ANALYSIS OF YIELDED STEEL PLATE USING TOTAL STRAIN ENERGY THEORY

E. I. Adah[1], L. O. Onundi[2], A. A. Adedeji[3], and D. E. Ewa[4],

[1]Department of Civil and Environmental Engineering, University of Calabar, Calabar

[2]Department of Civil and Water Resources Engineering, University of Maiduguri, Maiduguri

[3]Department of Civil Engineering, University of Ilorin, Ilorin

[4]Department of Civil Engineering, University of Cross River State, Calabar.

edwardadah@unical.edu.ng

ABSTRACT

This study aims to formulate an improved total strain energy yield theory equation for plane elements. This was done by formulating an applied stress equation for the analysis of plane elements such as rectangular plates. The mathematical Equation of total strain energy theory was modified to obtain a new general applied stress equation in terms of yield stress and stress factor. Polynomial shape parameters for the plate types considered were solved for n-values (plane vectors) which were substituted into the new general stress factor equation to obtain the specific stress factor equations for each plate type. Numerical application was made with mild steel as the plate material. With the values of the stress factor for each of the twelve plate types obtained, the applied stress for each plate type was determined. Based on the values obtained, it was noticed that the values of the stress factors for the six plate types without a free edge were less than 1, while those values for the other six plate types with a free edge were greater than 1. As a result of these values of stress factors, the applied stresses for the plate types with a free edge were greater than the yield stress of 250MPa, which implies that failure is imminent. This indicates the weak point of this yield theory in its original form. For the structure not to fail, a factor of safety is introduced thereby resulting in the design stress which is less than than the yield stress of the material, thereby providing a new equation for predicting the design stress of the plate types considered. Hence, this new design stress equation is adequate for predicting stress for the design of plane elements such as plates.

Keywords: Total Strain Energy, Yield, Design stress, Stress Factor, Plane Element

1.0 INTRODUCTION

Yield criterion analysis is a fundamental concept in materials science and engineering that is used to predict plastic deformation or yield under certain loading conditions at a given time (Mandalawi and Sabry 2020). Different materials respond differently under load. Hence, engineers consider this criterion very important when designing structures or components to ensure the elements can withstand the expected stresses without experiencing permanent deformation. Muhammad (2023) states that yield occurs, when a material's stress exceeds a certain threshold, known as the yield strength. As such, yield criteria help engineers predict the point at which materials will start to deform plastically. Benhan and Warnock (1976) show that yielding is not caused by hydrostatic pressure or tension. They further stated that any complex stress can be regarded as a combination of hydrostatic stress and a function of the difference of principal stresses. In yield criterion analysis, stress is typically compared to the yield criteria to determine whether the material will yield under the given conditions.

The process involves calculating the stress components and using mathematical models specific to each criterion (Abspoel et al., 2017). They further stated that yield criteria analysis often involves the use of safety factors to ensure that the material does not fail even, if it has reached the yield condition. This is because materials in real life can exhibit variations and uncertainties that the theoretical yield criteria might not fully captured. Generally, yield criterion analysis is a crucial tool in materials engineering to ensure the safe and efficient design of structures and components subjected to various types of loading (Team Xometry, 2016).

The importance of yield strength in materials has led scholars to the formulation of some yield theories such as maximum distortion energy theory. M.T.Huber who in 1904 as cited by Karol Frydrych (Mechanical, web of mechanics and mechanicians) and Ross (1987), formulated the maximum distortion energy theory of yield, which was further developed by R. von Mises in 1913. This is generally called the von Mises criterion and is highly applicable to ductile materials. It doesn't take into account the hydrostatic (uniform) stress component and assumes that yielding depends on the deviatoric stress (Lew, 2021).

Also, the Tresca criterion or maximum shear stress theory, predicts that yielding will occur when the maximum shear stress in the material reaches a critical value called in stability analysis as critical buckling load. Unlike the von Mises criterion, Tresca considers the role of the hydrostatic stress component in yielding (Roostaei and Jahed, 2022). Comanici and Barsanescu, (2018) and Juvinal & Marshek (1991) mentioned Mohr-Coulomb Criterion, which is used for brittle materials like concrete and rock. The criterion is based on the concept of effective stresses and takes into account the angle of internal friction and cohesion of the material. Here, yielding will occur when the Mohr-Coulomb failure envelope is exceeded.

