VARIATION OF THE ACCURACY OF MANNING'S FORMULA WITH PIPE EQUIVALENT ROUGHNESS HEIGHT

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ABSTRACT

The accuracy of simplified friction loss relationships such as Manning's formula depends on the pipe material but the relationship has so far not been obtained. This paper develops and then presents in tabular and graphical form, the variation of the accuracy of head loss, pipe diameter and discharge estimates obtained by the Manning's formula, with pipe equivalent roughness height. Thus once engineers in practice, have an idea of the type of pipe material to be used, they will be able to determine if accuracy of the Manning's formula will be suitable for the application they have on hand.

The results show that within the range of flow of practical interest in the water supply industry, accuracy of head losses, diameters and discharges estimated with the Manning's formula improves with increasing roughness. They can be estimated within factors of 3.71, 1.28 and 1.93 respectively for equivalent roughness height of 1 μm. These factors decline to 1.22, 1.04 and 1.10 respectively for equivalent roughness height of 1 cm.

1. INTRODUCTION

It can be shown from Darcy's friction loss relationship for flow of fluids in pipes with friction factor obtained from Colebrook's formula [1], that

$$Q = -6.957 D^{5/2}\sqrt{S}\,\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{0.567\nu}{\sqrt{D^3 S}}\right) \tag{1}$$

where ε is the pipe equivalent roughness height, D is pipe diameter, S is the friction slope i.e. head loss per unit pipe length, Q is pipe discharge and ν is the fluid kinematic viscosity. Equation 1 typically needs to be solved for either S, Q or D and obviously, only Q of these can be found explicitly.

Determination of S or D requires iteration. This is why simpler equations that can be solved explicitly for any of these were developed such as Hazen-Williams' Formula. Another one which is the focus of this paper is Manning's formula which can be written as

$$Q = 0.3117\frac{D^{8/3}}{n}S^{1/2} \tag{2}$$

where n is Manning's coefficient. However, previous studies [1,2] relating to the accuracy of the Manning's formula indicate that this depends on the pipe material, but the degree of accuracy that could be expected from Manning's formula for different pipe equivalent roughness heights (hence different pipe materials) have so far not been explored.

For instance, a relevant study related to this issue by Khatibi et al. [3] was concerned mainly with estimation of friction factors for flow in nearly flat tidal channels. Although, Aiyesimoju [4] investigated the variation of the accuracy of a simplified friction loss formula with pipe roughness, the study was limited to the Hazen-Williams formula.

The aim of this paper is to examine the variation of the accuracy of head loss, pipe diameter and discharge estimates from Manning's formula, with pipe equivalent roughness height. The accuracy of a particular estimate is defined as the factor of the estimate relative to the estimate from Colebrook-White formula. A factor of accuracy of unity implies a perfect estimate and the larger the factor, the worse the estimate. The factor obviously cannot be less than unity.

For this study, there must be a clearly defined range of flow of interest which here will be as is typical in the water supply practice, and are as follows [1, 2, 5, 6]:

1. Pipe equivalent roughness height ε between 1 μm (less than the value for drawn tubing) and 1 cm (corresponding to riveted steel).

2. Pipe diameter D between 20mm and 2000mm.

3. Pipe relative roughness ε/D up to 0.05

4. Flow Reynold's Number R above 4000 (lower limit for turbulent pipe flow)

5. Maximum Reynolds's Number in a pipe of diameter D is $R_{\max} = V_{\max}*D/\nu_{\min}$. Maximum velocity $V_{\max}$ of water which is the fluid of interest will be limited to 10m/s here (usually less than 3m/s in practice). Minimum kinematic viscosity $\nu_{\min}$ of water is a little above 3×10^{-7} m^2/s (at 100°C). These justify limiting $R_{\max}$ to 10^8D/3.

