ANALYSIS OF TELECOMMUNICATION TOWERS USING THE STIFFNESS METHOD.

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ABSTRACT:

It has been observed that most of the telecommunication towers being erected in Nigeria now by the GSM operators are the free standing type with various types of lattice arrangement of the internal bracing members. In this paper, we explore the powerful tool offered by the stiffness matrix method in conjunction with computer-aided solution to conduct the Structural stability analysis of two types of lattice arrangement of telecommunication towers: the triangular and the cross lattice arrangements. A computer model using Structural Analysis Programme SAP-2000 was developed to carry out the analysis.
As a demonstration problem, a 40m high telecommunication free standing tower was analysed for the two different arrangements of the Internal bracing members at wind speeds which were varied between 15m/s and 120m/s. Results of the analysis confirm the presence of significant overturning moments, shaft up-lift and axial forces at the base of the towers. The analysis also revealed the deflection pattern at the top of the towers.
It was interesting to observe that for the same level and magnitude of wind load, the cross lattice arrangement of internal bracing members proved to be more stable than the triangular arrangement as it was discovered that the axial forces induced in the members, the overturning moment at the base as well as the deflection at the top of the tower were generally less in the former than in the latter. It is therefore recommended that for more effective and stable free standing telecommunication towers, the cross lattice arrangement of internal bracing members should be preferred to the triangular lattice arrangement.

1.0 INTRODUCTION:

The advent of Global Mobile Telecommunication System (GSM) in the country today has popularised the erection of Guyed and Free Standing Telecommunication towers. Furthermore, the indiscriminate localization of these complex structures in the nook and cranny of various states across the nation, and the inherent danger that may accompany their failure or accidental collapse, imperatively demands that adequate attention should be focused on their analysis in order to guarantee their safe design and erection.

Besides, it is expected that the indigenous engineers and analysts would sufficiently take active part in the analysis, design and the erection of such structures rather than being restricted to the erection of already fabricated structures as it is being practiced presently.

A study of some of the existing literature reveals that as far back as the middle of 19th century, researchers from various locations in the world had made commendable efforts in finding solutions to the challenge posed by the need to use Masts and Towers for various purposes. Till date, extensive works have been published in the area of analysis and design of Guyed Masts and telecommunication towers.

Using the parabolic approximation and the assumption of small vibrations, Kolousek(1947) determined the first dynamic solution for massive guy cables. The method of solution was to represent the deflected mode shape by Fourier series of sinusoidal components and to solve for the amplitude of each component by substitution into the governing differential equation.

Building on the above, other researchers also considered the parabolic representation of guyed cables. Prominent researchers in this respect include Cohen and Perrin (1957), Davenport (1959) and Scholt et al (1954). Exploring the advantages presented by the linear behaviour of Masts, earlier researchers adopted linear model which made analysis simpler. Davenport and Steels (1965) proposed a linear model to describe the vibration of guys based on the assumption that be parabolic and the guy amplitude would be small.

Also worthy of note is the work of Vellozi (1975) which involved a detailed computer study for determining the dynamic response of a tall tower based on non-deterministic approach.
In a bid to address the challenges posed by non-linear analysis of towers with probabilistic approach to account for wind variations, a few other researchers have attempted this by idealizing the tower using truss elements. Jayaraman et al (1980) developed a centenary cable element and used it for static and dynamic analysis of Guyed tower. Also, in the work of Surya Kumar and Raman (1986) where a comparison of responses was obtained using explicit and implicit methods for non-linear dynamic analysis. The tower was idealised using beam - column and the guys were idealised as assemblage of truss elements.

George (1996) worked on Large Deformation Analysis of Space Truss Structures. In his work, static analysis of space truss structures, including inelastic material and large deformation geometric non-linearity were considered. Member material behaviour modes include buckling, yielding, unloading and reloading. He based the large deformation behaviour on the linear updated LaGrange space truss displacement equation. He observed that due to interaction of the non-linear member behaviour modes, force redistribution in a space truss structure can lead to progressive structural failure. Then he developed a model which could trace the sequence of member bucking, yielding and inelastic member response.

Eric Savory, et al (2000) at the Department of Civil Engineering, University of Surrey, Guildford, collectively presented an extensive work on modelling of tornado and microburst - induced wind loading and failure of lattice towers. The paper described models for the wind velocity time - history of transient tornado and microburst events and the resulting loading on lattice towers. A dynamic structural analysis was undertaken for two High Intensity Wind (HIW) events, predicting a tornado induced shear failure, as observed on the field. However, the microburst does not produce failure due to its lower intensity and long duration. In the analysis of the modelled tower, a structural analysis method using the finite element code ABACUS had, previously been implemented. In the same vein the research also included monitoring in full scale, the wind induced foundation loads. A relatively small value for the moment of inertia was used for the elements of the tower in order to keep the bending and torsional stiffness at both ends of the members close to zero. This simulates almost pinned connections for the structural model that is close to the real connection response exhibited in the actual structure. By using a low bending stiffness and the true axial stiffness, the tower members were modelled as truss elements, the behaviour of which incorporated a full post bucking response.

