

SOFTWARE DEVELOPED FOR POLYNOMIAL CAMS DESIGN AND ANALYSIS

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Abstract

The numerical design of cams that leads to their final manufacture has always been associated with much tedium and time consuming mathematical and graphical procedures. This is due to the number of lengthy equations and many design options involved in such design processes. Therefore, in this work, a significant contribution has been made in cam system design and analysis, by developing a software package and applying it to the analysis of polynomial cams. Two critical parameters associated with the design and manufactures of cam systems have been analyzed; the prime circle radius of the cam and face width of the follower.

Precisely, the effect of using the 3-4-5 and 8th- order polynomial functions on the two parameters while designing plate cams with reciprocating flat face followers has been analyzed in this work. Results show that these two parameters are directly influenced by the type of cam function used rather than the cam angular segments in the optimum design and final manufacture of cam system. In this work reliable software capable of performing extensive optimum design analyses when properly utilized has been developed.

**Key Words; Cams; Software; Analyses;
Optimum-Design**

NOTATION

- R_o : is Prime circle radius i.e. radius of the smallest circle drawn with center at the cam rotation axis and tangent to the pitch curve.
- F_{fw} : follower face width
- β : cam angle interval for motion segments
- ϵ : offset of the follower face
- θ : angle of cam rotation
- C : the instantaneous center of curvature.
- ρ : radius of curvature corresponding to the current contact point of cam and follower
- ρ_{min} : minimum radius of curvature
- $\{u, v\}$: coordinates of the contact point in a coordinate system attached to the cam
- s : distance of travel of the point of contact either side of the cam rotation center
- Y : displacement of follower with respect to θ
- Y' : first derivatives of displacement of follower with respect to θ
- Y'' : second derivatives of displacement of follower with respect to θ
- r : distance of the instantaneous center of curvature to the cam center
- L : maximum displacement of follower (lift)

1.0 Introduction

The impact of newly developed computer aided method, in form of CAD software and packages in the industrial and engineering world today cannot be over stated. As a follow up to this trend, the objectives of this work are; (i) to develop a software package for the design and simulation of plate cams with flat-face follower (ii) perform cam design analysis using different polynomial cam functions and derive inferential

conclusions on how they affect the final manufacture of such cam systems. The numerical and mathematical procedures of cam systems design are tedious, cumbersome and time involving. This work overcomes all associated tedium with cam profile design and simulation analysis for optimum design and final manufacture of such systems and in shorter time.

The synthesis and application of relevant design concepts and equations in the

development of software for selected options were based on works done by various researchers. Shigley and Uicker, (1980) expounded on the theory of cam mechanisms and dwelt much on different types of follower motions with emphasis on the roller and flat face follower cam mechanisms. This has led to the design and analysis using the flat-face follower as carried out in this work. Tesar and Mathew, (1976) were able to classify cam systems based on values of the dynamic factor (μ^d) and made recommendations for cam designers based on these values. Andrezej Oledzki, (1980) approached cam design from dynamic analysis of the mechanism.

Virgil Moires, (1965) was able to identify and reckon with all the forces acting on the plate cam in his analysis of force and torque on cam systems. Concepts and conclusions obtained from all the above-mentioned works contributed immensely to software algorithm developed and applied in this work.

Particularly, the present computer aided design method employed is an improvement on previous works done by Olaniyi, (1977) and Udoh, (2001). Other works that have

emanated from the present work include; the development of a profile simulation software for both trigonometric and polynomial cams (Simolowo and Bamiro, 2007); the development of an analysis-intensive software for improved cams systems design (Simolowo and Bamiro, 2008).

Also, presented in this paper are numerical detailed design procedures for cam systems based on the generalized fundamental functions presented by most researchers mentioned above. In addition, the result of analysis brings out valuable inferences on the optimum design of cam profiles and presents clearer pictures in the design trend for such systems using 3 – 4 -5 and 8th order polynomial motions.

2.0 DESIGN THEORY OF POLYNOMIAL CAMS

Although there are other basic cam motions that could be used in the numerical design and commercial production of cams generally, the Polynomial motion curves have been chosen in this design analysis. They are one of those motions that address high-speed applications such as obtains in automobiles.

2.1 The Polynomial Motion

The polynomial family of motion curves has their basic equation as presented in equation (1)

$$y = \left[C_0 \left(\frac{\theta}{\beta} \right)^3 + C_1 \left(\frac{\theta}{\beta} \right)^4 + C_2 \left(\frac{\theta}{\beta} \right)^5 \right] + \dots \quad (1)$$

Synthesizing a full-rise curve with boundary conditions

$$\theta = 0; \quad Y = 0; \quad Y' = 0; \quad Y'' = 0$$

$$\theta = \beta; \quad Y = L; \quad Y' = 0; \quad Y'' = 0$$

equation (1) becomes equation (2) with six unknown constants.

