DISCHARGE VERSUS DEPTH CURVE FOR A GIVEN SPECIFIC ENERGY APPLICABLE TO ALL RECTANGULAR CHANNELS

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ABSTRACT

The aim of this paper is to present a universal discharge versus flow depth curve for any given Specific Energy, which is applicable to all rectangular channels. This is achieved by non-dimensionalizing the Specific Energy versus depth relationship for rectangular channels in such a way as to result in a single non-dimensional discharge versus non-dimensional depth curve. Unlike in the case of universal Specific Energy curve, this curve never approaches infinity and thus covers the entire theoretically possible range of flow depths and is still as useful as the unified Specific Energy curve, in determining flow depth for given discharge, Specific Energy and channel width, without iteration. It will also be shown that the universal discharge curve is easier to use in analyzing sudden width change problems and also gives a better intuitive feel for these problems whereas the universal Specific Energy curve is similarly better for sudden bed elevation change problems. For instance the discharge curve confirms the well-known fact that when channel width suddenly contracts, flow depth decreases for subcritical flow and increases for supercritical flow. Such exercise with universal Specific Energy curves is inconclusive.

Keywords: *discharge, depth, specific energy, rectangular channels*

INTRODUCTION

Whenever there is a rapid transition in open channel flow such as at a sudden change in the bottom elevation and/or the width of the channel, this may imply that the flow also must change rapidly such that energy is conserved. This would imply that the Specific Energy of flow after the transition can be obtained from the transition characteristics and the Specific Energy of flow before the transition. The problem then is often to determine after the transition, the depth of flow for the given discharge and Specific Energy.

However for a given discharge, whereas the Specific Energy varies explicitly with the flow depth, the reverse is not the case. The Specific Energy versus flow depth curve for the given discharge thus has to be produced and used to obtain the flow depth corresponding to the given discharge. However the Specific Energy versus depth curve differs with cross-sectional shapes and flow discharge. Thus to tackle channel transition problems, separate curves have to be plotted for the given discharges that may be of relevance in the analysis.

Alternately, use could be made of the relationship between discharge and flow depth for a given Specific Energy. However in this case also, whereas the discharge varies explicitly with the flow depth, the reverse is not the case. The discharge versus flow depth curve for the given Specific Energy has to be produced and used to obtain the flow depth corresponding to the given Specific Energy. However, the discharge versus depth curve differs with cross-sectional shapes and Specific Energy. Thus here too separate curves have to be plotted for the given Specific Energies that may be of relevance in the analysis.

The only alternative left is to obtain the depth by iteration (see Henderson, 1989; Streeter, 1971). Any of these is time-consuming. To avoid these problems, Aiyesimoju (2006, 2007) has presented unified Specific Energy versus flow depth curves applicable to all rectangular and triangular channels. However the presented curves show Specific Energies approaching infinity as flow depth approaches zero or infinity. Practical curves must have a limited range, thus they have to leave out flow depths that are either very small or very large.

Other workers in this area have been mostly concerned with transitions involving the specific force, as in hydraulic jumps, rather than the specific energy. For example Alhamid and Negm (1996) considered rapidly varied flow in sloping and rough rectangular channels. Mossa (1999) focused on the oscillating characteristics that accompany hydraulic jumps in rectangular channels.

The aim of this paper is to obtain a universal discharge versus flow depth curve for any given Specific Energy, which is applicable to all rectangular channels. Unlike in the case of universal Specific Energy curve presented by Aiyesimoju (2006), this curve covers the entire theoretically possible range of flow depths. The obtained curve will be used to explain flow depth changes that are known to occur in sudden open channel transition flow problems, especially when channel width suddenly changes.

Also, although either of the universal discharge or Specific Energy curves can be used to solve specific problems, the discharge curves give a better intuitive feel for sudden width change problems whereas the universal Specific Energy curves are similarly better for sudden bed elevation change problems. For instance the discharge curve confirms the well-known fact that when channel width suddenly contracts, flow depth decreases for subcritical flow and increases for supercritical flow. Such exercise with universal Specific Energy curves shows that the result would depend on the specific case.