Furthermore, with the maturity of the lattice tower engineering profession and the advances in P.C analytical solution power, the engineer can now push the margin of analysis beyond the simple truss model. The use of the simple truss model is still the main analysis tool of the engineer, but advanced tool for evaluating performance of space truss towers, including effects other than simple truss action are now being utilized. In this respect, the above mentioned authors presented three - advanced tower analysis programmes viz AK TOWER programme, Modular Reliability Analysis MORENA Programme and the LIMIT programme. These computer programmes offer the engineer with effective facilities to determine tower failure mechanism and member loads. Furthermore, the programme successfully modelled non-linear bahviour, post buckling performance and capacity variation of the lattice towers structural system.

Sadiq et al (2005) presented an investigation into the dynamic analysis of multi-level guyed masts using the Stiffness Method. The displacements and reactions of the masts were obtained from a linear dynamic analysis of the structure based on the Stiffness Matrix Method. The mast shaft and guy ropes problems were solved separately, forcing compatibility of displacements at the points of guy attachment. The model was then used to analyse a 60m two level guyed mast at wind speeds of 80km/h, 160Km/h and 240Km/h in the positive X-direction. Their analysis showed that the bending moment values for the mast shaft at the third node points decreased gradually between zero and 3 seconds elapsed time, thereafter increased sharply. The axial forces on the shaft also decreased gradually from a peak value at about 0.2 seconds elapsed time to a minimum value at about 3.0 seconds, increasing sharply thereafter.

In this paper, the Stiffness Matrix Method is used in conjunction with computer-aided solution to conduct the structural stability analysis of self supporting telecommunication towers subjected to wind loads. Two types of lattice arrangement (the Triangular and the Cross Lattice) were considered for the diagonal bracing. Graphs showing the character of the base moments, shaft base reactions as well as joint displacements at some node points with respect to various wind velocities are presented.

2.0 THE STIFFNESS MATRIX METHOD

The stiffness matrix method was extensively developed during the 1950's for the analysis of skeletal structures, however, as soon as the computer was available to solve large sets of simultaneous equations, it became a powerful tool for structural analysis. The 1960s saw the extension of the method to encompass structural continuum such as plates and shells and the name Finite Element Analysis was applied to this development.

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \vdots \\ P_n \end{bmatrix} = \begin{bmatrix} \frac{\partial P_1}{\partial \Delta_1} & \frac{\partial P_2}{\partial \Delta_1} & \cdots & \frac{\partial P_n}{\partial \Delta_1} \\ \frac{\partial P_1}{\partial \Delta_2} & \frac{\partial P_2}{\partial \Delta_2} & \cdots & \frac{\partial P_n}{\partial \Delta_1} \\ \frac{\partial P_1}{\partial \Delta_3} & \frac{\partial P_2}{\partial \Delta_3} & \cdots & \frac{\partial P_n}{\partial \Delta_3} \\ \vdots & \vdots & & \vdots \\ \frac{\partial P_1}{\partial \Delta_n} & \frac{\partial P_2}{\partial \Delta_n} & \cdots & \frac{\partial P_n}{\partial \Delta_n} \end{bmatrix} \times \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \vdots \\ \Delta_n \end{bmatrix} \quad (1)$$

The left hand side (LHS) of equation (2.1) above represents external forces applied on the system (including reactions) while the column matrix on the right hand side (RHS) represents the displacement of the nodal point (including those at the supports).

If the elements $\frac{\partial P_i}{\partial \Delta_j}$ of the square matrix are all known the solution of this equation will yield the unknown joint displacement Δ_i. The whole problem then reduces to the determination of the square matrix and subsequently determining the joint displacement.

Mathematically, the term $\frac{\partial P_i}{\partial \Delta_j}$ for a specified i and j represents the variation of P_i with respect to Δ_j while other Δs are kept unchanged. Physically, it represents the holding force required (or developed) at point "i" in order to keep the body in equilibrium when a unit displacement is introduced at point "j".

