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THE ASYMMETRICAL DESIGN OF RC COLUMNS BY RIGOROUS APPROACH

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ABSTRACT

The symmetrical arrangement of reinforcement is justifiable for the columns of a building where the axial loads are the dominant forces and where any moments due to wind can be acting in either direction. But some members are required toresist axial forces combined with large bending moments, or have irregular cross-sections. In such cases, it is not economical to have equal areas of steel in both faces, and as such the usual design charts for columns found in the Codes of Practice cannot be applied. This paper proposes a rigorous approach for such cases.

NOTATION

A_s - Tension steel

A'_s - Compression steel

d - Depth to tension steel

d' - Depth to compression steel

x - Depth to the Neutral Axis, N.A.

F_{cc} - Compressive force developed in the concrete and acting through the centroid of the concrete stress block, K_2X

F_{sc} - Compressive force in the reinforcement area A'_s and acting through its centroid

F_{st} - Tensile (or compressive) force in the reinforcement area A_s and acting through its centroid

K_1 - Mean concrete stress

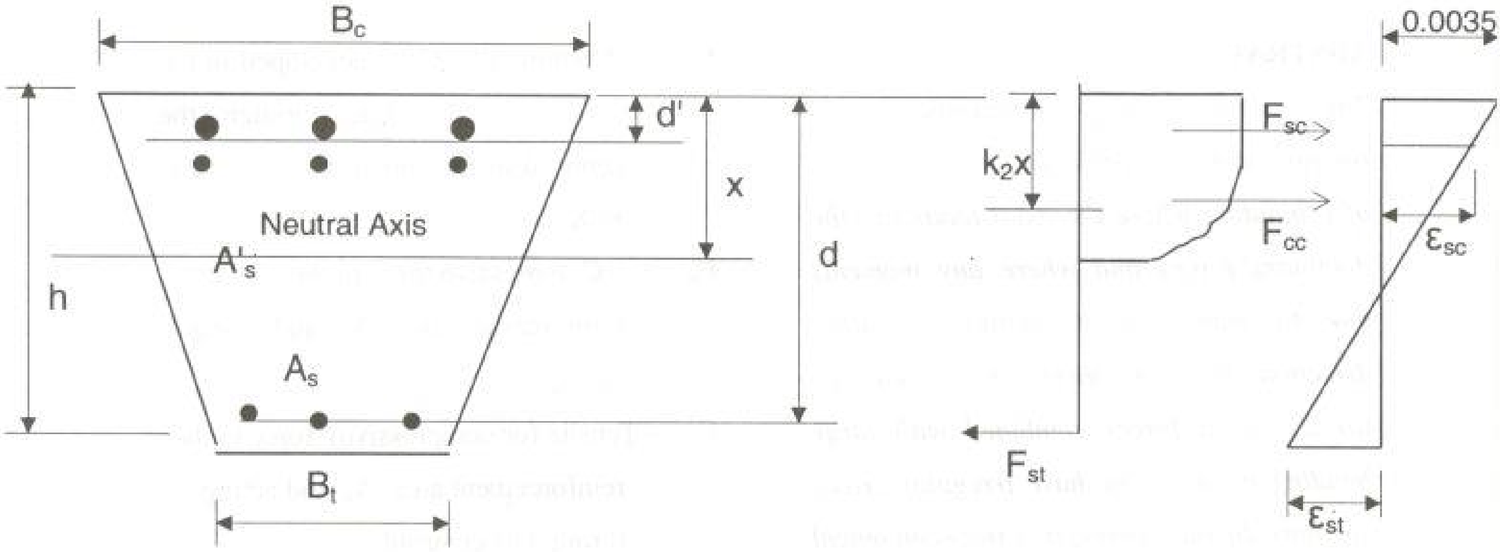
A_{cc} - Area of concrete in compression

INTRODUCTION

In cases where the effects of bending are considerable and a reversal of bending is impossible, it may be worthwhile examining the effects of disposing the reinforcement unequally in a column section. Appropriate charts and tables are not available in the design codes of practice for the design of such a section. Also, design charts are not usually available for columns of other than a rectangular or a circular cross-section, [**1,2**].

This paper proposes a rigorous design procedure for unsymmetrically-reinforced and irregular column sections that is readily programmed for solution by a micro-computer, which could find the optimum steel areas without the necessity of plotting a graph.