The extension of the Mohr-Coulomb criterion known as the Drucker-Prager criterion, is often used for materials that exhibit both ductile and brittle behaviour. Both tensile and compressive strength are considered with the introduction of a pressure-dependent term to account for the effect of hydrostatic stress. Roscoe et al. (1958), highlighted the Modified Cam-Clay Model criterion, use often in geotechnical engineering to predict the yield behaviour of soils. The effects of stress history and pore water pressure on soil yield are considered. The total strain energy theory proposed by Beltrami & Haigh as cited by Benhan & Warnock (1976) and Ross, (1987), states that elastic failure will occur, when the total strain energy per unit volume, at a point, reaches the total strain energy per unit volume of a specimen made from the same material, when it is subjected to a simple uni-axial test.

However, the total strain energy criterion is not being used due to the discrepancy experienced in its predicted result compared with the experimental values. The reason may be that the stress factor of the criterion may be less than zero in most cases, leading to higher stress than the material yield strength. Also, this criterion is generic and mostly applicable to line element and shell structures such as cylinders. Benhan & Warnock(1976) said this method is unsafe. Therefore, the aim of this study is to improve this theory by formulating a new stress equation applicable to plane elements such as plates to help predict the design stress of ductile materials using polynomial shape parameters. This will ensure the use of the theory for yield analysis with ease and with safety and confidence in mind.

2.0 MATHERIALS AND METHOD

In this study, the existing general total strain energy equation as proposed by Beltrami & Haigh will be started. Based on the equation, a new applied stress equation for plane element such as rectangular plates will be formulated in terms of stress factor. And using the polynomial shape parameters given by Ibearugbulem et al. (2014) for twelve plate types based on edge support conditions, with the consideration of the point of maximum deflection on the plate, the stress factor will be determined for each of the plate type under consideration.

A rectangular plate with four edges and numbered from the top edge in an anti-clockwise direction is shown in Figure 1.0.

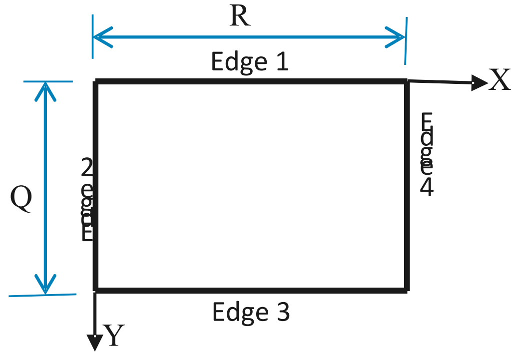
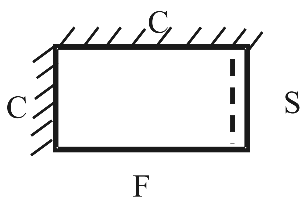


Figure 1.0: Edge numbering of rectangular plate

Figure 2.0: shows a plate clamped (C) at edge 1 and 2, free (F) at edge 3 and simply supported (S) as edge 4, named CCFS plate type.



CCFS

Figure 2.0: A CCFS rectangular plate

Based on the above numbering in the Figure 1.0 and considering the type of edge support as depicted in Figure 2.0, the twelve plate types with their deflected shape parameters are presented in Table 1.

Table 1: Plate types and their deflected shape parameters for twelve plate types under consideration

PLATE TYPE	SHAPE PARAMETER, H
CCCC	$(R^2 – 2R^3 + R^4) (Q^2 – 2Q^3 + Q^4)$
SSSS	$(R – 2R^3 + R^4) (Q – 2Q^3 + Q^4)$
CSSS	$(R – 2R^3 + R^4) (1.5Q^2 – 2.5Q^3 + Q^4)$
CSCS	$(R – 2R^3 + R^4) (Q^2 – 2Q^3 + Q^4)$
CCSS	$(1.5R^2 – 2.5R^3 + R^4) (1.5Q^2 – 2.5Q^3 + Q^4)$
CCCS	$(1.5R^2 – 2^3 + R^4) (Q^2 – 2Q^3 + Q^4)$
SSFS	$(R – 2R^3 + R^4) (7/3\ Q – (10/3\ Q^3 + (10/3\ Q^4 – Q^5)$
SCFS	$(1.5R^2 – 2.5R^3 + R^4) (7/3\ Q – (10/3\ Q^3 + 10/3\ Q^4 – Q^5)$
CSFS	$(R – 2R^3 + R^4) (2.8Q^2 – 5.2Q^3 + 3.8Q^4 – Q^5)$
CCFS	$(1.5R^2 – 2R^3 + R^4) (2.8Q^2 – 5.2Q^3 + 3.8Q^4 – Q^5)$
SCFC	$(R^2 – 2R^3 + R^4) (7/3\ Q – 10/3\ Q^3 + 10/3\ Q^4 – Q^5)$
CCFC	$(R^2 – 2R^3 + R^4) (2.8Q^2 – 5.2Q^3 + 3.8Q^4 – Q^5)$