Since, by Castigliano's theorem, it can be shown that

$$\frac{\partial P_i}{\partial \Delta_j} = \frac{\partial P_j}{\partial \Delta_i} \quad (2)$$

The square matrix shown in equation (1) is obviously a symmetric system. Thus, interchanging rows with corresponding columns, equation (1) can be re- written as:

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \vdots \\ P_n \end{bmatrix} = \begin{pmatrix} \frac{\partial P_1}{\partial \Delta_1} & \frac{\partial P_1}{\partial \Delta_2} & \cdots & \frac{\partial P_1}{\partial \Delta_n} \\ \frac{\partial P_2}{\partial \Delta_1} & \frac{\partial P_2}{\partial \Delta_2} & \cdots & \frac{\partial P_2}{\partial \Delta_n} \\ \frac{\partial P_3}{\partial \Delta_1} & \frac{\partial P_3}{\partial \Delta_2} & \cdots & \frac{\partial P_3}{\partial \Delta_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial P_n}{\partial \Delta_1} & \frac{\partial P_n}{\partial \Delta_2} & \cdots & \frac{\partial P_n}{\partial \Delta_n} \end{pmatrix} \times \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \vdots \\ \Delta_n \end{bmatrix} \quad (3)$$

If a small displacement □i is introduced at point i while the other load points are prevented from varying or from undergoing any displacement, then from equation (2), the force required due to this displacement is

$$P_i = \frac{\partial P_1}{\partial \Delta_1} \text{ x } \Delta_1 + \frac{\partial P_2}{\partial \Delta_2} \text{ x } \Delta_2 + \frac{\partial P_2}{\partial \Delta_2} \text{ x } \Delta_1 + \frac{\partial P_n}{\partial \Delta_n} \text{ x } \Delta_1 \tag{4}$$

If we designate $\frac{\partial P_i}{\partial \Delta_j}$ as K_{ij}, then equation (3) can be re-written in a short form as,

$$P = K\Delta \tag{5}$$

Where,

P = Vector of the forces including those at the supports
Δ = Vector of displacement including those at the supports.
K = Stiffness matrix.

The formation and solution of equation. (5) is the main objective of the stiffness method. P, K, and Δ are the generalised force vector, stiffness matrix of the structure and the generalised displacement respectively

2.1 Stiffness of a Structural Member

The stiffness of a structural member is often understood as the amount of force required to introduce a certain amount of deflection. The stiffness K of an entire structure is normally expressed in a common co-ordinate system (GLOBAL SYSTEM).

Before assembly of the stiffness of the entire structure, the member stiffnesses evaluated in the local co-ordinate frames are expressed in the Global co-ordinate frame by co-ordinate transformation.
In a more compact form, the relationship between the displacement and the generalized forces in the local coordinate can be represented in matrix form as;

$$\begin{pmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \\ P_{10} \\ P_{11} \\ P_{12} \end{pmatrix} \begin{pmatrix}
\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{2EA}{L^3} & 0 & 0 & 0 & \frac{-6EA}{L^2} & 0 & \frac{12EA}{L^2} & 0 & 0 & 0 & \frac{6EA}{L^2} \\
0 & 0 & \frac{12EA}{L^2} & 0 & \frac{6EA}{L^2} & 0 & 0 & 0 & \frac{12EA}{L^3} & 0 & \frac{-6EA}{L^2} & 0 \\
0 & 0 & 0 & \frac{GL}{L} & 0 & 0 & 0 & 0 & 0 & \frac{GL}{L} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{4EI_Y}{L} & 0 & 0 & 0 & \frac{-6EA}{L^2} & 0 & \frac{-2EI_Y}{L} & 0 \\
0 & \frac{6EI_Z}{L} & 0 & 0 & 0 & \frac{4EI_Z}{L} & 0 & \frac{6EA}{L^2} & 0 & 0 & 0 & \frac{-2EI_Y}{L} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{12EA}{L^2} & 0 & 0 & 0 & \frac{6EA}{L^2} & 0 & \frac{2EA}{L^3} & 0 & 0 & 0 & \frac{-6EA}{L^2} \\
0 & 0 & \frac{12EA}{L^3} & 0 & \frac{-6EA}{L^2} & 0 & 0 & 0 & \frac{12EA}{L^2} & 0 & \frac{6EA}{L^2} & 0 \\
0 & 0 & 0 & \frac{GL}{L} & 0 & 0 & 0 & 0 & 0 & \frac{GL}{L} & 0 & 0 \\
0 & 0 & \frac{-6EA}{L^2} & 0 & \frac{-2EI_Y}{L} & 0 & 0 & 0 & 0 & 0 & \frac{4EI_Y}{L} & 0 \\
0 & \frac{6EA}{L^2} & 0 & 0 & 0 & \frac{-2EI_Y}{L} & 0 & \frac{6EI_Z}{L} & 0 & 0 & \frac{4EI_Z}{L} &
\end{pmatrix} \begin{pmatrix} \\ \\ \\ \delta_4 \\ \delta_5 \\ \delta_6 \\ \delta_7 \\ \delta_8 \\ \delta_9 \\ \delta_{10} \\ \delta_{11} \\ \delta_{12} \end{pmatrix}$$

In a reduced form, the Matrix represented in equation (6) can be written as:

$$\begin{bmatrix} P_{ij} \end{bmatrix} \quad K_{ij} \quad = \quad \begin{bmatrix} K_{ij} \end{bmatrix} \begin{bmatrix} \delta_{j} \end{bmatrix}$$

x

(7)

P_{ij} K_{ij}

K_{ij} δ_{j}

Where the first subscript designate the end of the member under consideration i.e. P_{ij} represents the force at the i end while δ_{ji} represents the displacement at j end. For Kij, the first subscript represents the end where the forces are developed and the second subscript denotes the end where the displacements are introduced.