DESIGN EQUATIONS



(a) Section (b) Stresses (c) Strains

Fig. 1 - Non-rectangular Column Section

Fig. 1 represents the cross-section of a column member with typical stress and strain distributions for a position of the Neutral Axis $x<h$, [**3,4**]. The cross-section is subject to a moment M and an axial compressive force N, and in the figure the direction of the moment is such as to cause compression on the upper part of the section and tension on the lower part.

The applied force (N) must be balanced by the forces developed within the cross-section. Therefore

$$N = F_{cc} + F_{sc} + F_{st} \tag{1}$$

F_{st} is negative whenever the position of the NA is such that the reinforcement A_s is in tension, as in Fig. 1.

Substituting the terms for the stresses [3] and areas into equation (1):

$$N = K_1 A_{cc} + f_{sc} A'_s + f_{st} A_s \qquad (2)$$

or $$A_s = \frac{(N - K_1 A_{cc} - f_{sc} A'_s)}{f_{st}} \qquad (3)$$

where

f_{sc} = compressive stress in reinforcement A'_s

f_{st} = tensile or compressive stress in reinforcement A_s

The design moment M must be balanced by the moment of resistance of the forces developed within the cross-section.

Hence, taking moments about the mid-depth of the section:

$$M = F_{cc} (h/2 - K_2x) + F_{sc} (h/2 - d') + F_{st} (h/2- d)$$

i.e. $M = K_1 A_{cc} (h/2 - K_2x) + f_{sc} A'_s (h/2 - d') + f_{st} A_s (h/2 - d)$

But $$A_s = \frac{(N - K_1A_{cc} - f_{sc} A'_s)}{f_{st}}$$

$$\therefore M = K_1 A_{cc} (h/2 - K_2x) + (N - K_1 A_{cc}) (h/2 - d) + f_{sc} A'_s (d - d')$$

or $$A'_s = \frac{M - K_1 A_{cc} (h/2 - K_2x) - (N - K_1 A_{cc}) (h/2 - d)}{f_{sc} (d - d')} \qquad (4)$$

ITERATIVE SOLUTION

A_s and A'_s may be obtained from equations (3) and (4) using an iterative procedure: [3,5,9]

(i) select a depth of Neutral Axis, X.

(ii) determine the steel strain ε_{sc} and ε_{st} from the strain distribution.

From Fig. 1 (c) :

Compression steel strain,

$$\varepsilon_{sc} = \frac{0.0035 (X- d')}{X} \qquad (5)$$

Tension steel strain,

$$\varepsilon_{st} = \frac{0.0035 (d - X)}{X} \qquad (6)$$

(iii) determine the steel stresses f_{sc} and f_{st} from the stress-strain curve for the reinforcing bars:

$$\text{Steel Stress} = \frac{0.2f_y\ (0.008 + \varepsilon)}{0.0023 + f_y \times 10^{-6}} \qquad (7)$$

where f_y = characteristic strength of steel

ε = tension or compression steel strain given by equation (5) or (6)

(iv) calculate the compression steel A'_s using equation (4) :

$$A'_s = \frac{M - K_1 A_{cc}\,(h/2 - K_2X) - (N - K_1 A_{cc})(h/2 - d)}{f_{sc}\,(d - d')}$$

(v) calculate the tension steel A_s using equation (3) :

$$A_s = \frac{(N - K_1 A_{cc} - f_{sc} A'_s)}{f_{st}}$$

(vi) select further values of X and repeat steps (i) to (v) until a minimum value for $(A'_s + A_s)$ is obtained.