VARIATION OF SPECIFIC DISCHARGE WITH DEPTH

The specific energy E for a given depth of flow y is given by Henderson (1989) as

$$E = y + \frac{V^2}{2g}$$

This can be written as

$$E = y + \frac{Q^2}{2gb^2y^2} \qquad (1)$$

where b is the channel width and Q is the discharge. For a given Q, there is a relationship between E and y. Thus different values of Q result in different E-y curves.
Equation (1) can be written as

$$Q^2 = 2gb^2y^2(E - y)$$

Dividing through Equation (1) by a scaling parameter a, yields

$$\frac{Q^2}{a} = 2gb^2\frac{y^2}{a}E - 2gb^2\frac{y^3}{a}$$

which implies

$$\frac{Q^2}{a} = 2gb^2Ea\left(\frac{y}{a}\right)^2 - 2gb^2a^2\left(\frac{y}{a}\right)^3 \qquad (2)$$

Above is simplified if

$$2gb^2Ea = 2gb^2a^2$$

in which case

$$a = E$$

Substituting this in Equation (2) gives

$$\frac{Q^2}{E} = 2gb^2E^2\left(\left(\frac{y}{E}\right)^2 - \left(\frac{y}{E}\right)^3\right)$$

or

$$\frac{Q^2}{2gb^2E^3} = \left(\frac{y}{E}\right)^2 - \left(\frac{y}{E}\right)^3$$

This implies

$$\frac{Q}{b\sqrt{2gE^3}} = \sqrt{\left(\frac{y}{E}\right)^2 - \left(\frac{y}{E}\right)^3} \qquad (3)$$

Let $y' = \frac{y}{E}$ and $Q' = \frac{Q}{b\sqrt{2gE^3}}$

then $Q' = \left(y'^2 - y'^3\right)^{1/2}$

For the maximum and minimum values of Q'

$$\frac{dQ'}{dy'} = \frac{d}{dy'}\left(y'^2 - y'^3\right)^{1/2}$$

$$= \frac{1}{2}\left(y'^2 - y'^3\right)^{-1/2}\left(2y' - 3y'^2\right)$$

$$= \frac{1}{2}\left(y'^2(1 - y')\right)^{-1/2}y'(2 - 3y')$$

$$= \frac{1}{2}(1 - y')^{-1/2}(2 - 3y')$$

$$= \frac{2 - 3y'}{2\sqrt{1 - y'}} = 0$$

which implies a maximum value for $\frac{Q}{b\sqrt{2gE^3}}$ of $\frac{2}{\sqrt{27}}$ (i.e. 0.3849) at $\frac{y}{E} = ⅔$ as is well-known and this in fact corresponds to critical flow, with $\frac{y}{E} > ⅔$ and $\frac{y}{E} < ⅔$ corresponding to subcritical and supercritical flows respectively.

RESULTS AND DISCUSSION

From Equation (3), a relationship exists between a dimensionless unit discharge $\frac{Q}{b\sqrt{2gE^3}}$ and a dimensionless unit depth irrespective of the specific values of discharge Q, Specific Energy E and channel width b. The least possible value of flow depth y is zero and from Equation (1), it can never exceed E thus implying the practical range of interest is $0 < \frac{y}{E} < 1$. Computed values of $\frac{Q}{b\sqrt{2gE^3}}$ for various values of $\frac{y}{E}$ ranging from 0 to 1 are plotted in Figure 1. These confirm that the unit discharge $\frac{Q}{b\sqrt{2gE^3}}$ has a peak value of 0.3849 at a unit depth $\frac{y}{E}$ of ⅔. Over the range of interest 0< $\frac{y}{E}$ <1, they also show minima for $\frac{Q}{b\sqrt{2gE^3}}$ of 0 at $\frac{y}{E}$ =0 and 1. These results are consistent with the results that are obtained using the longer methods as in Chow (1959), Streeter (1971) and Henderson (1989) as well as the Specific Energy curve method in Aiyesimoju (2006).

To demonstrate the utility of Figure 1, consider a rectangular channel with water discharge Q of 4.427 m³/s flowing with a depth y of 0.6m and width b of 1.409m. If width rapidly but smoothly contracts to 1m such that energy losses are negligible, determine the new water depth. This is a rapidly varied flow problem and the backwater equations (gradually varied flow) do not apply. With no energy losses, the Specific Energy remains the same immediately before and after the width contraction. The Specific Energy from Equation1 is

$$E = 0.6 + \frac{(4.427)^2}{2*9.8*(1.409)^2*(0.6)^2}$$ i.e. E is 2.000 m.

After transition, Specific Energy E remains the same which implies

$$\frac{Q}{b\sqrt{2gE^3}} = \frac{4.427}{1\sqrt{2*9.8*2^3}} = 0.35354$$

This corresponds to of $\frac{y}{E}$ 0.5 from Figure 1 which implies $y=0.5E=0.5*2=1$m.