K^{j}_{ii} represents the stiffness of the member at i - end and will be referred to as the direct stiffness of member i j. It actually relates the displacement and the forces at the same end (i.e. i end) of the member. The superscript ^{ij} is added to designate that it belongs to member $_{ij}$

K_{ij} relates the forces at i - end to the displacement at j end; it is referred to as CROSSED STIFFNESS of member ij.

From equation (7), if there is a member lying between i and j, then.

$$P_{ij} = K^{j}_{ij}\,\delta_{i} + K_{ij}\,\delta_{ji}$$

$$P_{ji} = K_{ji}\,\partial + K_{ii}^{\,i}\,\delta_{ii} \qquad \delta_{ii}$$

(8)

It is worthy of note that for Prismatic members,

$$K_{ji} = K_{ij}^{T} \quad \text{and} \quad K_{ii}^{i} = K_{ii}^{j} \qquad (9)$$

These relations are not true for non prismatic members and for members with different end conditions.

When a structural member does not have all the six degrees of freedom, certain rows and columns of equation (6) will not be needed for the member. For instance, the members in plane rigid frames possess only 3-degrees of freedom and only rows and columns corresponding to δ_1, δ_2 and δ_6 are needed.

2.0 STRUCTURAL BEHAVIOUR OF TELECOMMUNICATION LATTICE TOWER STRUCTURE

In the analysis of tower structures, the largest uncertainty is accurate knowledge of the wind load. When lattice tower structures are subjected to lateral loads due to wind or other meteorological factors, two principal actions, namely overturning moment and horizontal shear, are produced in the structure. The external shear forces due to the environmental loading create axial compression in some of the main members of the horizontal plan bracing. This axial load is normally resisted by the structural members until the limit point load of the individual member is exceeded, thereby giving way for local buckling, which may result in the ultimate failure of the whole structure .Consequently, the bracing system in any lattice tower structure must be designed to be stiff enough to withstand the wind-induced horizontal shear force and the resulting member axial compression or tensile forces.

Also, when lattice tower structures are subjected to dynamic loading due to gust wind, the towers undergo some oscillatory motion the frequency of which depends on the configuration, the dynamic characteristic of the structure as well as the return period of the gust wind. However, unlike the guyed Mast, the free standing lattice tower is less sensitive to dynamic load, as the vibration induced in it has lower frequency due to the formidability of its foundation.

For most lattice tower structures a static linear three dimensional structural analysis is normally sufficient. However, for lattice tower structures with large complicated panel bracing, the secondary bracing forces can be significantly altered by non-linear effects caused by curvatures of the panels under the influence of the design load. Generally, the rules in the code are sufficient to take care of this effect but where structures are of particular importance or where there is much repetition of a design, a non-linear analysis may be necessary.

3.0 LOADING:

Basically, the loads acting on the tower include dead loads and environmental loads. The dead load is made up of the dead weight of the structural members and all the ancillaries while environmental loads on tower structures are usually due to wind and ice, some times, the combination of the two. Occasionally, specification requires that towers be designed for full wind and ice loads acting together. However, most codes do not require this since ice is unlikely to build up under maximum wind. Earthquake forces are significant in the design of some towers of high mass, especially those supporting water tanks, loading from climatic

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Generally, the forces due to wind load are critical in tall structures. These forces are dependent on a number of factors which include: the shape of the tower (triangular or rectangular), its orientation, the shape of the cross section of members and the ratio of the effective to total area of the face.

The action of wind can be classified into static and dynamic effects. The static wind loading is the steady average wind pressure acting on the tower and tending to give it a displacement in the wind direction. The dynamic loading on the other hand, tends to set the tower into an oscillatory motion in accordance with the dynamic properties of the wind.

Most wind codes use simple quasi-static method of assessing wind loads, which has some limitations, for calculating the long wind response, but it is adequate for the majority of structures. Wind loads are specified in terms of a 50 - years return mean hourly wind pressure together with gust factors which convert the forces to an equivalent static gust. This explains why the forces near the top of towers are relatively higher due to an allowance for dynamic response. Though, wind loads are generally distributed over the length of the Mast, they are replaced by equivalent nodal forces applied at the joint, for the purpose of analysis.

