_R\frac{dL_{RA}}{dt} + L_{RA}\frac{di_R}{dt} \quad (15)$$

and

$$0=R_R i_R+\left(L_{AR}+L_{RR}+L_{RE}\right)\frac{di_R}{dt}+i_R\frac{d\left(L_{AR}+L_{RR}+L_{RE}\right)}{dt}-L_{RE}\frac{di_E}{dt}-i_E\frac{dL_{RE}}{dt}-L_{RA}\frac{di_A}{dt}-i_A\frac{dL_{RA}}{dt} \quad (16)$$

If (10) to (13) are substituted into (15) and (16), and the derivatives of the state variables i_A, i_E i_R are set to zero, they will yield two equations which can be algebraically solved for the main and rotor winding currents as:

$$i_A = I_{AM}\cos(2\theta_r - \varphi) \quad (17)$$

$$i_R = \frac{L_{re}}{L_{rr}} I_E \sin\theta_r - \frac{L_{ra}}{2L_{rr}} I_{AM}\left(\cos(3\theta_r-\varphi)+\cos(\theta_r-\varphi)\right) \quad (18)$$

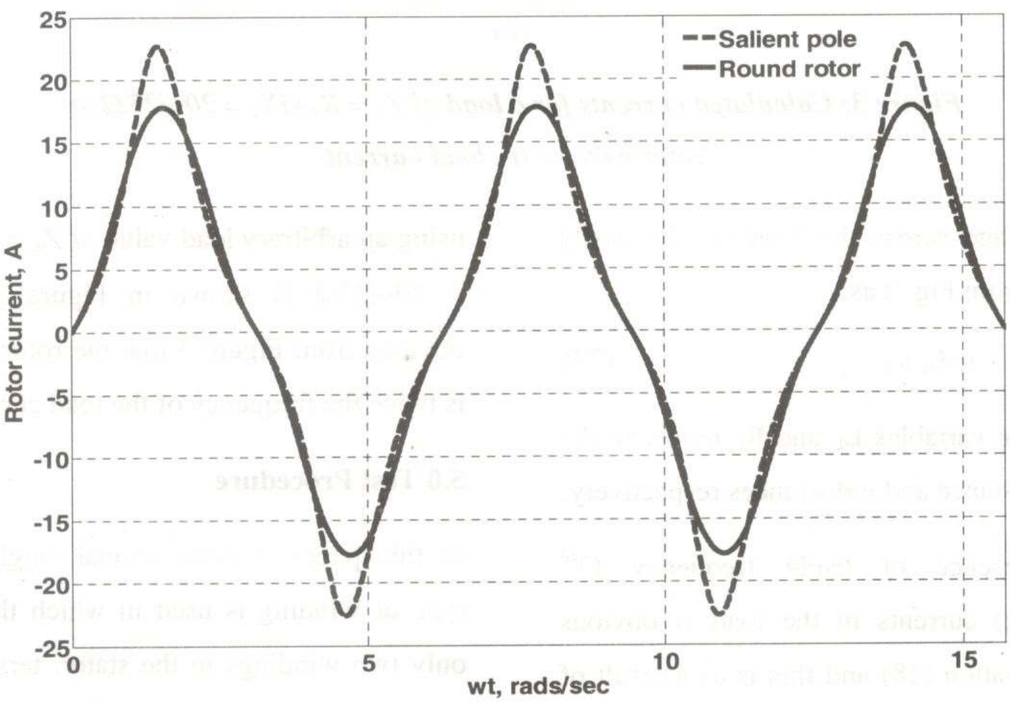
where:

$$I_{AM}=\frac{L_{ra}L_{re}I_E}{L_{rr}\sqrt{R_L^{\,2}+L_L^{\,2}+\left(L_Y+L_Z\cos2\theta_r\right)^2}} \quad (19)$$