$$y = \left[C_0 + C_1 \left(\frac{\theta}{\beta} \right) + C_2 \left(\frac{\theta}{\beta} \right)^2 + C_3 \left(\frac{\theta}{\beta} \right)^3 + C_4 \left(\frac{\theta}{\beta} \right)^4 + C_5 \left(\frac{\theta}{\beta} \right)^5 \right] \quad (2A)$$

$$y' = \frac{1}{\beta} \left[C_1 + 2C_2 \left(\frac{\theta}{\beta} \right) + 3C_3 \left(\frac{\theta}{\beta} \right)^2 + 4C_4 \left(\frac{\theta}{\beta} \right)^3 + 5C_5 \left(\frac{\theta}{\beta} \right)^4 \right] \quad (2B)$$

$$y'' = \frac{1}{\beta^2} \left[2C_2 + 6C_3 \left(\frac{\theta}{\beta} \right) + 12C_4 \left(\frac{\theta}{\beta} \right)^2 + 20C_5 \left(\frac{\theta}{\beta} \right)^3 \right] \quad (2C)$$

Solving these equations simultaneously gives the results to the six constants

$$C_0 = 0; \quad C_1 = 0; \quad C_2 = 0; \quad C_3 = 10L$$

$$C_4 = -15L \quad C_5 = 6L$$

Substituting these constants in equation (2a) and applying a parallel procedure, equations

(3) and (4) are obtained for the 3-4-5 polynomial equations. Similar boundary conditions are also chosen to obtain the 8th order polynomial equations presented in equations (5) and (6).

(RISE):

$$y = h \left[10 \left(\frac{\theta}{\beta} \right)^3 - 15 \left(\frac{\theta}{\beta} \right)^4 + 6 \left(\frac{\theta}{\beta} \right)^5 \right]$$

$$y' = \frac{h}{\beta} \left[30 \left(\frac{\theta}{\beta} \right)^2 - 60 \left(\frac{\theta}{\beta} \right)^3 + 30 \left(\frac{\theta}{\beta} \right)^4 \right] \dots\dots\dots (3)$$

$$y'' = \frac{h}{\beta^2} \left[60 \left(\frac{\theta}{\beta} \right) - 180 \left(\frac{\theta}{\beta} \right)^2 + 120 \left(\frac{\theta}{\beta} \right)^3 \right]$$

Return:

$$y = h \left[1 - 10 \left(\frac{\theta}{\beta} \right)^3 + 15 \left(\frac{\theta}{\beta} \right)^4 - 6 \left(\frac{\theta}{\beta} \right)^5 \right]$$

$$y' = -\frac{h}{\beta} \left[30 \left(\frac{\theta}{\beta} \right)^2 - 60 \left(\frac{\theta}{\beta} \right)^3 + 30 \left(\frac{\theta}{\beta} \right)^4 \right] \dots\dots (4)$$

$$y'' = -\frac{h}{\beta^2} \left[60 \left(\frac{\theta}{\beta} \right) - 180 \left(\frac{\theta}{\beta} \right)^2 + 120 \left(\frac{\theta}{\beta} \right)^3 \right]$$

Rise

$$y = h \left[6.0975 \left(\frac{\theta}{\beta} \right)^3 - 20.7804 \left(\frac{\theta}{\beta} \right)^5 + 26.7315 \left(\frac{\theta}{\beta} \right)^6 - 13.6096 \left(\frac{\theta}{\beta} \right)^7 + 2.5609 \left(\frac{\theta}{\beta} \right)^8 \right]$$

$$y' = \frac{h}{\beta} \left[18.2926 \left(\frac{\theta}{\beta} \right)^2 - 103.902 \left(\frac{\theta}{\beta} \right)^4 + 160.389 \left(\frac{\theta}{\beta} \right)^5 - 95.2675 \left(\frac{\theta}{\beta} \right)^6 + 20.4876 \left(\frac{\theta}{\beta} \right)^7 \right] \dots\dots (5)$$

$$y'' = \frac{h}{\beta^2} \left[36.5853 \left(\frac{\theta}{\beta} \right) - 415.608 \left(\frac{\theta}{\beta} \right)^3 + 801.946 \left(\frac{\theta}{\beta} \right)^4 - 571.605 \left(\frac{\theta}{\beta} \right)^5 + 143.4132 \left(\frac{\theta}{\beta} \right)^6 \right]$$

Return:

$$y = h \left[1 - 2.6341 \left(\frac{\theta}{\beta} \right)^2 + 2.7805 \left(\frac{\theta}{\beta} \right)^5 + 3.1706 \left(\frac{\theta}{\beta} \right)^6 - 6.8779 \left(\frac{\theta}{\beta} \right)^7 + 2.5609 \left(\frac{\theta}{\beta} \right)^8 \right]$$

$$y' = \frac{h}{\beta} \left[5.2683 \left(\frac{\theta}{\beta} \right) - 13.9027 \left(\frac{\theta}{\beta} \right)^4 - 19.0236 \left(\frac{\theta}{\beta} \right)^5 + 48.1456 \left(\frac{\theta}{\beta} \right)^6 - 20.4876 \left(\frac{\theta}{\beta} \right)^7 \right] \dots (6)$$

$$y'' = \frac{h}{\beta^2} \left[+5.2683055.6110 \left(\frac{\theta}{\beta} \right)^3 - 95.1180 \left(\frac{\theta}{\beta} \right)^4 + 288.8739 \left(\frac{\theta}{\beta} \right)^5 - 143.4132 \left(\frac{\theta}{\beta} \right)^6 \right]$$