This depth can be checked by computing the specific energy there (b=1m, y=1m) as

$$E = 1 + \frac{(4.427)^2}{2*9.8*1.^2*1^2} = 2.000\text{ m}$$ which is correct.

The maximum value that $\frac{Q^2}{2gb^2E^3}$ can have is 0.3849 and the breadth for this to occur is given by

$$\frac{Q}{b\sqrt{2gE^3}} = \frac{4.427}{b\sqrt{2*9.8*2^3}} = 0.3849$$ which implies b is 0.919 m. Contraction to a width below this value would result in flow choking whereby the flow at the contraction will be forced to be critical flow corresponding to the discharge.

If on the other hand, in the above transition, rather than a width contraction we have a rapid but smooth increase in bed elevation by say 0.3 m, such that energy losses are negligible. Total energy is a sum of bed elevation (potential energy) and Specific Energy, and for it to be conserved, the Specific Energy of the flow would reduce to 1.7 m which implies

$$\frac{Q}{b\sqrt{2gE^3}} = \frac{4.427}{1.409\sqrt{2*9.8*1.7^3}} = 0.320$$

This implies from Figure 1 that $\frac{y}{E}$ = 0.42 or y = 0.42*1.7=0.714 m. Again, this depth can be checked by computing the specific energy there (b=1.409 m, y=0.714 m) as

$$E = 0.714 + \frac{(4.427)^2}{2*9.8*(1.409)^2*(0.714)^2}$$

i.e. E is 1.702 m, which is correct within round off error.

Again, since the maximum value that $\frac{Q^2}{2gb^2E^3}$ can have is 0.3849. The Specific Energy for this to occur is given by

$$\frac{Q}{b\sqrt{2gE^3}} = \frac{4.427}{1.409\sqrt{2*9.8*E^3}} = 0.3849$$

which implies E of 1.504 m. Then a step increase above (2 – 1.504)= 0.496m would result in flow choking whereby the flow at the transition will be forced to be critical flow corresponding to the discharge.

Figure 1 can in fact be used in a more general way to explain phenomena where total energy is constant such as sudden changes in channel width and/or channel bed elevation. For instance when channel width suddenly contracts, this implies b decreases and hence $\frac{Q}{b\sqrt{2gE^3}}$ increases which implies from Figure 1, that $\frac{y}{E}$ (and hence depth y) decreases for subcritical flow and increases for supercritical flow.

If there is a sudden step up in bed elevation, the total energy (specific energy plus potential energy) must still be conserved. Since the potential energy has been suddenly increased, this implies a sudden decrease in flow Specific Energy E. This in turn implies an increase in $\frac{Q}{b\sqrt{2gE^3}}$ which means that the flow depth y decreases for subcritical flow because $\frac{y}{E}$ will decrease (Figure 1) and E also has decreased. Conversely, the depth y increases for a sudden step down for subcritical flow. These phenomena are consistent with discussions in Chow (1959), Streeter (1971) and Henderson (1989). A summary of results using similar reasoning as above are as in Table 1. No general conclusion can however be reached for sudden bed elevation changes for supercritical flow using Figure 1. Universal Specific Energy curve (Aiyesimoju, 2006) has a similar problem analyzing sudden width change problems. Thus although universal discharge or Specific Energy curves can be used to solve specific problems, the discharge curve gives a better intuitive feel for sudden width change problems whereas the Specific Energy curve is similarly better for sudden bed elevation change problems.

CONCLUSION

From the Specific Energy equation, a relationship between discharge and flow depth was obtained. This was then non-dimensionalized to yield a relationship between a unit discharge $\frac{Q}{b\sqrt{2gE^3}}$ and a unit depth $\frac{y}{E}$ that is applicable irrespective of the specific values of discharge Q, Specific Energy E and channel width b. Given the Specific Energy E, discharge Q and channel width b, $\frac{Q}{b\sqrt{2gE^3}}$ can be computed and the corresponding $\frac{y}{E}$ read from Figure 1. Once $\frac{y}{E}$ is so obtained, y can be readily calculated from the given value of E. The practical use of Figure 1 to rapidly obtain flow choking conditions as well as the flow depth given the specific energy, were demonstrated.

Furthermore, Figure 1 was easily used to generally analyze well-known phenomena in open channel flows such as flow depth changes that occur when there are sudden changes in channel width. This is in general not as easy to analyze with universal Specific Energy curves. The analysis confirms that when there is a sudden contraction in channel width, the flow depth decreases for subcritical flow and increases for supercritical flow. The converse is true when the channel width suddenly expands. Flow depth changes when there are sudden changes in bed elevation, which are easily analyzed with universal Specific Energy curves, was also easily analyzed here for general subcritical flows but not for general supercritical flows although specific supercritical flow cases could still be analyzed.