Ibearugbulem et al. (2014)

2.1 Mathematical Formulation

The limit state or the ultimate bearing capacity of steel structural component subjected to combined axial, bending and shear stresses under the most unfavourable conditions of loadings is usually determined by the "equivalent stress". This is based on the assumptions that the transition of steel from elastic to plastic state is equivalent to the effect of these complex combinations at the moment it reaches the plastic state. Assuming that steel is an ideal elastoplastic material in which the development of changes in the shape does not simultaneously affect or change in any way its gross volume. In addition, during this transition, the work done in connection with this change in shape reaches its maximum value but the work done in connection to the change in volume is equal to zero. Therefore, Beltram and Hyber's energy theory (Wright 1983), the work done to change the volume, $W_{vol.}$ is

$$W_{vol.} = \frac{1}{6E} (\sigma_x + \sigma_y + \sigma_z)^2 (1 - 2?) = 0 \quad \ldots\ldots\ldots\ldots\ldots \quad 1$$

Therefore, the maximum possible value of the Poisson ratio 'u' for steel is 0.5
$\sigma_{x,}$ $\sigma_{y,}$ σ_z are principal stresses along the three axes x, y and z
E = Modulus of elasticity of steel.
The average value for the Poisson ratio for steel is 0.3.
Equation (1) can only be useful for practical purposes if (1 – 2?) = 0
Similarly, the work done to change the shape

$$W_{sh} = \frac{1+?}{3E} \left[(\sigma_x^2 + \sigma_y^2 \;\; \sigma_z^2) - (\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x)\right] \quad \ldots\ldots\ldots\ldots\ldots \quad 2$$

Since ? = 0.5

$$W_{sh} = \frac{1}{2E} \left[(\sigma_x^2 + \sigma_y^2 \;\; \sigma_z^2) - (\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x)\right] \quad \ldots\ldots\ldots\ldots\ldots \quad 2$$

The maximum work done is expected when the steel has yielded,
Therefore,

$$maxW_{sh} = \frac{f^2y}{2E} \geq \frac{1}{2E} \left[(\sigma_x^2 + \sigma_y^2 \;\; \sigma_z^2) - (\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x)\right] \quad \ldots\ldots\ldots\ldots\ldots \quad 3$$

Similarly, for an elastic limit state, the three principal stresses of three-dimensional stresses, the total strain energy per unit volume for steel considered according to (Rose, 1987) is given as Equation (1d).

$$U_T/\text{vol} = \frac{1}{2} (\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \sigma_z\varepsilon_z) \quad \ldots\ldots\ldots\ldots\ldots \quad 4$$

But strains for isotropic plates are given as

$$\varepsilon_x = \frac{1}{E} (\sigma_x - ?(\sigma_y + \sigma_z) \quad \ldots\ldots\ldots\ldots\ldots \quad 5a$$

$$\varepsilon_x = \frac{1}{E} (\sigma_y - ?(\sigma_x + \sigma_z) \quad \ldots\ldots\ldots\ldots\ldots \quad 5b$$

$$\varepsilon_x = \frac{1}{E} (\sigma_z - ?(\sigma_x + \sigma_y) \quad \ldots\ldots\ldots\ldots\ldots \quad 5c$$

E is the Young modulus of elasticity, and ? is the Poisson ratio.
Substituting Equations (5) in Equation (1d) yields

$$U_T/\text{vol} = \frac{1}{2E} \left[\sigma_x(\sigma_x - ?\,(\sigma_y + \sigma_z) + \sigma_y(\sigma_y - ?\,(\sigma_x + \sigma_z) + \sigma_z(\sigma_z - ?\,(\sigma_x + \sigma_y)\right] \ldots\ldots\ldots \quad 6$$