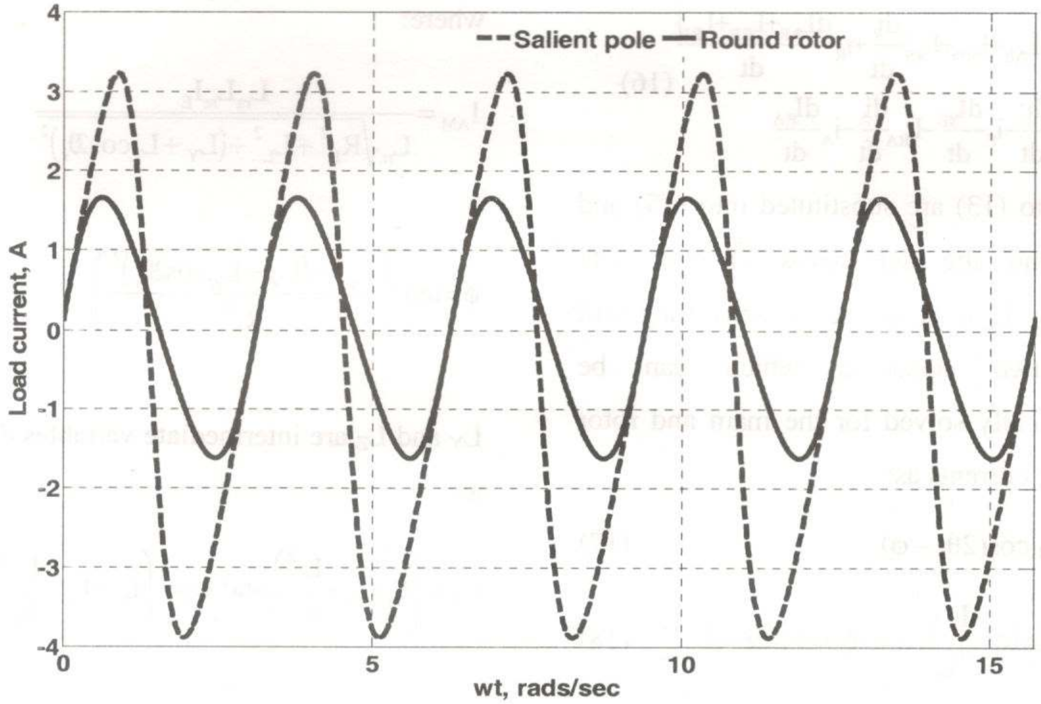
$$\phi=\tan^{-1}\left(\frac{L_L^{\,2}+\left(L_Y+L_Z\cos2\theta_r\right)^2}{R_L^{\,2}}\right) \quad (20)$$

L_Y and L_Z are intermediate variables defined as:

$$L_Y=\left(L_d+L_q-\frac{L_{ra}^{\,2}}{L_{rr}}\right) \text{ and } L_Z=\left(L_d-L_q-\frac{L_{ra}^{\,2}}{L_{rr}}\right) \quad (21)$$



(a)



(b)

Figure 3: Calculated currents for a load of $Z_L = R_L + jX_L = 20+j25\Omega$ (a) rotor current (b) load current.

The voltage across the load can be easily written from Fig. 2 as:

$$E_A = (R_L + j\omega L_L)i_A \qquad (22)$$

where the variables L_L and R_L represent the load resistance and inductances respectively.

The presence of triple frequency (3rd harmonic) currents in the load is obvious from equation (18) and this is as a result of negative-sequence revolving flux Φ_{RN}. The positive sequence flux Φ_{RP} is responsible for the current component with $\cos(\theta_r - \varphi)$. Plots of load current and rotor current for machine parameters shown in section (8) using an arbitrary load value of $Z_L = R_L+jX_L = 20+j25\Omega$ is shown in Figure 3. It is obvious from Figure 3 that the rotor current is twice the frequency of the load current.

5.0 Test Procedure

In this paper, a conventional single-phase type of winding is used in which there are only two windings in the stator, termed the power and the exciting windings as shown in Fig.1. The winding is of the same form as that of a conventional single-phase motor; the experimental motor was actually fabricated out of an existing single-phase induction motor. Performance of the round-rotor machine was achieved by applying a dc voltage to the second stator winding of the induction motor, and for a salient pole machine, another rotor was fabricated by removing parts of the original rotor (of another identical single-phase motor) by milling. In all the tests, the machine was driven by a dc motor at synchronous speed and the load connected to the main winding. Laboratory rheostats were used for resistive loads and fans were used as RL loads. The current in the exciting winding is maintained constant each time load in the main winding is adjusted.

6. Load characteristics

We proceed to the machine behaviour under different load conditions in the steady state. Two cases (R-L and pure resistance (R) loads) will be considered. The round rotor machine is idealized by setting $L_q = L_d$ and $g_2 = g_2$ in all the equations above.

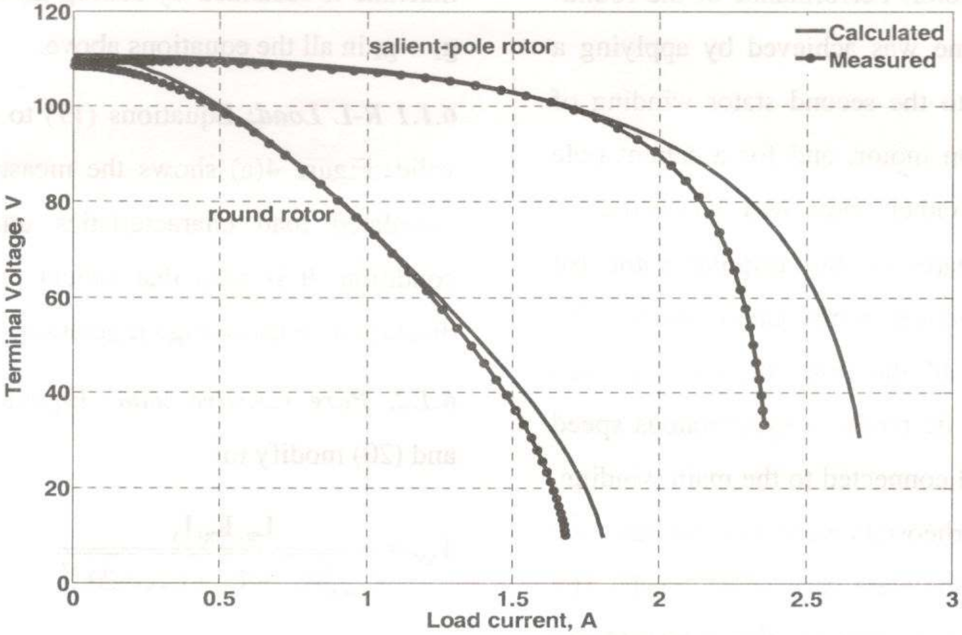
6.1.1 R-L Load: Equations (17) to (22) are valid. Figure 4(a) shows the measured and calculated load characteristics under this condition. It is seen that salient pole rotor displays a better voltage regulation.