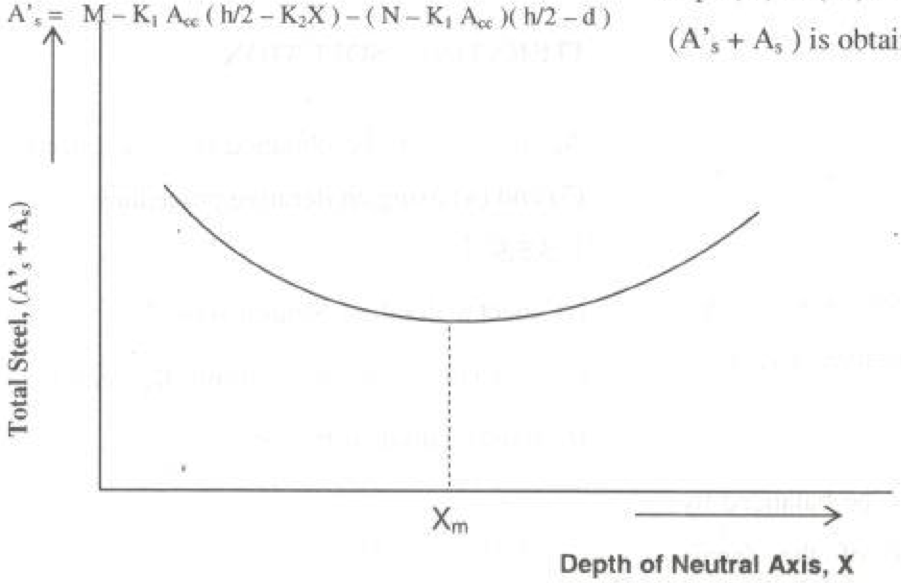


Fig.2. Plot of $(A'_s + A_s)$ against depth of Neutral Axis, X

(vii) using a depth of neutral axis X_m (Fig. 2), corresponding to the minimum value of $(A'_s + A_s)$, repeat steps (i) to (v) to calculate the actual values of A'_s and A_s.

In the case of irregular sections, K_2 will need to be calculated to determine the centroid of the compression zone. The term f_{sc} in the equation may be modified to $(f_{sc} - 0.45\ f_{cu})$ to allow for the area of concrete displaced by the reinforcement A'_s, [3]. Stress f_{st} has a negative sign whenever it is tensile.

TRAPEZOIDAL COLUMN SECTIONS

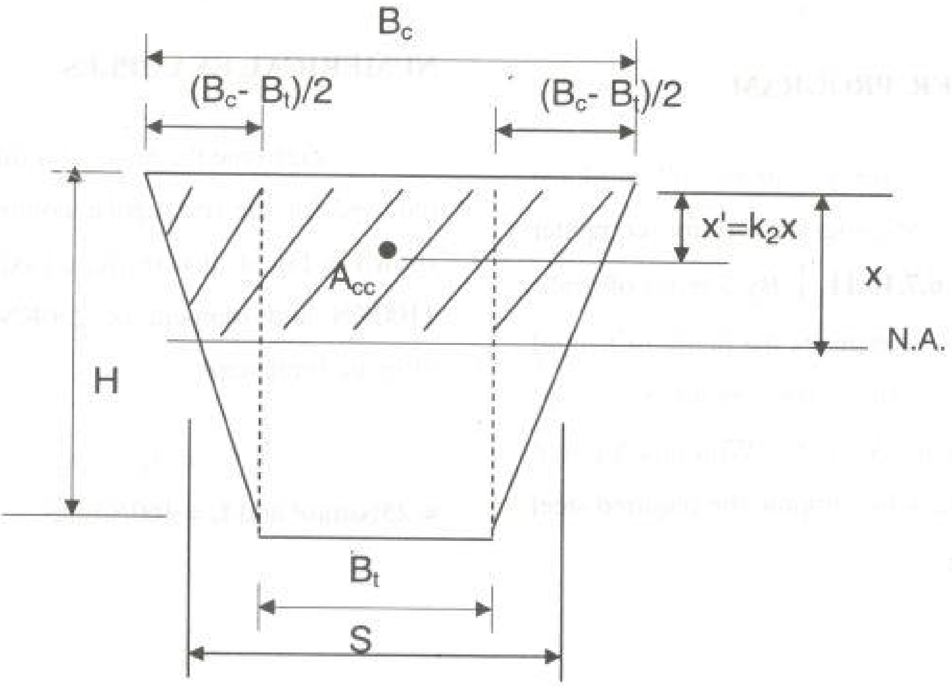


Fig.3: Trapezoidal Column Section with longer side in compression

From similar triangles,

$$S = B_t + (B_c - B_t)\frac{(H - X)}{H} \qquad (8)$$

where S is the width of section at the chosen depth of N.A.

Hence area of compression zone is

$$A_{cc} = \frac{(B_c + S)}{2}X \qquad (9)$$

To locate the centroid of the compression zone, take moments about the extreme fibre:

$$A_{cc}\;x' = \frac{(B_c - S)}{4}(X)\frac{(X)}{3} + (S)(X)\frac{(X)}{2} + \frac{(B_c - S)}{4}(X)\frac{(X)}{3}$$

i.e. $$Acc\;x' = \frac{(B_c + 2S)}{6}X^2$$

But from (9), $$A_{cc} = \frac{(B_c + S)}{2}X$$

$$\therefore\; x' = \frac{(B_c + 2S)}{3(B_c + S)}X \equiv K_2X$$

$$\therefore K_2 = \frac{(B_c+2S)}{3(B_c+S)} \qquad (10)$$

COMPUTER PROGRAM

The iterative design procedure outlined above may be programmed for computer solution, [**6,7,10,11**]. By a series of trials, the program determines the depth of neutral axis that yields the minimum total reinforcement, $(A'_s + A_s)$. With this depth, it then goes back to compute the required steel areas separately for tension and compression (see Flow Chart in Fig.5 and the program "COLUMN" in Appendix A)

NUMERICAL EXAMPLES

1. Determine the areas of reinforcement required for the rectangular column section shown in Fig. 4, if it resists an axial load of 1100KN and moment of 230KNm at the ultimate limit state.