4.1 ***Load Cases:***

Loads represent actions upon the structure. A spatial distribution of loads upon the structure is referred to as a load case. Primarily, for the purposes of analysis in the present model, two load cases were considered- wind load and the dead load for static analysis.

The wind load is the lateral load on the structure due to the effect of wind velocity. Lateral loads resulting from the impact of wind moving at various velocities (15 m/s, 30m/s, 45m/s, 60m/s, 75m/s, 90m/s, 105m/s, 120 m/s) were computed using a sub-programme developed spread sheet using the equation listed in Sadiq et al (2005). The lateral loads due to wind velocity were converted into point loads incident on the nodes of the modelled tower in the direction perpendicular to the Z - Y plane and in the positive X-direction. Tables 1 and 2 show a summary for the computation of lateral nodal loads applied to the tower for wind velocity of 45m/s and 60 m/s respectively. Figures 1 and 2 depict the masts being considered.

Figure 1: Mast Model 5a

Figure 2: Mast Model 6

Table 1: Summary Table of Load Conversion when wind velocity is 45m/s

H	V_o	L	V_z	V_s	q	W	$\acute{\alpha}^{I}$	$\acute{\alpha}^{II}$	W_u	W_L	W_z
(m)	(m/s)	(m)	(m/s)	(m/s)	KN/m^2				KN	kN	kN
2	45	2	35.757	46.4124	1.3204719	0.5942124	1	0	1.188425	0	1.84129
4	45	2	39.479	51.2435	1.6096733	0.724353	0.5493493	0.4506507	0.795846	0.6528603	1.56211
6	45	2	41.833	54.2993	1.8073802	0.8133211	0.5289294	0.4710706	0.860379	0.7662632	1.70711
8	45	2	43.588	56.5774	1.9622137	0.8829962	0.5205372	0.4794628	0.919265	0.8467277	1.8304
10	45	2	45	58.41	2.0913893	0.9411252	0.5159334	0.4840666	0.971116	0.9111345	1.93675
12	45	2	46.187	59.9513	2.2032212	0.9914495	0.51302	0.48698	1.017267	0.9656321	2.03055
14	45	2	47.216	61.2862	2.3024264	1.0360919	0.511009	0.488991	1.058905	1.0132792	2.11476
16	45	2	48.125	62.4665	2.3919653	1.0763844	0.5095368	0.4904632	1.096915	1.0558539	2.19141
18	45	2	48.942	63.5265	2.4738302	1.1132236	0.5084123	0.4915877	1.131953	1.0944941	2.26193
20	45	2	49.684	64.4899	2.5494321	1.1472445	0.5075252	0.4924748	1.164511	1.129978	2.32737
22	45	2	50.365	65.374	2.6198109	1.1789149	0.5068074	0.4931926	1.194966	1.1628641	2.38853
24	45	2	50.995	66.1916	2.6857567	1.2085905	0.5062148	0.4937852	1.223613	1.1935683	2.44602
26	45	2	51.582	66.9529	2.7478859	1.2365487	0.5057171	0.4942829	1.250688	1.2224098	2.50033
28	45	2	52.131	67.6654	2.8066892	1.2630101	0.5052932	0.4947068	1.276381	1.2496393	2.55184
30	45	2	52.647	68.3357	2.8625643	1.2881539	0.5049279	0.4950721	1.30085	1.2754581	2.60088
32	45	2	53.135	68.9686	2.9158384	1.3121273	0.5046098	0.4953902	1.324224	1.3000301	2.64772
34	45	2	53.597	69.5685	2.9667845	1.335053	0.5043302	0.4956698	1.346615	1.3234909	2.69257
36	45	2	54.036	70.1389	3.0156328	1.3570348	0.5040827	0.4959173	1.368115	1.3459542	2.73563
38	45	2	54.455	70.6827	3.0625793	1.3781607	0.5038619	0.4961381	1.388805	1.3675161	2.77706
40	45	2	54.856	71.2026	3.1077926	1.3985067	0.5036637	0.4963363	1.408754	1.3882591	1.40875

H = Vertical height of Tower
Vo = Prevalent Wind Velocity
L = Projected height of bar
$\mathbf{V_z}$ = Wind Velocity at vertical height, z
$\mathbf{V_s}$ = Calculated design Speed
q = Calculated Wind Pressure
ά = Distribution Factor
$\mathbf{W_u}$ = Upper Nodal force on Tower member
$\mathbf{W_L}$ = Lower Nodal Force on Tower Member
$\mathbf{W_z}$ = Calculated Point Load on Tower member.