2. ESTIMATION OF ACCURACY

Equivalence of Manning's formula and Darcy's formula [1], implies

$$n = \frac{\sqrt{D^{1/3} f}}{11.16} \quad (3)$$

The relative roughness ε/D and Reynolds Number R determine the friction factor f and thus for a given roughness height ε, $D^{1/3}f$ depends on D and R. Thus from Eqn. 3 the value of n that will give equivalent results to Darcy's formula will depend only on D and R.

For a given pipe material, and hence a given ε, the problem here is to find that n that will perform best over the entire practical range of flow of interest.

Let the flow condition yielding minimum value of Manning's roughness coefficient

$$n_{min} = \frac{\sqrt{(D^{1/3} f)_{min}}}{11.16} \quad (4)$$

be termed as minimum extreme flow condition and that yielding its maximum value

$$n_{max} = \frac{\sqrt{(D^{1/3} f)_{max}}}{11.16} \quad (5)$$

be termed as maximum extreme flow condition.

For a given ε, a single value of n must be selected and this will obviously be between n_{min} and n_{max}. Let the proposed value n for the given ε be such that the ideal roughness coefficient for the minimum extreme flow condition (n_{min}) is estimated within a factor of F_1 and the value for the maximum extreme flow condition (n_{max}) is estimated within a factor of F_2, then

$$F_1 = \frac{n}{n_{min}} \quad (6)$$

$$F_2 = \frac{n_{max}}{n} \quad (7)$$

It should be noted that F_1 may be larger or smaller than F_2 depending on the proposed n. If the proposed value n for a given ε, is accurate within a factor of F_{max} over the entire range of flow conditions of interest, the worst accuracy will occur at one or both of the extreme flow conditions which implies

$$F_{max} = (F_1, F_2)_{max} \quad (8)$$

From Eqns. (6) and (7), as n increases, F_1 increases and F_2 decreases. F_{max} which is the larger of the two values will thus be minimum when F_1 equals F_2, The lower F_{max} is, the more accurate n is, which implies that the most accurate is obtained by equating Eqns. (6) and (7) which gives

$$n = \sqrt{n_{min} n_{max}} \quad (9)$$

Substituting Eqns. (4) and (5) in Eqn. (9) yields the best value of n for the given ε as

$$n = \frac{\left((D^{1/3} f)_{\min} (D^{1/3} f)_{\max}\right)^{1/4}}{11.16} \quad (10)$$

Since as stated earlier, for a given ε, $D^{1/3}f$ depends only on D and R, then if R is also given, as D increases, ε/D decreases which implies f decreases, thus making it unclear as to how a change in D affects $D^{1/3}f$. But for a given ε and D, $D^{1/3}f$ is maximized when R is minimum and vice versa. Thus for each given ε, to obtain $(D^{1/3}f)_{\max}$, it is sufficient to try different values of D and keep R at the minimum allowed (i.e. 4000) and to obtain $(D^{1/3}f)_{\min}$, it is sufficient to try different values of D and keep R at the maximum allowed (i.e. $10^8D/3$). Based on the above explanation, Aiyesimoju [1] obtained values of n using Eqn. 10 as well as the extreme flow situations yielding them as in Table 1, for various values of ε.

The accuracy of Manning's roughness coefficient for a particular ε can be measured by the factor of accuracy F defined as the worst ratio over the entire range of practical interest as stated earlier, of the actual value of n (from Eqn. 10) to the minimum ideal value $n_{\min}$ for that ε, i.e.

$$F = \frac{n}{n_{\min}} \quad (11)$$

For the given pipe equivalent roughness, substitution of Equations 4 and 5 in Equation 11 gives factor of accuracy F as

$$F = \sqrt{\frac{n_{\max}}{n_{\min}}} = \left(\frac{(D^{1/3} f)_{\max}}{(D^{1/3} f)_{\min}}\right)^{1/4} \quad (12)$$