The characteristic material strengths are f_{cu} = 25N/mm² and f_y = 460N/mm²

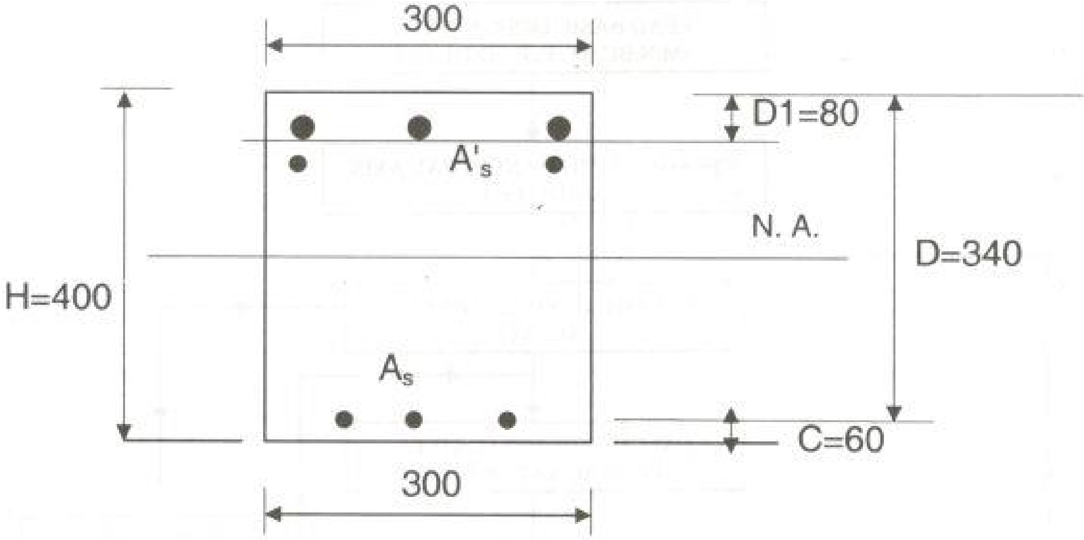


Fig. 4 - Rectangular Column Section

INPUT DATA	
BC=300	MMAX=2.3E+08
BT=300	NMAX=1.1E+06
H =400	FY =460
C =60	FCU =25

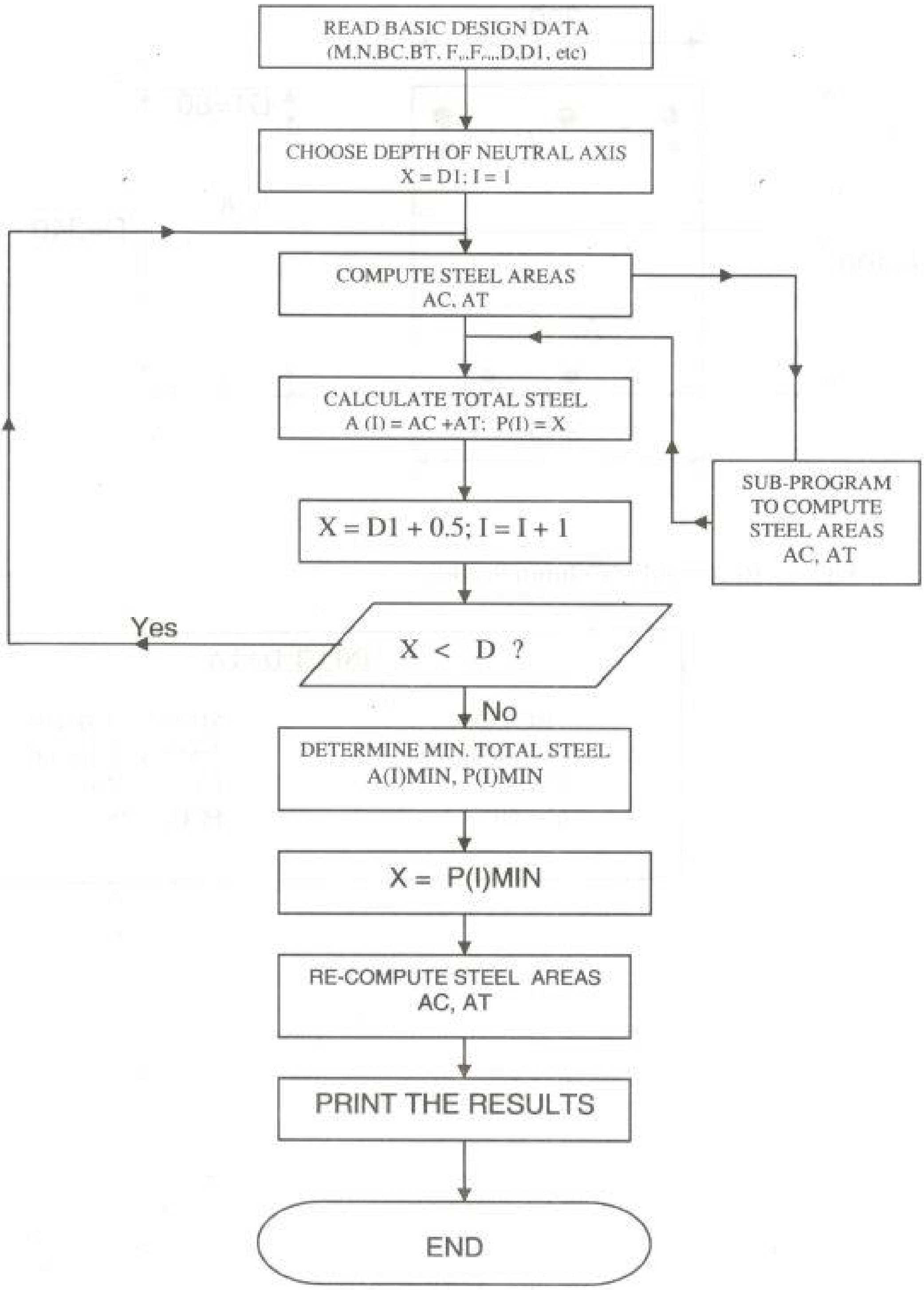


Fig. 5 - Flow Chart for Iterative Solution

2. Design the reinforcement for the trapezoidal section shown in Fig. 6. , given that M = 320KNm and N = 1200KN at the ultimate limit state. The characteristic material strengths are f_{cu} = 30N/mm² and f_y = 425N/mm²