Opening up internal bracket yields

$$U_T/vol = \frac{1}{2E}\left[\sigma_x^{2} - ?\sigma_x\sigma_y - ?\sigma_x\sigma_z + \sigma_y^{2} - ?\sigma_x\sigma_y - ?\sigma_y\sigma_z + \sigma_z^{2} - ?\sigma_x\sigma_z - ?\sigma_y\sigma_z\right] \quad .... \quad 7$$

Collecting like terms yields

$$U_T/vol = \frac{1}{2E}\left[\sigma_x^{2} + \sigma_y^{2} + \sigma_z^{2} - 2?(\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z)\right] \quad .... \quad 8$$

For simple uni-axial test

$$\sigma_x = f_y; \;\; \sigma_y = \sigma_z = 0 \quad .... \quad 9$$

where f_y is the yield stress of a material
Thus, Equation (8) reduces to

$$U_T/vol = \frac{1}{2E}\left[f_y^{2}\right] = U_{0max}/vol. \quad .... \quad 10$$

Equating Equation (8) and Equation (10) yields

$$U_T/vol = \frac{1}{2E}\left[\sigma_x^{2} + \sigma_y^{2} + \sigma_z^{2} - 2?(\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z)\right] = \frac{1}{2E}\left[f_y^{2}\right] \quad .... \quad 11$$

That is

$$\sigma_x^{2} + \sigma_y^{2} + \sigma_z^{2} - 2?(\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z) = f_y^{2} \quad .... \quad 12$$

For plane stress case, $\sigma_z = 0$

Equation (12) becomes

$$\sigma_x^{2} + \sigma_y^{2} + - 2?\sigma_x\sigma_y = f_y^{2} \quad .... \quad 13$$

$$\sigma_x^{2}\left[1 - 2?\frac{\sigma_y}{\sigma_x} + \frac{\sigma_y^{2}}{\sigma_x^{2}}\right] = f_y^{2} \quad .... \quad 14$$

$$\sigma_x^{2}\left[1 - 2?m_1 + m_1^{2}\right] = f_y^{2} \quad .... \quad 15$$

Where

$$m_1 = \frac{\sigma_y}{\sigma_x} \quad .... \quad 16$$

$$\sigma_x^{2} = \frac{f_y^{2}}{\left[1 - 2?m_1 + m_1^{2}\right]} \quad .... \quad 17$$

$$\sigma_x = \frac{f_y}{\left[F_{TST}\right]} \quad .... \quad 18$$

Where

$$F_{TST} = \sqrt{1 - 2?m_1 + m_1^{2}} \quad .... \quad 19$$

But stresses in a two-dimensional plane from the theory of elasticity are given as

$$\sigma_x = -\frac{EZ}{1-?^{2}}\left(\frac{\partial^2 \mathbf{w}}{\partial \mathbf{x}^2} + ?\frac{\partial^2 \mathbf{w}}{\partial \mathbf{y}^2}\right) \quad .... \quad 20$$

$$\sigma_y = -\frac{EZ}{1-?^{2}}\left(\frac{\partial^2 \mathbf{w}}{\partial \mathbf{x}^2} + ?\frac{\partial^2 \mathbf{w}}{\partial \mathbf{y}^2}\right) \quad .... \quad 21$$

Where Z is the plate thickness.
The displacement shape function, w, is given as

$$x = aR,\ y = bQ \quad 0 \le P \le 1.0 \le Q \le 1, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 23$$

Substitute Equation (22) and Equation (23) in Equation (20) and Equation (19) yield equations Equation (24) and Equation (25) respectively.

$$\sigma_x = -\frac{EAZ}{\left(1 - ?^2\right) a^2}\left(\frac{\partial^2 h}{\partial R^2} + \frac{?\,\partial^2 h}{\text{Ƹ}^2 \partial Q^2}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 24$$

$$\sigma_y = -\frac{EAZ}{\left(1 - ?^2\right) a^2}\left(?\,\frac{\partial^2 h}{\partial R^2} + \frac{1\,\partial^2 h}{\text{Ƹ}^2 \partial Q^2}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 25$$

Where

Aspect ratio $\text{Ƹ} = \frac{a}{b}$ $\quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 26$