Also, since from Eqn. 2,

$$h \propto n^2$$

then the factor F_h of Manning's head loss estimate (compared to the ideal i.e. Colebrook-White) is

$$F_h = F^2 \quad (13)$$

Again from Eqn. 2,

$$D \propto n^{3/8}$$

which implies that the factor F_D of Manning's diameter estimate will be

$$F_D = F^{3/8} \quad (14)$$

and the factor F_Q of Manning's discharge estimate will be

$$F_Q = F^{|-1|} = F \quad (15)$$

3. DISCUSSION

For any flow condition, there is an ideal Manning's coefficient which ensures equivalence between Manning's formula and Colebrook's formula. Any proposed estimate for n for a given roughness height ε, will be accurate within a factor which is the ratio of the larger of the actual n and the ideal n to the smaller of the two. The estimation of factor of accuracy F (for the Manning's coefficient) was obtained here as in Eqn.12. This was then used to obtain the variation F of the with pipe equivalent roughness height ε (see Table 2) assuming Manning's coefficient n for various ε obtained by Aiyesimoju [1].

Furthermore, the factors F_h, F_Q, and F_D within which Manning's formula can be expected to estimate head loss, pipe diameter and discharge respectively, were related to the factor F (Equations 13 to 15). The computed factors for different pipe equivalent roughness heights are presented in Figures 1 to 3 respectively and also in Table 2.

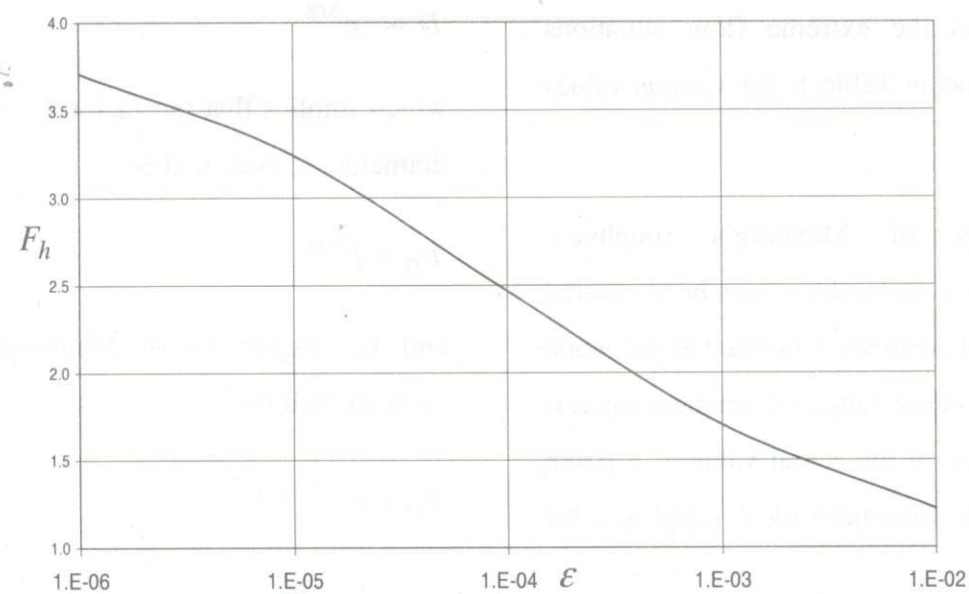

Figure 1 - Factor within which head loss is predicted Vs. Equivalent Pipe Roughness Height