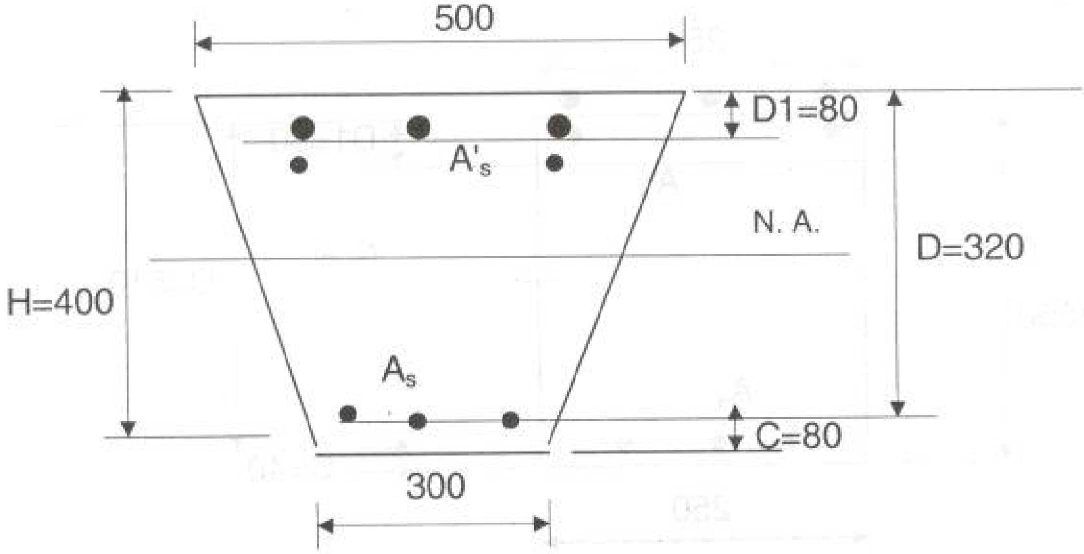


Fig. 6: Trapezoidal Column Section

INPUT DATA

BC=500	MMAX=3.2E+08
BT=300	NMAX=1.2E+06
H =400	FY =425
C =80	FCU =30

3. Determine the areas of reinforcement required for the rectangular column section shown in Fig. 7, if it resists an axial load of 250KN and moment of 400KNm at the ultimate limit state. The characteristic material strengths are f_{cu} = 25N/mm² and f_y = 460N/mm²