Substitute Equation (24) and Equation (25) into Equation (16) yields

$$m_1 = \frac{?\,\frac{\partial^2 h}{\partial R^2} + \frac{1\,\partial^2 h}{\text{Ƹ}^2 \partial Q^2}}{\frac{\partial^2 h}{\partial R^2} + \frac{?}{\text{Ƹ}^2}\frac{\partial^2 h}{\partial Q^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 27$$

$$m_1 = \frac{n_2}{n_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 28$$

Where,

n_i , i=1, 2 are vectors normal to principal x and y axes of the plate, therefore,

$$n_1 = \frac{\partial^2 h}{\partial R^2} + \frac{?}{\text{Ƹ}^2}\frac{\partial^2 h}{\partial Q^2} = \frac{\partial^2 h}{\partial R^2} * h_y + \frac{?}{\text{Ƹ}^2} * h_x * \frac{\partial^2 h_y}{\partial Q^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 29$$

$$n_2 = ?\,\frac{\partial^2 h}{\partial R^2} + \frac{?}{\text{Ƹ}^2}\frac{\partial^2 h}{\partial Q^2} = ?\left(\frac{\partial^2 h_x}{\partial R^2} * h_y\right) + \frac{?}{\text{Ƹ}^2}\left(h_x * \frac{\partial^2 h_y}{\partial Q^2}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 30$$

Equation (29) and Equation (30) are the general n-value equations.
Substituting Equation (28) into Equation (19) yields

$$F_{TST} = \sqrt{\left[1 - 2?\,\frac{n_2}{n_1} + \frac{n_2^{\,2}}{n_1^{\,2}}\right]} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 31$$

Substituting Equation (31) into Equation (18) yields

$$\sigma_x = \frac{f_y}{\sqrt{\left[1 - 2?\,\frac{n_2}{n_1} + \frac{n_2^{\,2}}{n_1^{\,2}}\right]}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 32$$

Note: if $\sigma_x < f_y$ (Elastic limit); $\sigma_x = f_y$ (Purely Plastic i.e yield has occurred);

$\sigma_x > f_y$ (Not attainable, since this violates the allowable stress conditions for steel)

Therefore, allowable stress, σ allow is

$$\sigma \text{ allow} = \left\{\left(\frac{f_y}{F_{TST}}\right)\right\}\frac{1}{F_s} = \frac{\sigma_x}{F_s} \le f_y \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 33$$

Equation (33) is the new formulated general allowable yield criterion equation for plane elements based on the total strain energy theory equation.

2.2 Estimation of 'n-Values' and Formulation of Stress Factor Equations

The various n-values (known as plane vectors) that is, n_1, and $n_{2,}$ for the different rectangular plate types will be evaluated using the polynomial displacement shape profiles in Table 1.

The n-values for the various plate types will be evaluated as follows.

For plates without a free edge, F, maximum deflection will occur at the center of the plate where R= Q = 0.5.

For CCCC Plate for instance;

From Table 1,

$$h = (R^2 - 2R^3 + R^4)\ (Q^2 - 2Q^3 + Q^4) = h_x * h_y \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 34$$

$$\frac{\partial^2 \mathbf{h}}{\partial \mathbf{R}^2} = \frac{\partial^2 \mathbf{h}}{\partial \mathbf{Q}^2} * h_y = (2 - 12R + 12R^2)\ (Q^2 - 2Q^3 + Q^4) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 35$$

$$\frac{\partial^2 \mathbf{h}}{\partial \mathbf{R}^2} = h_x * \frac{\partial^2 \mathbf{h}}{\partial \mathbf{Q}^2} = (R^2 - 2R^3 + R^4)\ (2 - 12Q + 12Q^2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 36$$

Substitute Equations (28) into Equations (23)- (24) yields

$$n_1 = (2 - 12R + 12R^2)\ (Q^2 - 2Q^3 + Q^4) + \frac{?}{Ƶ^2}\ (R^2 - 2R^3 + R^4)\ (2 - 12Q + 12Q^2) \quad \ldots\ldots\ldots\ldots \quad 37$$

$$n_2 = ?\ (2 - 12R + 12R^2)\ (Q^2 - 2Q^3 + Q^4) + \frac{1}{Ƶ^2}\ (R^2 - 2R^3 + R^4)\ (2 - 12Q + 12Q^2) \quad \ldots\ldots\ldots\ldots \quad 38$$