2.2 Cam/Follower Design Parameters

The vectors present at the point of contact between a plate cam and a flat-face follower, typical of reciprocating internal combustion engines, as considered in this work are shown in figure 1. Using complex polar notations (Shigley and Uicker, 1980) in figure 1, the applicable cam profile design equations were obtained. The prime circle radius of the cam geometry is determined by equation (7)

$$R_o > (p_{min} - Y - Y'') \max \dots (7)$$

The width of the follower face is obtained by equation (8)

$$F_w > Y'(\max) - Y'(\min) \dots (8)$$

The coordinates of the cam are also obtained using the equations (9a) and (9b)

$$u = (R_o + Y) \sin \theta + Y' \cos (\theta) \dots (9a)$$

$$v = (R_o + Y) \cos \theta - Y' \sin (\theta) \dots (9b)$$

The overall software and necessary compiler files developed this work has a considerable memory seize. A large number of codes of C/C++ high level language were written in the development of the cam design software.

Software Features

Two of the features among other common features that enhance proper design analysis include.

- (i) The ability to resize very large profiles and thereby make the scaled profile visible on the computer interface.
- (ii) Distinct graphics demarcation of different motion segments on all generated cam profile.

These features make the developed software peculiar. Easy correction and re-design of

the affected cam profile segments can easily identified. They also make the developed software more extensive in design analyses, synthesis and stimulation based on its ability to give precise segment defects such as profile discontinuities (cursps) and carry out faster and more accurate re-design procedures based on graphics distinction for each segment. Screen shot 1 shows the graphics distinctions on a generated profile while screen shot 2 presents the graphics distinction and profile resizing features for a generated cam profile.

3.2 Functions of Software Algorithms.

The developed software is based on two categories of algorithms, namely: (a) Arithmetic algorithm. (b) Visual display algorithm. Figure 2 depicts the types of algorithms employed in the development of the software.

Arithmetic algorithm

The arithmetic algorithm comprises (i) Design-analysis, calculations and logic (F1-F4). (ii) Design procedure and Methodology (F5-F7). Making references to fig. 2, the functions of algorithms for design-analysis, calculation and logic in the software are herein described. **F1:-** Calculations and

plotting of cam coordinate u, v for all degree intervals using all follower and cam motion equations. **F2:-** Calculations of displacements and its derivatives in the corresponding displacement-time profile for every cam profile design. **F3:-** Determination of minimum values of velocity, acceleration and displacement values. **F4:-** Calculation of output design parameters such as Face width (F_w), and prime circle radius (R_o). The algorithm for design procedure and methodology are responsible for the following: **F5:-** Sequence of calculations to be done. **F6:-** The transfer of result from one part of the program to other part of the program to be used as input for further calculations. **F7:-** The control of overall program flow as software is operated by the user.

Visual Display Algorithm

The visual algorithms depicted in fig. 2 are responsible for the following; **F8:-** All graphics display on the screen. **F9:-** Screen selection. **F10:-** Making the program a self operating program outside complier environment. Four basic modes as described below were used in the visual display algorithm for the development of the software. They are the Text mode, Graphics

mode, Text in graphics modes, and the ANSI cursor control.

MDI:- Text Mode: This mode was used for all words and texts in the whole program and also appearing on the screen. **MD:** This mode was used for the following: (i) All the color designs appearing on various screen on the VDU. (ii) Displaying strings of text at the cursor position or at the 'x' 'y' position on the screen. **MD4:-** ANSI cursor control:

This cursor system is used to directly control the position of cursor on the screen. This type of system generally comes with most IBM PC family of computers as well as most MS-DOS compatibles. However, since the system cursor control does not exist on the read-only memory (ROM) of the computers, software algorithms have to be written in connection with the existing ANSI files located in the operating system so as to make the cursor control operative.

3.3 Description of Algorithm for Cam Design

A step-by-step description of the computer algorithm involved in a typical profile design is presented in this section using a sample case study design with the following criteria: (i) plate cam; (ii) reciprocating flat-face follower; (iii) eight- order Polynomial;

(iv) Dwell: - 0° - 220° ; Rise: - 220° - 280° ; Dwell: - 280° - 300° Return: - 300° - 360° ;
 (v) Maximum Lift: - 9.57mm.

(1) Dwell (0° - 220°) or (0 - $11\pi/9$ radians)

Since at this stage, the follower dwells for $11\pi/9$ radians, i.e. for $0^\circ < \theta < 220^\circ$, $Y=0$; thus, $Y = Y' = Y'' = 0$.