Table 2 Summary of Load Conversion when wind velocity is 60m/s

H_0	V_o	L	V_z	V_s	q	W	α^{I}	α^{II}	W_a	W_L	W_z
(m)	(m/s)	(m)	(m/s)	(m/s)	kN/m^2				kN	kN	kN
2	60	2	47.7	61.883246	2.347506	1.056378	1	0	2.1128	0	3.273396
4	60	2	52.6	68.324643	2.861641	1.287739	0.54935	0.45065	1.4148	1.160641	2.777082
6	60	2	55.8	72.399116	3.21312	1.445904	0.52893	0.47107	1.5296	1.362246	3.034856
8	60	2	58.1	75.436522	3.48838	1.569771	0.52054	0.47946	1.6342	1.505294	3.254043
10	60	2	60	77.88	3.718025	1.673111	0.51593	0.48407	1.7264	1.619795	3.443108
12	60	2	61.6	79.935105	3.916838	1.762577	0.51302	0.48698	1.8085	1.716679	3.60986
14	60	2	63	81.714923	4.093202	1.841941	0.51101	0.48899	1.8825	1.801385	3.75957
16	60	2	64.2	83.288673	4.252383	1.913572	0.50954	0.49046	1.9501	1.877074	3.895838
18	60	2	65.3	84.701957	4.39792	1.979064	0.50841	0.49159	2.0124	1.945767	4.021211
20	60	2	66.2	85.986491	4.532324	2.039546	0.50753	0.49247	2.0702	2.00885	4.137556
22	60	2	67.2	87.165268	4.657442	2.095849	0.50681	0.49319	2.1244	2.067314	4.246283
24	60	2	68	88.255511	4.774679	2.148605	0,50621	0.49379	2.1753	2.121899	4.348484
26	60	2	68.8	89.270476	4.885131	2.198309	0.50572	0.49428	2.2234	2.173173	4.445026
28	60	2	69.5	90.22059	4.98967	2.245351	0.50529	0.49471	2.2691	2.221581	4.536603
30	60	2	70.2	91.114212	5.089003	2.290051	0.50493	0.49507	2.3126	2.267481	4.623786
32	60	2	70.8	91.95815	5.183713	2.332671	0.50461	0.49539	2.3542	2.311165	4.70705
34	60	2	71.5	92.758027	5.274284	2.373428	0.50433	0.49567	2.394	2.352873	4.78679
36	60	2	72	93.518542	5.361125	2.412506	0.50408	0.49592	2.4322	2.392807	4.863345
38	60	2	72.6	94.243666	5.444585	2.450063	0.50386	0.49614	2.469	2.43114	4.937003
40	60	2	73.1	94.936783	5.524965	2.486234	0.50366	0.49634	2.5045	2.468016	2.504452

5.0 THE EXAMPLE MODEL PROBLEM

In the present work, a model of a 40-m high free standing tower is analyzed using the SAP 2000 application software in conjunction with the stiffness matrix method briefly described above. The description of the sample model and the result obtained from the analysis are presented and discussed below.

The tower has a total vertical height of 40m. At the base, the shafts are positioned at the four angle of a square of length 4m. The shafts are slightly inclined such that the top, the horizontal brace forms a square of 0.70m in length.

Horizontal braces are connected to the four shafts to form a square of varying lengths in plan view, at different vertical height. The length of the horizontal brace reduce with height due to the inclination of the shaft. The bracing closest to the base has the maximum length of 4m while the top most brace has the minimum length 0.7m (i.e. at vertical height of 40-m). Between the base of the mast and vertical height of 30m, the horizontal brace are spaced at constant distance of 2m, beyond height 30m to the top of the mast, the space between successive horizontal bracing is reduced to a constant distance of 1m. See figures 1 and 2

By means of diagonal bracing, the shafts are connected in such a way as to form a lattice structure, as shown in figures 1 and 2 in order to produce a mast with adequate stiffness.

In the analysis, two forms of arrangements were considered for the diagonal bracing. The first is as shown in Fig 2 in which case the diagonal bracing are connected diagonally between two successive horizontal bracing. On the other hand, Fig 2 shows the second option in which case the bracing are connected between the edge and the middle of successive horizontal bracing to form series of triangular arrangement. These arrangements are commonly adopted in the telecommunication free- standing towers used for supporting the GSM Infrastructures.

5.1: Properties Assignment:

The application package used in the analysis of the model allows material properties to be assigned to all elements in the structure with great facility. Table 3 below shows the properties assigned for the two tower models i.e **model 5a and model 6a.**

Table 3: Parameters of the Tower

Model Name	SHAFT	HORIZONTAL BRACE	DIAGONAL BRACE
Model Mast 5a (FIG. 1)	Equal angle 120x120x10	Equal angle 70 x 70 x 8	Equal angle 70 x 70 x 8 Cross lattice arrangement.
Model Mast 6a (FIG.2)	Equal angle 120x120x10	Equal angle 70 x 70 x 8	Equal angle 70 x 70 x 8 Triangular lattice arrangement

As shown in the table 3 above, the properties of the material constituting the members used in the model are assigned and then the models identified as MAST 5a and MAST 6a respectively.