At the point of maximum deflection, R = Q = 0.5. Substitute these values of R and Q in Equation (37) and Equation (38), we have

$$n_1 = \left(-0.0625 - \frac{0.0625\ ?}{Ƶ^2}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 39$$

$$n_2 = \left(-0.0625? - \frac{0.0625}{Ƶ^2}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 40$$

Substituting Equation (39) and Equation (40) into Equation (31) yields the stress factor as Equation (33)

$$F_{TST} - \left[1 - 2?\ \frac{\left(-0.0625? - \frac{0.0625}{Ƶ^2}\right)}{\left(-0.0625 - \frac{0.0625\ ?}{Ƶ^2}\right)} + \frac{\left(-0.0625? - \frac{0.0625}{Ƶ^2}\right)^2}{\left(-0.0625 - \frac{0.0625\ ?}{Ƶ^2}\right)^2}\right]^{½} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 41$$

Similarly, the rest of the 5 plate types contained in rows 2 to 6 of Table 1, were evaluated.

However, for plates with a free edge, F, maximum deflection will occur at the free edge of the plate where R= 0.5 and Q = 1.

+

For SSFS Plate for instance;

From Table 1,

$$h = (R - 2R^3 + R^4)\ \left(\frac{7}{3}Q - \frac{10}{3}Q^3 + \frac{10}{3}Q^4 - Q^5\right)\ h_x * h_y \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 42$$

$$\frac{\partial^2 \mathbf{h}}{\partial \mathbf{R}^2} = \frac{\partial^2 \mathbf{h}_x}{\partial \mathbf{R}^2} * h_x = (-12R + 12R^2)\left(\frac{7}{3}Q - \frac{10}{3}Q^3 + \frac{10}{3}Q^4 - Q^5\right) \text{.....................} \quad 43$$

$$\frac{\partial^2 \mathbf{h}}{\partial \mathbf{R}^2} = \frac{\partial^2 \mathbf{h}_y}{\partial \mathbf{R}^2} h_x * = (R - 2R^3 + R^4)(-20Q + 40Q^2 - 20^{Q3}) \text{.....................} \quad 44$$

Substitute Equations (35) into Equations (23)- (24) yields

$$n_1 = \left[(-12R+12R^2)\left(\frac{7}{3}Q - \frac{10}{3}Q^3 + \frac{10}{3}Q^4 - Q^5\right) + \frac{?}{\mathcal{Z}^2}(R-2R^3+R^4)(-20Q+40Q^2-20^{Q3})\right] \text{.......} \ 45$$

$$n_2 = \left[?(-12R+12R^2)\left(\frac{7}{3}Q - \frac{10}{3}Q^3 + \frac{10}{3}Q^4 - Q^5\right) + \frac{1}{\mathcal{Z}^2}(R-2R^3+R^4)(-20Q+40Q^2-20^{Q3})\right] \text{.......} \ 46$$

At the point of maximum deflection, R = 0.5 and Q = 1. Substitute these values of R and Q in Equation (45) and Equation (46), we have

$n_1 = -\mathbf{4}$ 47

$n_2 = -\mathbf{4}?$ 48

Substituting Equation (47 and Equation (48) into Equation (31) yields the stress factor as Equation (33)

$$F_{TST} = \left[1 - 2?\frac{-\mathbf{4}?}{(-\mathbf{4})} + \frac{(-\mathbf{4}?)^2}{(-\mathbf{4})^2}\right]^{\frac{1}{2}} = \left[1 - ?^2\right]^{\frac{1}{2}} \text{.....................} \ 49$$

Similarly, the rest of the 5 plate types with a free edge contained in roll 8 to 12 of Table 1, were evaluated.

2.3 Numerical Application

Consider a structural steel square plate with the following properties: $\upsilon = 0.3$ and $f_y = 250$MPa. Determine the corresponding parameters and stresses for different types of plates by substituting $\upsilon = 0.3$ into the stress factor equations in Table 3 for each plate type and evaluate the applied stress, σ_x

2.0 RESULTS AND DISCUSSIONS

Based on the present studies, the obtained result for a new general applied stress equation based on the total strain energy theory of yield is presented in Table 2 together with the stress factor.