(2) Rise (220° - 280°) or ($11\pi/9$ - $14\pi/9$ radians)

The values of Y , Y' and Y'' are generated by the computer algorithm using the rise equations (5) for 8^{th} order polynomial equations. Where, L is given as 9.57mm: $\beta = 100^\circ$

(3) Dwell (280° - 300°) or ($14\pi/9$ - $15\pi/9$ radians)

At this stage the follower dwells for $\pi/9$ radians, since for $280^\circ < \theta < 300^\circ$, $Y=L$
 $Y' = Y'' = 0$

(4) Return (300° - 360°) or ($15\pi/9$ - 2π radians)

The values of Y , Y' , and Y'' are also generated using the 8^{th} order polynomial return equation (6) for $300^\circ < \theta < 360^\circ$. $\beta = 60^\circ$ (the cam angle interval for the return-motion).

(5) The values of Y , Y' and Y'' are generated in stages 1-4. Shown in figure 2 are the motion profiles.

(6) From equation (7) and (8), R_o and F_w are obtained respectively. Y at Y'' min is also obtained from the values generated in stages 2 and 4;

(7) Cam geometry and motion coordinates of profile being designed are generated using equations 9a and 9b for the four stages of motions considered (D-R-D-R). The cam profile is obtained by plotting the generated values as shown in figure 3

The summary of the design output parameters, the follower motion profiles and cam profile as obtained for the sample computer-aided design described in stages 1-7 above is presented below in Table 1 and figure 3 respectively.

3.4 Software Application on Optimum Design Analysis

The developed software was used to carry out optimum design analysis from a total of 29 stimulations (Simolowo, 2004).

Ordinarily each of these simulations takes hours to accomplish using the conventional numerical procedure described in section 2.

Therefore, the development of a software procedure carried out in this work has enabled faster and more extensive researches to be carried out in this area. The four different aspects of simulations studied in this parametric analysis for optimum design are:

Case (I): Obtaining the values of prime circle radius of the cam (R_o) for a combination of cam angular interval using the 3-4-5 and 8th order polynomial functions for a Rise-Return (R-R) follower motion.

Case (II): Obtaining the values of follower face width (F_w) for a combination of cam angular intervals using the 3-4-5 and 8th order polynomial functions for a Rise-Return (R-R) Follower motion.

Case (III): Obtaining the values of prime circle radius of the cam (R_o) for a combination of cam angular intervals using the 3-4-5 and 8th polynomial functions for a Rise-Return (R-D-R) Follower motion.

Case (IV): Obtaining the values of face width (F_w) for a combination of cam angular intervals using the 3-4-5 and 8th order polynomial functions for a Rise-Dwell-Return (R-D-R) Follower motion.

For the four cases, different cam angle combinations were kept constant for each of the follower motion, while the polynomial cam motions were varied. The effects of the polynomial functions were then observed for the design output parameters, namely, prime circle radius of cam (R_O) and follower face width (F_w). It should be noted that more optimum, design analysis could also be extended to other cam functions.

4.0 RESULTS AND DISCUSSION

Consideration cases (I) and (IV); these sets of analyses show that the values of prime circle radius (R_O) obtained using the 8th order polynomial are generally higher for more cam angular combinations than those of the 3-4-5 polynomials as shown in figure 4 and 6. Thus lesser materials would be needed generally in the manufacture of the cam units when 3-4-5 polynomial is used due to the smaller values of (R_O) More so, in a situation where space available is a critical design criterion, 3-4-5 polynomial cams would be a better choice out of the two.

Shown in figure 5 and 7 are the simulation results for cases (II) and (III) respectively.

Herein it is also observed as in cases (I) and (IV) that the follower faces widths (F_w) are

particularly influenced by the type of cam functioned used rather the follower motion segments considered. This is because in figure 5 the values of F_w using 8th order polynomials are distinctively lower than those of 3-4-5 polynomials irrespective of either the D-R-D or R-R follower motion segments.

When the 3-4-5 polynomial is used, more materials will be required in the final manufacture of the flat face follower while lesser materials will be needed for the cam manufacture. Depending on the design requirements, a proper combination of these polynomials motions and others in cam design will give the most reliable and economical cam system when the results given herein are put into consideration and similar analysis also extended to other cam motions.

5.0 CONCLUSION

The objective of developing software with enhancing features for extensive for cam design and analysis has been achieved in this work. Also important inferences have been made from the application of the software in polynomial cam design as case study. The computer aided simulation package is highly user friendly, requiring

basically the knowledge of operating the computer by the cam designer. The design out-put parameters are much more reliable than those obtained numerically and can be used for all practical purposes such as the manufacturing of the emerging cam system.