The graphical presentations, showing the variations of parameters such as base reaction, member and shaft forces (axial and moment) and displacement or deformation with respect to increasing wind velocity from normal moving wind up to the effect of gust wind are demonstrated in Figs 3 – 10.

6.0: DISCUSSION OF RESULTS:

6.1 Base Reaction:

Analysis shows that the base of the free standing mast is usually subjected to a significant amount of overturning moment and uplift forces as wind of different velocities move against it in different directions. The magnitude of the moment is usually very significant and increases according to the velocity of the prevalent wind.

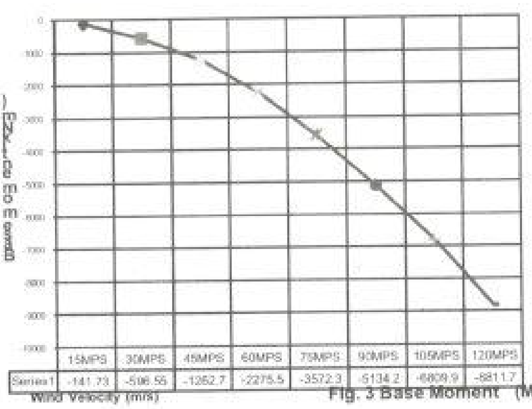

Fig. 3 Base Moment (Mast 5a)

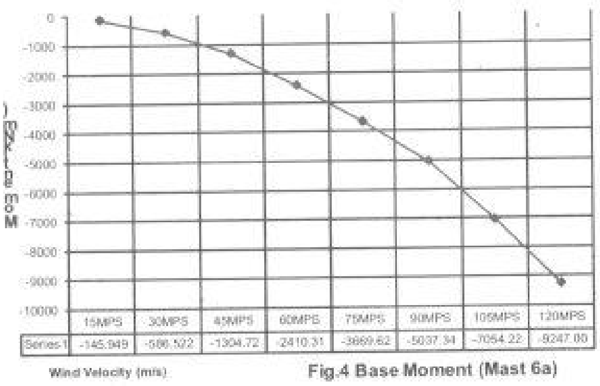

Fig.4 Base Moment (Mast 6a)

fact, in this analysis the effect of wind velocity up to a value of 120m/s was considered.

Besides, the base must also resist the resultant horizontal force due to the combined effect of lateral loads incident on different points on the wind ward face of the tower.

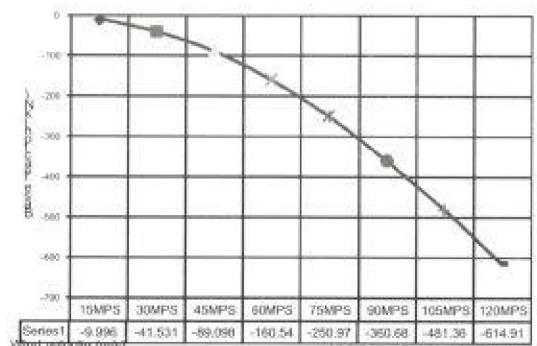

Fig. 5 Base Reaction (Mast 5a)

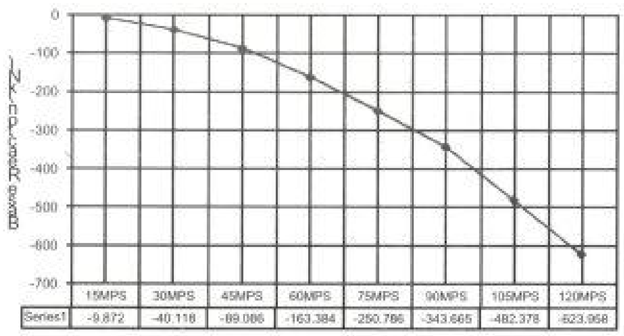

Fig.6 Base Reaction (Mast 6a)

Figs 7and 8 show the variation of shaft forces at the point of connection of the tower and the base with respect to the increase in wind velocity from 15m/s to 120m/s. The value of the reaction at these nodal points are plotted and shown in the referred figures

It will be seen that slope of the graphs are negative, this is due to the fact that the node points are located on the wind-ward direction, hence reaction of the shaft at this point will be downward as earlier noted. This implies that the design of the connection of the shaft base to the foundation of the structure must take adequate care of these negative forces which tend to cause uplift in the windward face of the tower as the tower is subjected to lateral loads due to wind moving at different velocities.

In summary the graphical presentation of the shaft base reaction for each of the cases for Mast 5a and Mast 6a show similar shape with general increase in the magnitude of the reaction as the velocity is increased from 15m/s to 120m/s.