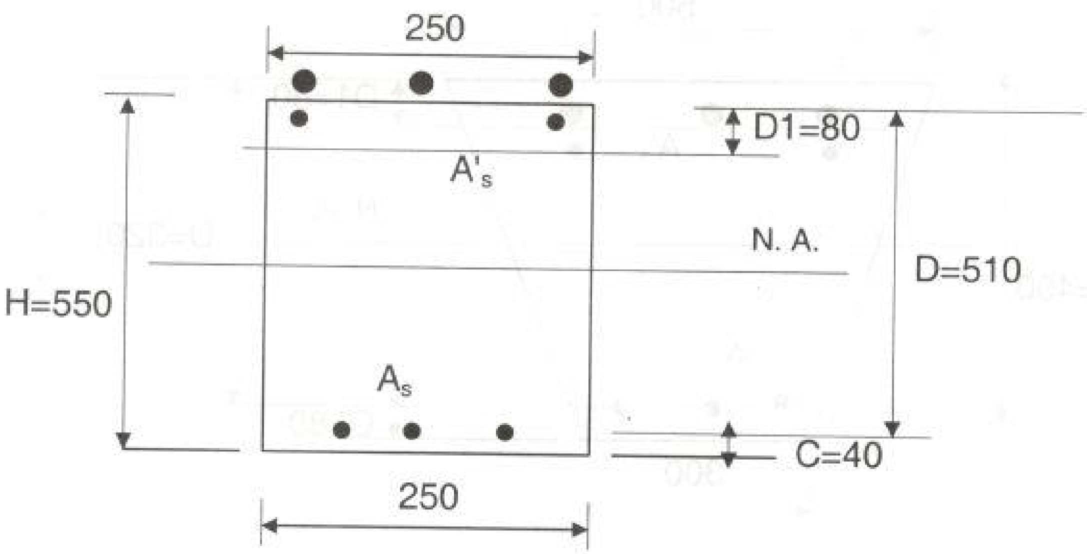


Fig. 7: Rectangular Column Section

INPUT DATA

BC=250	MMAX=4.0E+08
BT=250	NMAX=0.25E+06
H =550	FY =460
C =40	FCU =25

4. Determine the areas of reinforcement required for the rectangular column section shown in Fig. 8, if it resists an axial load of 300KN and moment of 500KNm at the ultimate limit state. The characteristic material strengths are f_{cu} = 25N/mm² and f_y = 460N/mm²

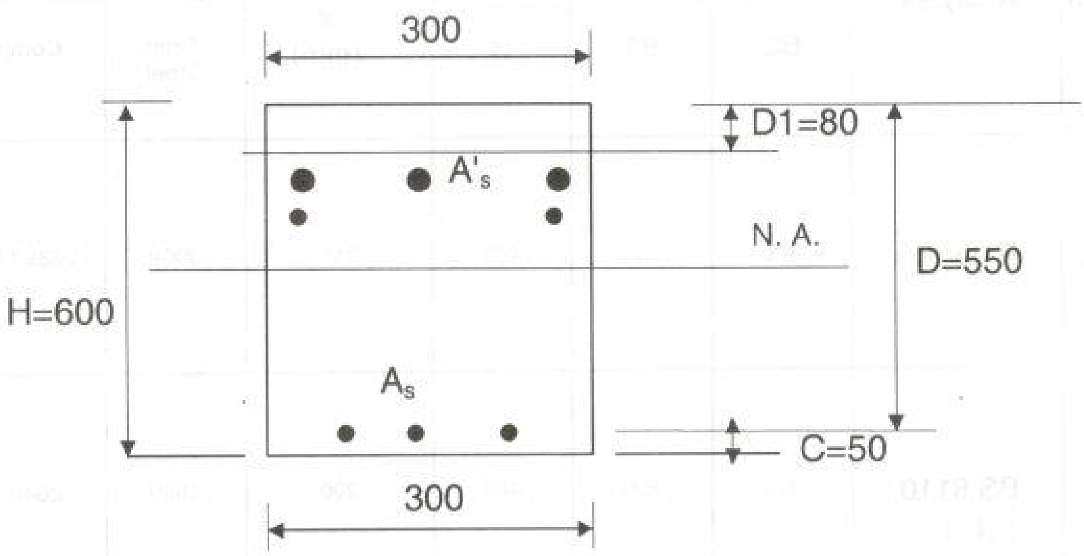


Fig. 8: Rectangular Column Section

INPUT DATA	
BC=300	MMAX=5.0E+08
BT=300	NMAX=0.3E+06
H =600	FY =460
C =50	FCU =25

The results are presented in Tables 1 and 2 below.