The simulation analysis of the polynomial cams performed in this work opens up to further analysis in cam design and manufacturing using other cam motions

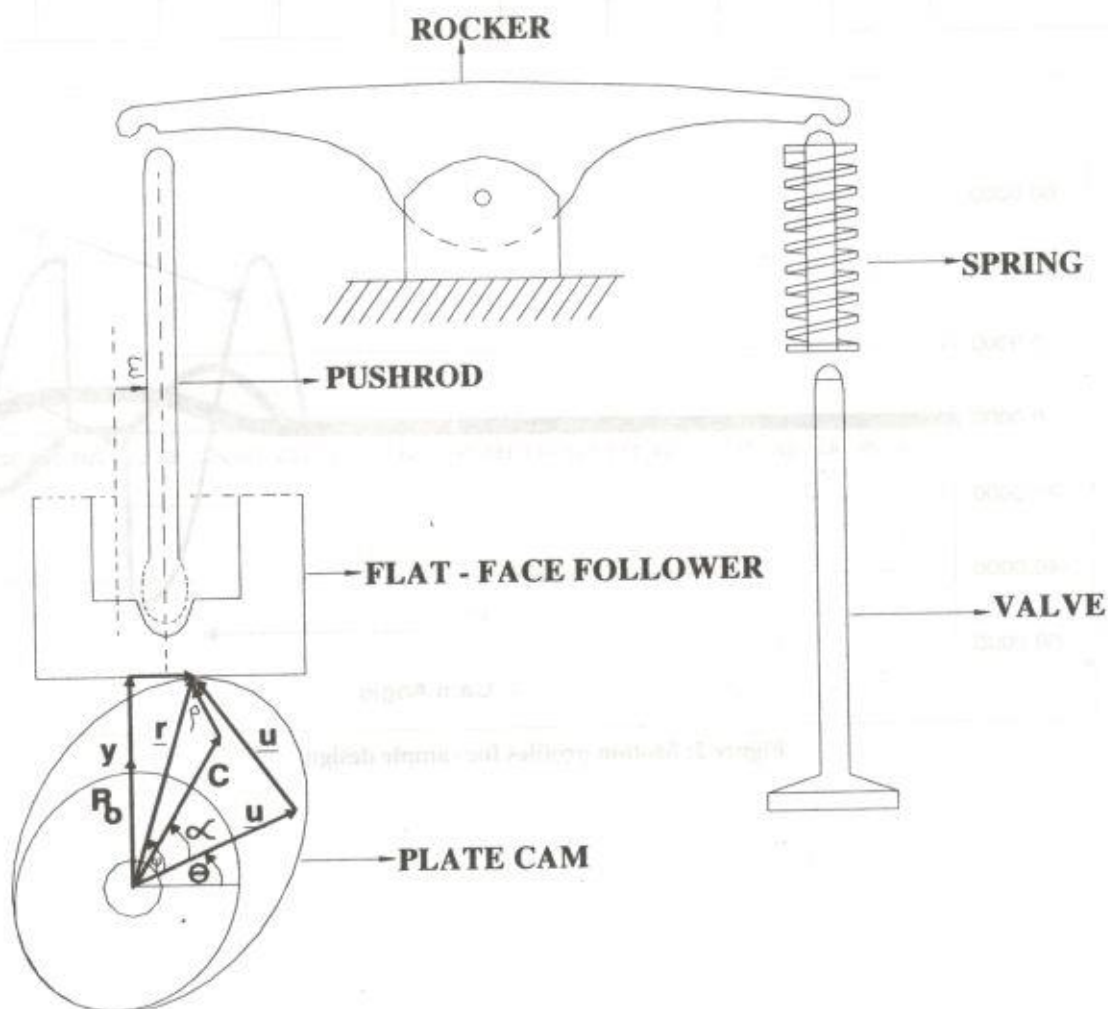


Fig. 1: Cam and Follower for Reciprocating Internal Combustion Engine at Point of Contact

Table 1: Summary of design output parameters for sample design

CASE NO	INPUT PARAMETERS				OUTPUT PARAMETERS					
	FOLLOWER MOTION				R_o	F_w	Y'_{MIN}	Y'_{MAX}	Y''_{MIN}	Y''_{MAX}
	Dwell D(deg.)	Rise R deg.)	Dwell D(deg.)	Return R(deg.)	mm	mm	mm/rad	mm/rad	mm/rad ²	mm/rad ²
1	220°	280°	300°	360°	46.00	34.00	-16.11	16.11	45.92	46.00