6.1.2. Pure resistive load: Equations (19) and (20) modify to:

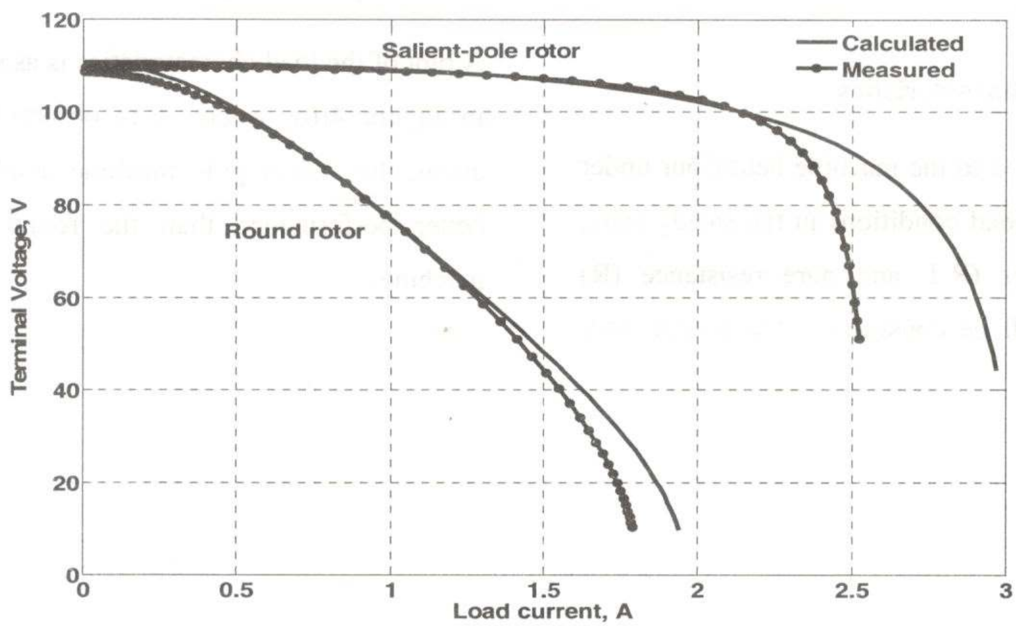
$$I_{AM} = \frac{L_{RA} L_{RE} I_A}{L_{RR}\sqrt{R_L^{\ 2} + (L_Y + L_Z \cos 2\theta_r)^2}} \tag{23}$$

$$\phi = \tan^{-1}\left(\frac{(L_Y + L_Z \cos 2\theta_r)^2}{R_L^{\ 2}}\right) \tag{24}$$

A plot of the load characteristics is as shown in Figure 4(b), where it is observed that again, the salient pole machine displays a better performance than the round rotor machine.



(a)



(b)

Figure 4: Measured and calculated load characteristics for a constant field current of I_e = 0.78A. (a) R-L load, (b) R load

7.0 Conclusion

An analysis of a pure single-phase stator excited generator has been presented. It is possible to generate an output voltage even in the absence of the rotor damper winding since the reluctance difference in L_d and L_q will bring about a magnetic flux which intersects the load winding. In other words, if $L_d = L_q$ (round rotor machine) the excitation voltage becomes the necessary condition for voltage generation.

10.0 Appendix: Machine parameters

$L_{ra} = 19mH$, $L_{re} = 19mH$, $L_{rr} = 7.5mH$, $L_d = 113mH$, $L_q = 29mH$, $R_A = 0.2\Omega$, $R_E = 0.3\Omega$, (R_A and R_B are the main and exciting winding resistances respectively and were neglected in the analysis).

Main winding voltage is at twice the frequency of rotor currents. This will be advantageous when the generator output is rectified and used to supply dc loads, since the ripples will be of a higher frequency and lower amplitude. Results shown in Figures 4(a) and (b) depict the superiority of the salient pole rotor to round rotor in terms of load characteristics.

9.0 References

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