Table 1: Computed Steel Areas for Columns Subjected to Axial Load and Bending Moment

Problem	Analysis	Column Section Dimensions (mm)			Depth to N.A., X (mm)	Computed Reinforcement (mm^2)		
		BC	BT	H		Total Steel	Comp	Tension
No. 1	Rigorous	300	300	400	211	3908	2685.14	1222.86
	BS 8110 [1]	300	300	400	200	4080	2040	2040
	% Diff.					4.2		
No. 2	Rigorous	500	300	400	175.5	5210.53	3134.33	2076.20
	BS 8110 [1]	500	300	400	No design charts for trapezoidal sections			

Table 2: Computed Steel Areas for Columns Subjected to Axial Load and Bending Moment

Problem	Analysis	Column Section Dimensions (mm)			Depth to N.A., X (mm)	Computed Reinforcement (mm^2)		
		BC	BT	H		Total Steel	Comp	Tension
No. 3	Rigorous	250	250	550	193	3628.04	1828.86	1799.18
	BS 8110 [1]	250	250	550	275	3782	1891	1891
	% Diff.					4.1		
No. 4	Rigorous	300	300	600	212.5	3940	1850	2090
	BS 8110 [1]	300	300	600	300	4122	2061	2061
	% Diff.					4.4		

DISCUSSION OF RESULTS

Tables 1 and 2 show that for the rectangular section, the Code simply shares the total steel equally for compression and tension, whereas the rigorous approach determines the compression and tension steel based on whether the section is more in compression than in tension, or vice versa. The results also indicate that there is some saving in the total steel obtained by asymmetrical design (more than 4%) over that needed when compression and tension reinforcement are assumed equal as prescribed by the Code.

CONCLUSION

The developed computer program will be found particularly useful in cases where it will not be advisable to design a column section as symmetrically reinforced. In certain practical situations it may be found that a column section has to withstand axial loads combined with large bending moments. Unsymmetrical reinforcement will be more economical in such cases, [8]. Since charts are not readily available for unsymmetrically reinforced columns and trapezoidal column sections, a rigorous design procedure using the computer becomes inevitable.