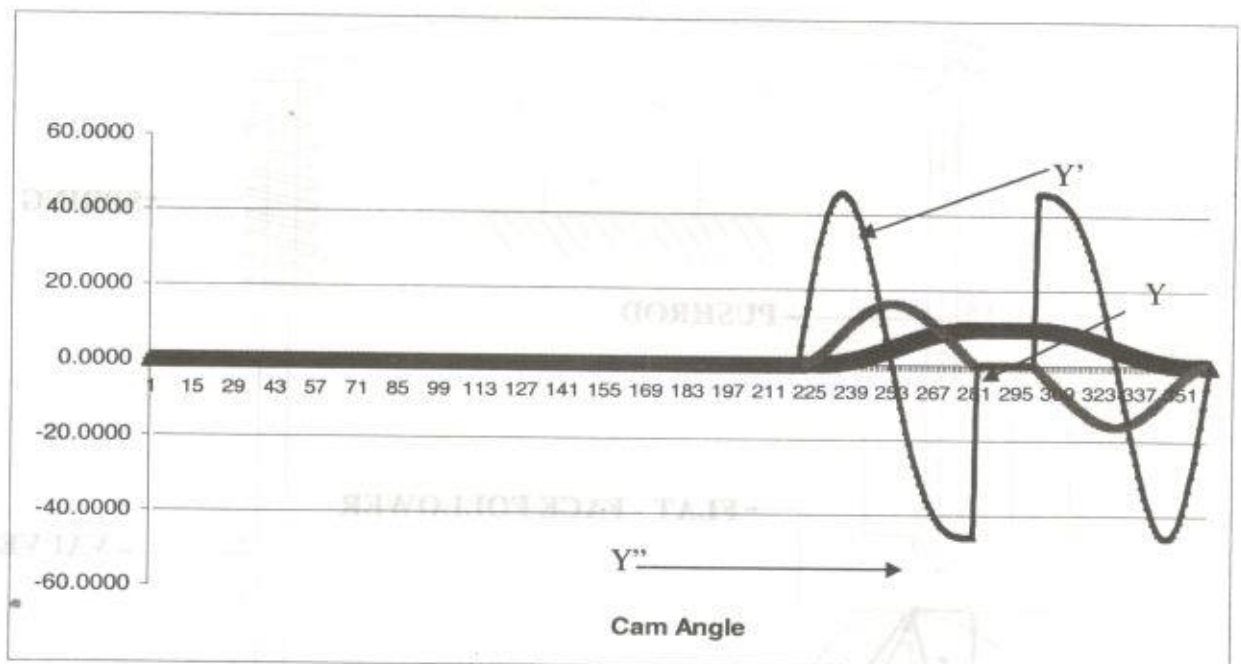
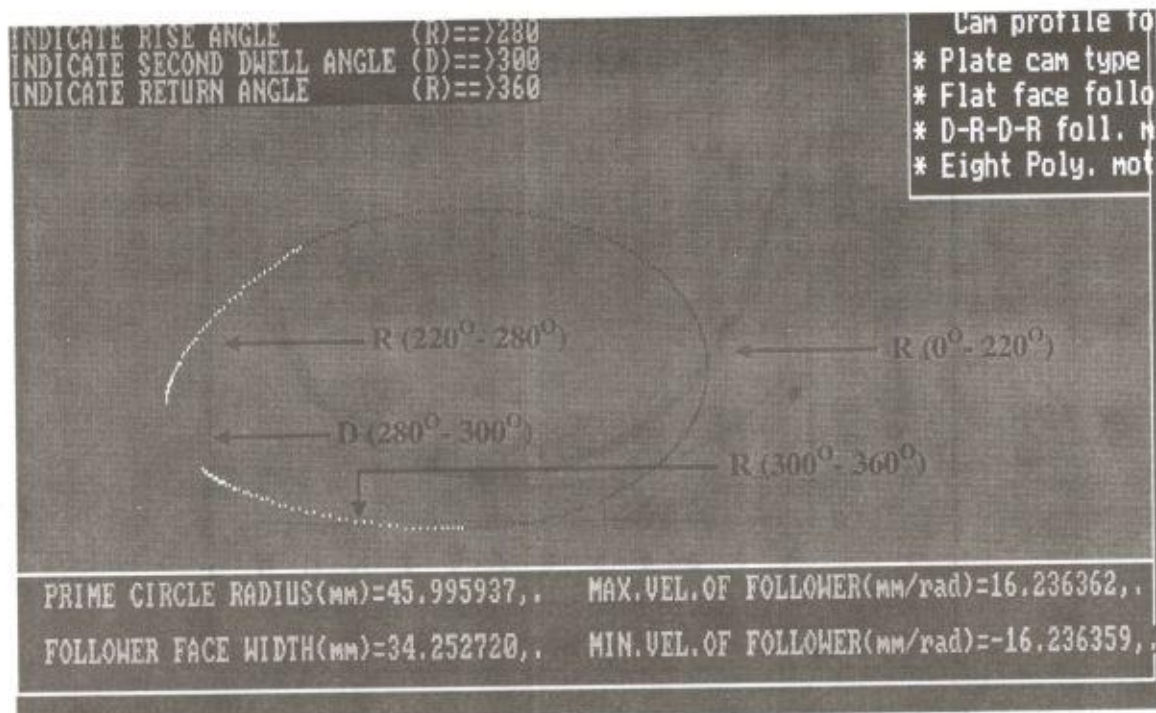
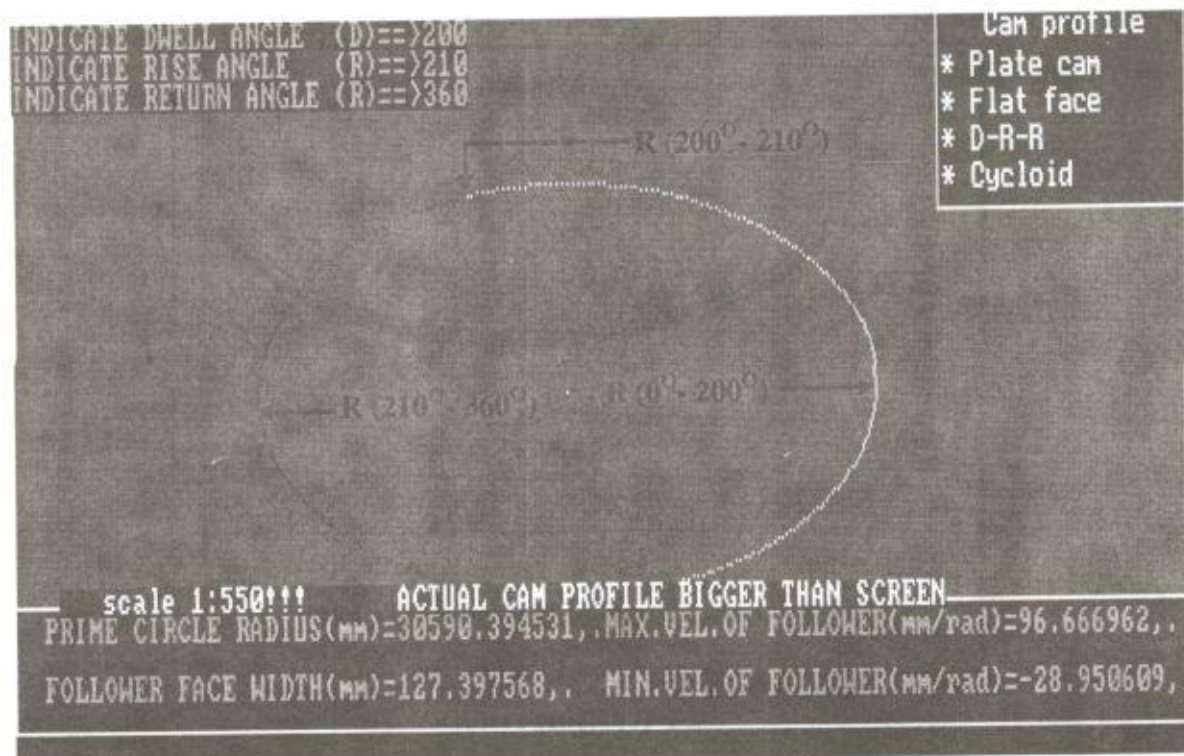