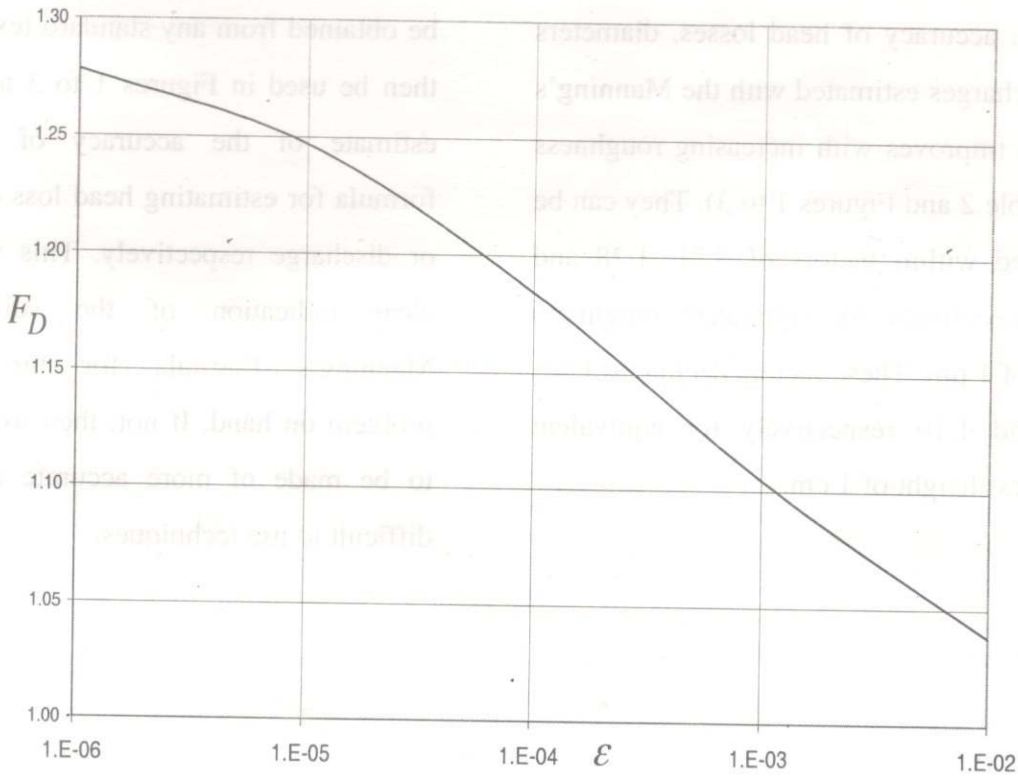

Figure 2 - Factor within which diameter is predicted Vs. Equivalent Pipe Roughness Height

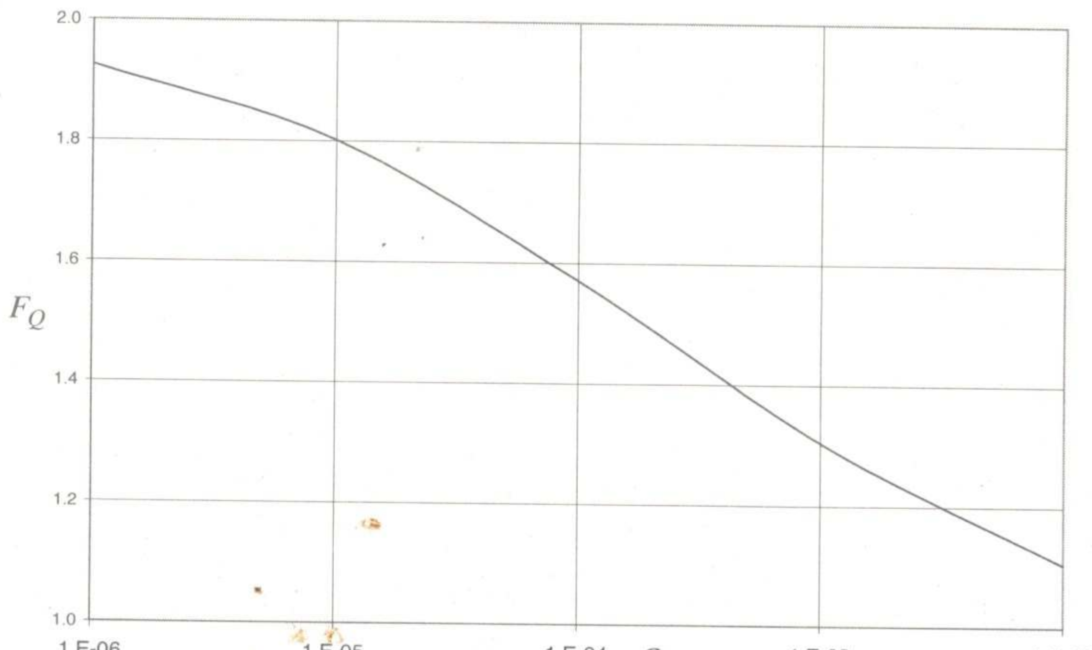

Figure 3 - Factor within which discharge is predicted Vs. Equivalent Pipe Roughness Height