Table 2: General Formulated Applied Stress Equation based on Total Strain Energy Theory

S/N	DESCRIPTION	EQUATIONS
1	Applied Stress	$\sigma_x = \frac{f_y}{F_{TST}}$
2	Stress Factor	$F_{TST} = \sqrt{\left[1 - 2?\frac{\mathbf{n}_2}{\mathbf{n}_1} + \frac{\mathbf{n}_2^2}{\mathbf{n}_1^2}\right]}$

Also, the n-values equations from Equations (39) and (40) and Equations (47) and (48) and for the rest of the other plate types under consideration were substituted into Equation (31) to obtain the stress factor, F_{TST}, equations for the twelve plates as presented in Table 3 (that is, rolls 1 to 6 for plate types without a free edge and rolls 7 to 12 for plate types with a free edge, F.

Table 3: Stress Factor, F_{TST}, Equations

Plate Type	$F_{TST} = \sqrt{\left[1 - 2?\frac{n_2}{n_1} + \frac{n_2^2}{n_1^2}\right]}$
CCCC	$\left[1 - 2?\frac{(-0.0625\text{Ƹ}^2? - 0.0625)}{(-0.0625\text{Ƹ}^2 - 0.0625?)} + \frac{(-0.0625\text{Ƹ}^2? - 0.0625)^2}{(-0.0625\text{Ƹ}^2 - 0.0625?)^2}\right]^{\frac{1}{2}}$
SSSS	$\left[1 - 2?\frac{(-0.9375\text{Ƹ}^2? - 0.9375)}{(-0.9375\text{Ƹ}^2 - 0.9375?)} + \frac{(-0.9375\text{Ƹ}^2? - 0.9375)^2}{(-0.9375\text{Ƹ}^2 - 0.9375?)^2}\right]^{\frac{1}{2}}$
CSSS	$\left[1 - 2?\frac{(-0.375\text{Ƹ}^2? - 0.46875)}{(-0.375\text{Ƹ}^2 - 0.46875?)} + \frac{(-0.375\text{Ƹ}^2? - 0.46875)^2}{(-0.375\text{Ƹ}^2 - 0.46875?)^2}\right]^{\frac{1}{2}}$
CSCS	$\left[1 - 2?\frac{(-0.1875\text{Ƹ}^2? - 0.3125)}{(-0.1875\text{Ƹ}^2 - 0.3125?)} + \frac{(-0.1875\text{Ƹ}^2? - 0.3125)^2}{(-0.1875\text{Ƹ}^2 - 0.3125?)^2}\right]^{\frac{1}{2}}$
CCSS	$\left[1 - 2?\frac{(-0.1875\text{Ƹ}^2? - 0.1875)}{(-0.1875\text{Ƹ}^2 - 0.1875?)} + \frac{(-0.1875\text{Ƹ}^2? - 0.1875)^2}{(-0.1875\text{Ƹ}^2 - 0.1875?)^2}\right]^{\frac{1}{2}}$
CCCS	$\left[1 - 2?\frac{(-0.09375\text{Ƹ}^2? - 0.125)}{(-0.09375\text{Ƹ}^2 - 0.125?)} + \frac{(-0.09375\text{Ƹ}^2? - 0.125)^2}{(-0.09375\text{Ƹ}^2 - 0.125?)^2}\right]^{\frac{1}{2}}$
SSFS	$\left[1 - ?^2\right]^{\frac{1}{2}}$
SCFS	$\left[1 - ?^2\right]^{\frac{1}{2}}$
CSFS	$\left[1 - ?^2\right]^{\frac{1}{2}}$
CCFS	$\left[1 - ?^2\right]^{\frac{1}{2}}$
SCFC	$\left[1 - ?^2\right]^{\frac{1}{2}}$
CCFC	$\left[1 - ?^2\right]^{\frac{1}{2}}$

The new general applied stress for rectangular plates of all types based on this new equation in Table 2, shows that it is the ratio of the yield stress of the material to the stress factor. This equation is applicable to all the rectangular plate types. The only thing that will change is the displacement shape profile of the particular plate type considered. For the safety of the structure, this applied stress should be less than the yield stress of the material, if not a factor of safety must be introduced. From the equations in Table 3, the stress factor is a function of the Poisson ratio which is a material property. This means that the type of material influences its ability to withstand stress. Also, while all the plate types are a function of the Poisson ratio; those with no free edge are a function of the aspect ratio of the plate. This means these plates are similarly influenced by the dimensions of the plate.