Figure 2: Motion profiles for sample design



Screen shot1: Software interface showing sample cam profile with distinct graphical demarcations



Screen shot2: - Software interface showing a re-scaled sample cam profile

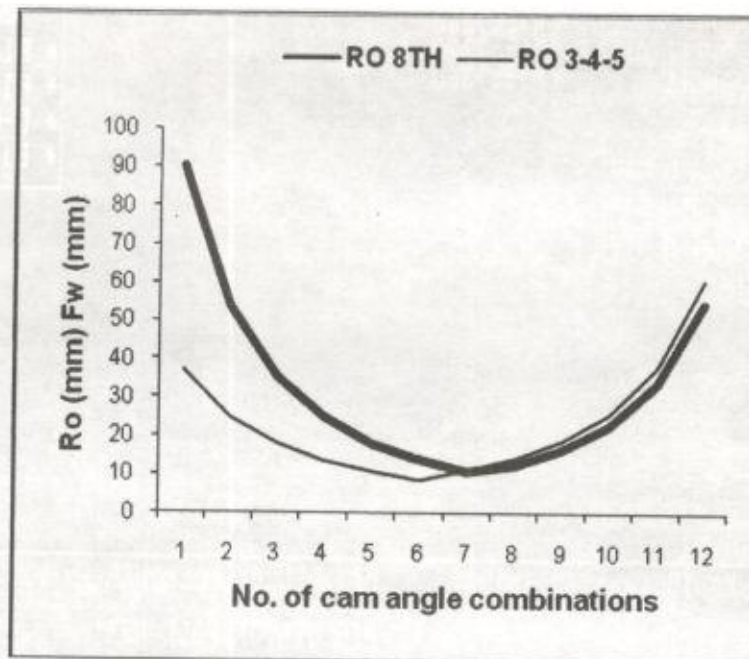


Fig 4: - Simulated values of R_o for 8th order and 3-4-5 Polynomials for R-D-R follower Profile