APPENDIX A - Program "COLUMN" for the Rigorous Design of Columns

```
' PROGRAM "COLUMN" – FOR THE RIGOROUS DESIGN OF COLUMNS WRITTEN BY DR. M.E. ONYIA
' DEFINITION OF VARIABLE AND ARRAY NAMES
' BC: BREADTH OF COLUMN ON COMPRESSION SIDE (mm)
' BT: BREADTH OF COLUMN ON TENSION SIDE (mm)
' H: TOTAL DEPTH OF COLUMN (mm)
' MMAX: DESIGN MOMENT (Nmm)
' NMAX: DESIGN AXIAL LOAD (N)
' D: DEPTH TO TENSION REINFORCEMENT (mm)
' D1: DEPTH TO COMPRESSION REINFORCEMENT (mm)
' C: COVER TO TENSION REINFORCEMENT (mm)
' FY: CHARACTERISTIC STRENGTH OF REINFORCEMENT (N/mm²)
' FCU: CHARACTERISTIC STRENGTH OF CONCRETE (N/mm²)
' A(I): ARRAY OF REINFORCEMENT AREAS
' P(I): ARRAY OF NEUTRAL AXIS POSITIONS

 CLEAR
 ' INPUT BASIC PROGRAM DATA
 READ MMAX,NMAX,BC,BT,H
 READ D,D1,C,FY,FCU
 IF BC=BT THEN 5
 ' INDICATE WHICH SIDE IS IN COMPRESSION, (L/S)
 READ Z$
5 N=D-D1+1
 N=2*N-1
 ' DIMENSION ANY ARRAYS
 DIM A(N),P(N)
 ' BEGIN THE ITERATIVE PROCESS
 J=1
 FOR I=D1 TO D STEP 0.5
 X=I
 GOSUB 30
 A(J)=ABS(AC)+ABS(AT)
 P(J)=X
 J=J+1
 NEXT I
```

```
' DETERMINE MINIMUM TOTAL AREA OF STEEL
AMIN=A(1)
X=P(1)
FOR I=2 TO N
IF A(I)<AMIN THEN 10
GOTO 20
10 AMIN=A(I) : X=P(I)
20 NEXT I
GOSUB 30

' SPECIFY BASIC DESIGN DATA
' ORDER OF DATA : MMAX,NMAX,BC,BT,H
' ORDER OF DATA : D,D1,C,FY,FCU,Z$
DATA 3.2E+08,1.2E+06,500,300,400
DATA 320,80,80,425,30,L

' PRINT RESULTS
' FIRST SET THE PRINTER WIDTH
WIDTH "LPT1:",180
PRINT : PRINT : PRINT
PRINT "PAGE ";A1
PRINT : PRINT
PRINT "BASIC DESIGN PARAMETERS"
PRINT "------------------------------------------------"
PRINT
PRINT "DESIGN MOMENT, (Nmm) –";MMAX
PRINT : PRINT "DESIGN AXIAL LOAD, (N) –";NMAX
PRINT : PRINT "CHARACTERISTIC STRENGTH OF STEEL, (N/mm²) –";FY
PRINT : PRINT "CHARACTERISTIC STRENGTH OF CONCRETE, (N/mm²) –";FCU
PRINT : PRINT "COVER TO REINFORCEMENT, (mm) –";C
PRINT : PRINT "------------------------------------------------------------------------------------------------"
PRINT : PRINT : PRINT
PRINT "COLUMN SECTION DIMENSIONS (mm)"
PRINT "------------------------------------------------------"
PRINT
PRINT TAB(3)"BC";TAB(13)"BT";TAB(23)"H";TAB(33)"X"
PRINT "------------------------------------------------------"
```

```
PRINT
PRINT TAB(2)BC;TAB(12)BT;TAB(22)H;TAB(32);X
PRINT "---------------------------------------------------------------------"
PRINT : PRINT  "Note -:"
PRINT "BC = Breadth of column on compression side"
PRINT "BT = Breadth of column on tension side"
PRINT "H   =Total depth of column"
PRINT "X   = Depth to the neutral axis"
PRINT : PRINT : PRINT
PRINT "REINFORCEMENT   (mm²)"
PRINT "-------------------------------------------"
PRINT
PRINT TAB(3)"COMPRESSION";TAB(18)"TENSION"
PRINT "-------------------------------------------------------------"
PRINT
AC=ABS(AC)
AT=ABS(AT)
PRINT TAB(2);USING "####.##";AC;
PRINT TAB (17);USING "####.##";AT
PRINT : PRINT  "----------- ---------------------------------------------"
END

30 ' SUB-PROGRAM TO COMPUTE STEEL AREAS
  ' FIRST CALCULATE WIDTH OF SECTION AT THE CHOSEN DEPTH OF NEUTRAL AXIS
  IF Z$="L" THEN 40
  S=BC+(BT-BC)*X/H
  GOTO 50
40 S=BT+(BC-BT)*(H-X)/H
50 ' LOCATE CENTROID OF THE COMPRESSION ZONE
   IF BC=BT THEN 60
   K2=(BC+2*S)/(3*(BC+S))
   GOTO 70
60 K2= 0.45
70 ' CALCULATE THE MEAN CONCRETE STRESS
  K1=0.4*FCU
  ' CALCULATE AREA OF COMPRESSION ZONE
  AZ=(BC+S)*X/2
```

```
' DETERMINE STEEL STRAINS
CS=0.0035*(X-D1)/X
TS=0.0035*(D-X)/X
' CALCULATE STEEL STRESSES USING THE ABOVE COMPUTED STEEL STRAINS
PC=0.2*FY*(8.000001E-03+CS)/(0.0023+FY*10^(-6))
PT=0.2*FY*(8.000001E-03+TS)/(0.0023+FY*10^(-6))
' MODIFY THE COMPRESSIVE STEEL STRESS TO ALLOW FOR DISPLACED CONCRETE
PC=PC-0.45*FCU
' COMPUTE AREAS OF TENSION AND COMPRESSION STEEL
M1=K1*AZ*(H/2-K2*X)
M2=(NMAX-K1*AZ)*(H/2-D)
AC=(MMAX-M1-M2)/(PC*(D-D1))
N1=K1*AZ
N2=PC*AC
AT=(NMAX-N1-N2)/PT
RETURN
```

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