The combined effects of dead load and wind load shows behaviour similar to that of exclusive wind effect. This shows that the wind load plays a significant role in the deformation of free standing telecommunication towers. At different nodal interval the shaft is also subjected to bending moment due to the lateral forces resulting from the effect of the wind load. These moments can be read from the table of results for member forces

6.3 Displacement:

The results for joint displacement (for the 2 cases- Mast 5a and Mast 6a) reveal that the displacement of the nodes is generally more significant in the windward direction. The graphs in fig 9 and 10 also show that displacement increases with height just as it increases with an increasing wind velocity. Although the increase is not directly linear, the graphs depict a distinct increase in the value of displacement as the wind velocity is varied from 15m/s to 120m/s

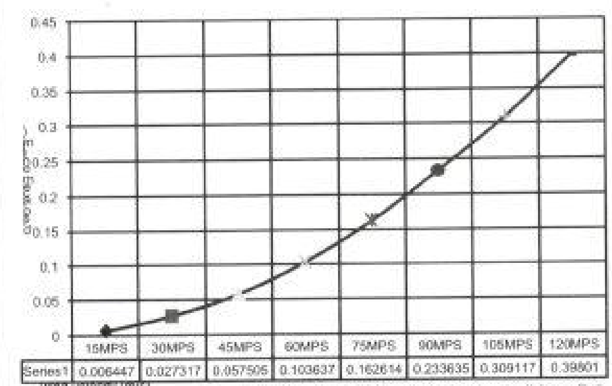

Fig 9 Displacement at Mast top (Mast 5a)

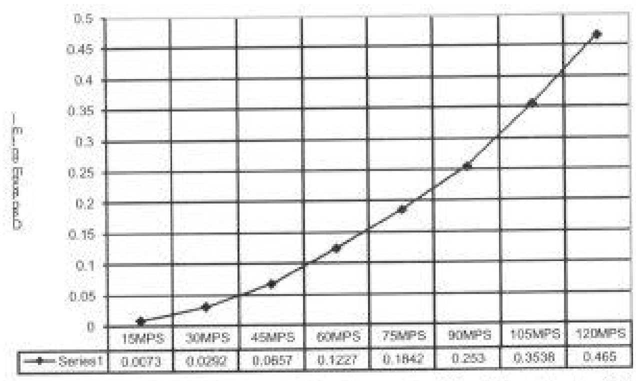

Fig. 10 Displacement at Mast Top (mast 6a)

Fig 11: Pictorial view of the deformed pattern (under lateral loads) of the tower Mast Model

Note that the deformation pattern is almost the same for the two models

7.0 CONCLUSION AND RECOMMENDATION

This work exhibits the great facilities offered by computer application in modelling of telecommunication free standing towers and studies the behaviour of such structure under lateral load resulting from the effect of wind moving at different velocities. It also determines the member forces, joint displacement, base reaction and other important parameters needed for optimum design of the structure.

As confirmed in the analysis results, unlike the guyed masts, the free standing masts depend primarily on the foundation for their stability. This is quite evident in the magnitude of shear forces and the moments at the base the modelled mast were subjected to.

In view of this fact it is hereby recommended that the free standing mast must be duly analysed before design. At the same time, the construction exercise must be carefully supervised to take adequate care of the reactions listed below as their effects are usually quite significant in ensuring the stability of the whole structure. Their magnitudes depend on the design wind velocity. These reactions are:

- Overturning moment
- Shaft uplift forces
- Shaft bending moment
- Axial forces transferred to the base through the shaft

In the same vein, the bracing members which give additional rigidity and enhance the stiffness of the tower must be carefully sized by ensuring that a proper analysis is carried out before finally selecting these elements. This would help to eliminate completely the occurrence of structural failure due to over stressing of members which may eventually lead to the ultimate failure of the whole mast.

The lateral deflection must also be controlled to the minimum bearable degree as this may lead to temporary lost of signals incase of telecommunication or transmission towers. This can be controlled by ensuring adequate bracing of the mast at the top section where the lateral deflection is likely to be maximum as shown in the result of the analysis. Analysis of the results presented reveals that the towers with cross lattice arrangement (model 5a) proves to be more stable in terms of resistance to deformation than the towers with triangular lattice arrangement (model 6a), as the values of the base moment, base reaction, shaft reaction at various wind velocities are generally lesser in the former than in the Latter.

In the final analysis, it was shown that a complete static analysis of telecommunication free standing towers of any given size and dimension can be achieved using properly developed computer model which will return all the necessary results needed for the design optimization. Telecommunication towers are no doubt a set of structures which analysis may be very complex. Their design should therefore be preceded with an extensive analysis backed up with a reliable 3-D model which perfectly represents the proposed structure. This would facilitate the determination of the structural behaviour of the intended tower under different loading conditions.

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