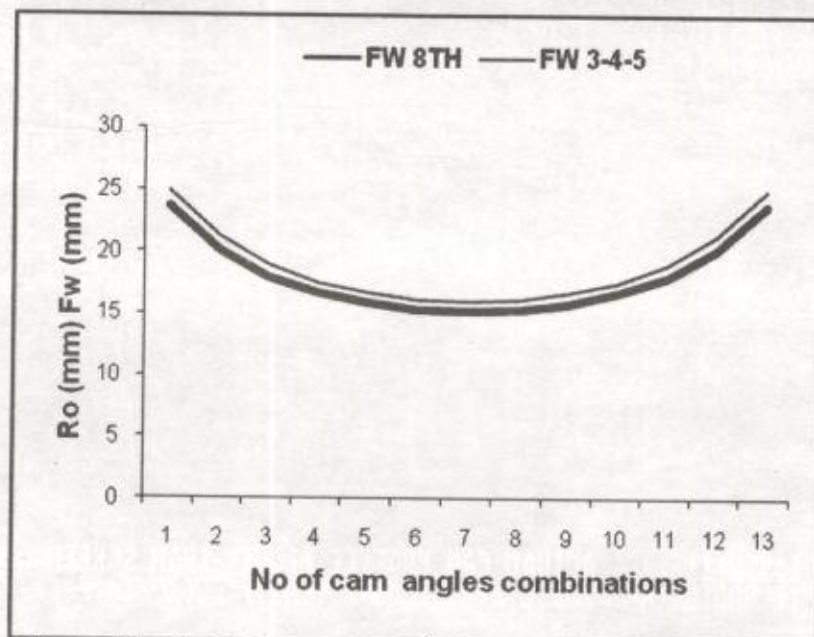


Fig 5: - Simulated values of F_w for 8th order and 3-4-5 Polynomials for R-D-R follower Profile

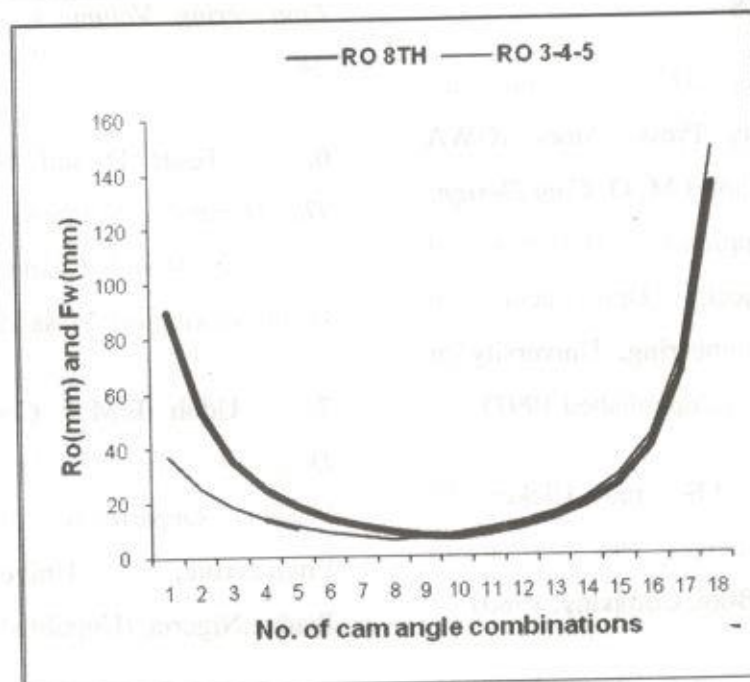


Fig. 6: Simulated values of R_o for 8th order and 3-4-5 Polynomials for R-R follower Profile

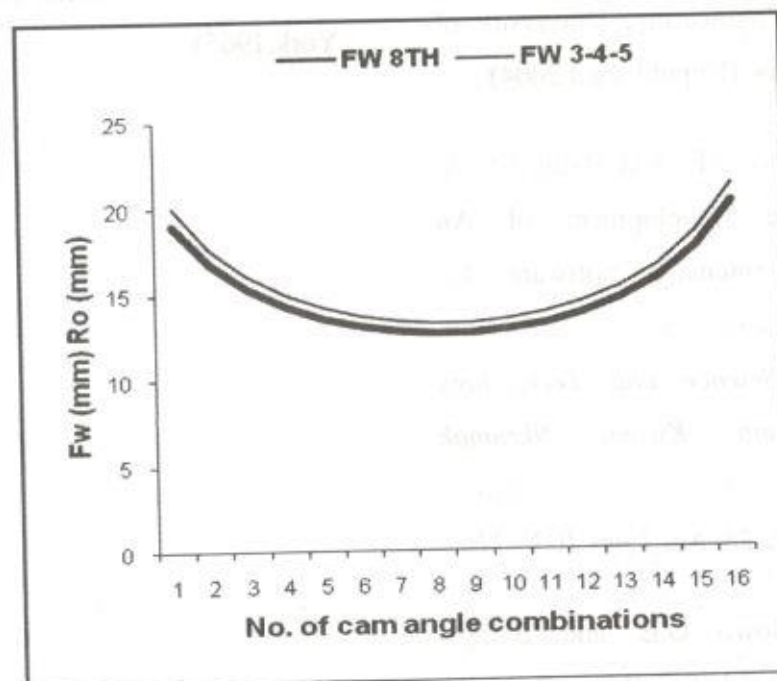


Fig. 7: Simulated values of F_w for 8th order and 3-4-5 Polynomials for R-R follower Profile

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