Table 1 – Explanation of sudden open channel transition flow problems

Phenomenon	Subcritical flow	Supercritical flow
Sudden width contraction	Depth decrease	Depth increase
Sudden width expansion	Depth increase	Depth decrease
Sudden step up	Depth decrease	Depends. (E decreases but y/E increases)
Sudden step down	Depth increase	Depends. (E increases but y/E decreases)

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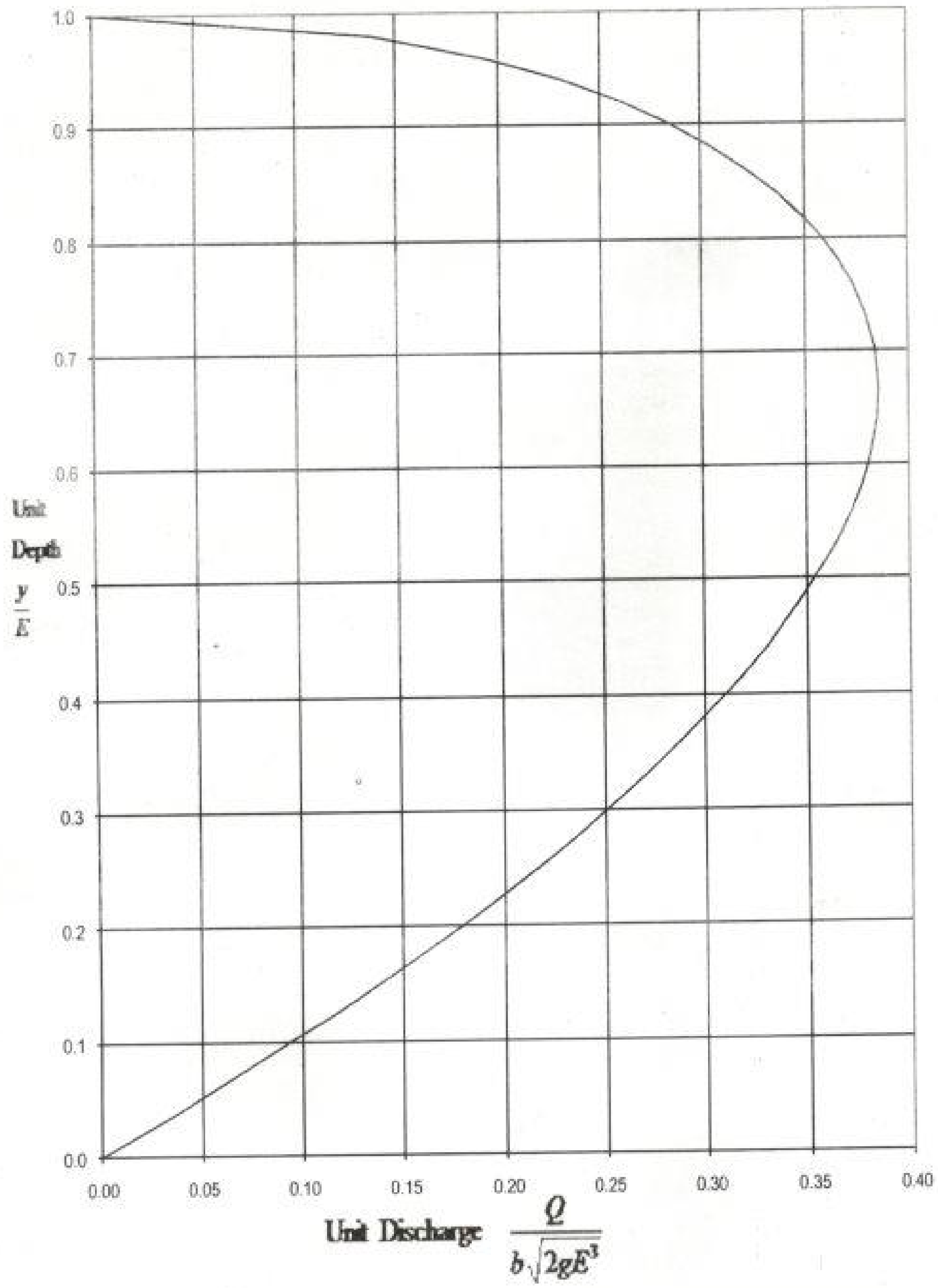

Figure 1 - Dimensionless Depth Vs. Dimensionless Discharge