4. CONCLUSION

This study reveals that within the range of flow of practical interest in water supply practice, accuracy of head losses, diameters and discharges estimated with the Manning's formula improves with increasing roughness (see Table 2 and Figures 1 to 3). They can be estimated within factors of 3.71, 1.28 and 1.93 respectively for equivalent roughness height of 1 μm. These factors decline to 1.22, 1.04 and 1.10 respectively for equivalent roughness height of 1 cm.

For a given pipe material to be used in practice, the equivalent roughness height can be obtained from any standard text. This can then be used in Figures 1 to 3 to obtain an estimate of the accuracy of Manning's formula for estimating head loss or diameter or discharge respectively. This will give a clear indication of the suitability of Manning's Formula for the particular problem on hand. If not, then use may have to be made of more accurate albeit more difficult to use techniques.

Table 1 - Extreme flow conditions and computed values of Manning's roughness coefficient for different pipe equivalent roughness heights

	Minimum extreme flow conditions							Maximum extreme flow conditions							Manning's n
Rough. height ε (m)	Pipe Dia. D (m)	Relative Rough. $\kappa = \varepsilon/D$	Reynold's Number R	Kinematic Viscosity ν m^2/s	Discharge Q m^3/s	Friction Factor f	$(D^{1/3}f)_{min}$	Pipe Dia. D	Relative Rough. $\kappa = \varepsilon/D$	Reynold's Number R	Kinematic Viscosity ν m^2/s	Discharge Q m^3/s	Friction Factor f	$(D^{1/3}f)_{max}$	Proposed (Eqn. 18)
1.E-06	0.02	5.00E-05	6.67E+05	3.E-07	3.14E-03	0.01335	3.62E-03	2	5.E-07	4.0E+03	3.E-07	1.89E-03	0.03956	4.98E-02	0.01039
1.E-05	0.02	5.00E-04	6.67E+05	3.E-07	3.14E-03	0.01744	4.73E-03	2	5.E-06	4.0E+03	3.E-07	1.89E-03	0.03957	4.99E-02	0.01111
1.E-04	0.02	5.00E-03	6.67E+05	3.E-07	3.14E-03	0.03051	8.28E-03	2	5.E-05	4.0E+03	3.E-07	1.89E-03	0.04002	5.04E-02	0.01281
1.E-03	0.11	9.09E-03	3.67E+06	3.E-07	9.50E-02	0.03673	1.76E-02	2	5.E-04	4.0E+03	3.E-07	1.89E-03	0.04047	5.10E-02	0.01551
1.E-02	1.10	9.09E-03	3.67E+07	3.E-07	9.50E+00	0.03671	3.79E-02	2	5.E-03	4.0E+03	3.E-07	1.89E-03	0.04475	5.64E-02	0.01927

Table 2 - Factors within which calculations of head loss, discharge and diameter are accurate for different pipe equivalent roughness heights

Case No.	Rough. height ε (m)	Pipe Dia. D (m)	Relative Rough. $\kappa = \varepsilon/D$	Factors of Accuracy: For Coefficient n F (Eqn. 8)	For Head Loss F_h	For Diameter F_D (Eqn. 11)	For Discharge F_Q (Eqn. 12)
1	1.E-06	0.14	7.23E-06	1.93	[illegible]	[illegible]28	1.93
2	1.E-05	1.26	7.92E-06	1.80	[illegible]	[illegible]25	1.80
3	1.E-04	2.00	5.00E-05	1.57	2.47	1.18	1.57
4	1.E-03	2.00	5.00E-04	1.30	1.70	1.10	1.30
5	1.E-02	2.00	5.00E-03	1.10	1.22	1.04	1.10

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