The numerical results obtained for the stress factor and the applied stress from the new equations are presented in columns 4 and 5 respectively of Table 4. The results show that the values of the stress factor for a plate with one free edge are less than 1, which results in the applied stress being higher than the yield stress. These results imply that the plate will fail when loaded. This is the limitation of the total strain theory of yield in its original form which make the theory not used for design purpose. This arises from the fact that the theory is not affected by hydrostatic pressure (Benchan & Warnock, 1976), elementary or residual shear stresses at the support or likely punching shear due to various service loading conditions. For design purpose, a safety factor, F_s that must be greater than one (i.e. F_s? 1) is introduced to obtain allowable or design stress less than the yield stress. Therefore, for the plates with a free edge, the factor of safety is taken as 1.1 which will reduce the allowable stress to 238MPa. This is less than the yield stress of 250MPa for structural steel, and is satisfactory. While applied stress for plate types with no free edge are all less than the yield stress. Therefore, their factor of safety is 1.0. This will ensure the safety of the structure. So, for uniform design for all plate types using this theory, Equation (33) become the allowable stress as given in simply written as Equation (50).

$$\sigma_{allow} = \frac{\sigma_x}{F_s} \leq f_y \quad \text{.....................} \quad 50$$

Where

For plates with one free edge,

$$F_s = 1.1 \quad \text{.....................} \quad 51a$$

And for plates with no free-edge

$$F_s = 1 \quad \text{.....................} \quad 51b$$

Equation (50) becomes the new allowable yield stress equation for a plane element based on the total strain energy theory of yield. This equation is new because the applied stress $\sigma_x = f_y / F_{TST}$ which is a function of the stress factor. The stress factor equation is newly formulated in this work and is not found in any existing literature to the knowledge of the authors. This will improve the total strain energy theory of yield for plane elements and will be useful to Analysts and designers of plated structures as it will ensure safety and save resources.

Table 4: Numerical Values of Applied Stress, σ_x and Allowable Stress, σ_{allow} from Yield

Criterion Analysis

Plate Type	$\sigma_x = \frac{f_y}{F_{TST}}$; ? = 0.3,	Ƶ = 1; f_y = 250MPa;			$\sigma_{allow} = \frac{\sigma_x}{f_s} \leq f_y$
	n_1	n_2	F_{TST}	σ_x	σ_{allow}
CCCC	-0.08125	-0.08125	1.18321596	211	211
SSSS	-1.21875	-1.21875	1.18321596	211	211
CSSS	-0.515625	-0.58125	1.26268768	198	198
CSCS	-0.28125	-0.36875	1.39008837	180	180
CCSS	-0.28125	-0.24375	1.10955447	225	225
CCCS	-0.13125	-0.153125	1.28884100	194	238
SSFS	-4	-1.2	0.95393920	262	238

SCFS	-2	-0.6	0.95393920	262	238
CSFS	-1.2	-0.36	0.95393920	262	238
CCFS	-0.6	-0.18	0.95393920	262	238
SCFC	-1.33333	-0.399999	0.95393920	262	238
CCFC	-0.4	-0.12	0.95393920	262	238

2.0 CONCLUSION

The new applied stress equation of a plane element based on the total strain energy theory has been formulated. The stress factor equations for the twelve plate types were given and numerically evaluated using polynomial shape parameters, and the numerical values obtained. Based on the values, a factor of safety obtained has been proposed. This led to a new allowable or design stress equation which is adequate and safe for yield analysis of two-dimensional plane structures. The predicted stresses from this new equation for all the plate types under consideration were below the yield stress of steel used in this work. Therefore, this study has eliminated the former limitation of this yield criterion thereby making the criterion useful for yield analysis of rectangular plates. This will ensure safety and prevent damage to plated structures.

Notations:

SSSS – a plate simply supported on all four edges
CCCC – a plate clamped on all four edges,
CCFC – a plate clamped at three edges and free at one edge, and so on.
X – horizontal axis and Y is vertical axis
R – a non dimensional parameter along the X-axis
Q – a non dimensional parameter along the Y-axis
R = X/a, **0**] R] 1; Q= Y/b, 0] R] 1; 0] Q] 1
a – plate dimension (length) along X-axis,
b – is plate dimension (Width) along Y-axis

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