A NON-LINEAR REGRESSION MODEL OF THE DEFORMATION BEHAVIOUR OF ALUMINIUM FOAMS SUBJECTED TO QUASI-STATIC COMPRESSION

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ABSTRACT

A nonlinear regression model to elucidate the elasto-plastic stress–strain deformation behaviour of aluminum foams subjected to quasi-static compression is presented. Compression tests were conducted on Aluminium foams with relative densities within the range of 0.1217 – 0.7558, and at a strain rate of 10^{-3}/s; also the data generated was modelled using Matlab software. The model developed was found to effectively fit the experimental data and predict reliably the stress-strain response within the relative density range of 0.1 – 0.8. The regression model proposed was found to also capture fully the three typical deformation sequence of the stress-strain response of Aluminium foams, namely: linearity, plasticity-like stress plateau, and densification phases. The proposed model will be invaluable for prescribing Aluminium foam relative densities for specific applications where impact safety and crashworthiness are important.

Keywords: Regression model; Elasto-plastic stress-strain behaviour; Aluminum foam; Impact Safety; compression test

1. Introduction

The development of artificial cellular materials grew from the observation of the efficient load bearing capacity of natural occurring cellular materials such as wood and coral bone (Li, 2008). Polymers and ceramics are among the engineered materials initially utilized for the development of foams as functional materials but their very low toughness and plasticity make them unsuitable for constructional/structural applications (Gibson and Ashby, 1997; Singace, 2000). The necessity to overcome these limitations has led to the development of foams based on the use of light weight metallic materials (Simancik et al., 1995). Aluminium remains the most utilized metallic material for the production of metallic foams. Aluminium foams as they are called are made up of a composite structure consisting of aluminium or aluminium alloy as the matrix and the pores dispersed throughout the matrix. This unique structure impacts unusual combination of properties, such as low density, high ratio stiffness and ratio-strength, low thermal conductivity, high impact energy absorption capacity, very high specific toughness and good acoustic properties (Idris et al., 2008; Fraunhofer, 2002; Wierzbicki and Doyoyo, 2003). It is also inflammable, ecologically harmless and easily recyclable (Baker et al., 1998). Some potential areas of application of aluminium foams are in automotive, railway and aerospace applications where they serve as energy absorbers for the internal padding and inside structural elements of the external body. In this regards, they help in reducing damage from sudden loading or impact (Fuganti et al., 2000; Ramamurty and Kumaran, 2004; Chen and Wierzbicki, 2001).

Optimizing high impact energy absorption and general mechanical properties of aluminium foams have attracted a lot of interest from researchers (Lopatnikova, et al., 2007; Santosa and Wierzbicki, 1998; Ruan et al., 2002; Harrigan et al., 1999). The progress made has been hinged on the understanding of the deformation mechanisms (behaviour) in aluminium foams. Aluminium foams have been reported to exhibit an elasto-plastic stress–strain behaviour quite different from that of conventional dense metallic materials. The deformation sequence is in three stages including: a linear elastic region, a plateau region over a large range of strains and a densification region where the stress rises sharply. The deformation profile has been reported to immensely influence the potential area of application of Aluminium foams as the relative densities and the pore sizes influence the plastic collapse stress and the energy of absorption during deformation (Idris et al., 2008).

This work is an attempt to provide a mathematical model which can comprehensively describe the deformation stress-strain behaviour of the Aluminium foams subjected to compressive loading under quasi-static strain rates. The dependency of the deformation behaviour on the densities of the foams is utilized as basis for formulating the model. The aim of the work is to proffer an easy and rapid basis of determining optimal density – absorption energy combinations for specific applications.

2. Materials and Methods

2.1 Experimental Procedure

AlSi6Cu4 closed cell foams were utilized for this research. The alloy was foamed inside a closed mould through the powder compact route and then solidified following procedures described in details by Mukherjee et al (2010). The relative density of the foams ranges from 0.12 to 0.76 and the average cell sizes are between 0.9 mm and 1.6mm.

Cube shaped specimens with dimensions 14 × 14 × 14mm were prepared from the Al foams for compression tests. Representative samples of the Aluminium foams utilized for the experiment are presented in Figure 1. Before the compression tests, each specimen was weighed and the volume measured in order to calculate its density. The effective relative density of the foam sample was determined by dividing the obtained density of the AlSi6Cu4 foam by the density of given solid (non porous) alloy making the cell wall of the foam. The density of the cell wall material was assumed equal to the density of pure aluminium and taken as 2.7 g/cm^3. The compression tests were then conducted at room temperature (25°C) using an Instron screw-driven mechanical test machine with a 10 kN loading cell, utilizing a strain rate of 10^{-3}/s. The specimens were compressed between two steel loading platens and the corresponding engineering stress and strain were recorded by a computerized software attachment of the machine. For each relative density of foam, at least three repetitions of the compression test were performed.

2.2 Modeling Procedure

The experimental data obtained from studying the compressive stress-strain behaviour under quasi-static strain rate of 10^{-3}/s for the different Al foam relative densities were collated for the model development with the use of Matlab software. In order to model the relationship between the stress, strain and relative density of the Al foams, the stress - strain relationship was first modeled. By inspecting the graph of the stress vs strain, it was discovered that the relationship can be modeled as a polynomial function. Although the order of the polynomial could be five (5) for some relative densities, it must be as much as seven (7) for others. In order to maintain parameter uniformity for future estimations, an order of seven was chosen for all relative densities.

The stress -strain relationship was then modeled as:

$$\sigma = a\varepsilon^7 + b\varepsilon^6 + c\varepsilon^5 + d\varepsilon^4 + e\varepsilon^3 + f\varepsilon^2 + g\varepsilon + h \qquad (2.1)$$

Where σ is the stress, ε is the strain, a, b, c, d, e, f, g and h are the polynomial parameters. Figures 2 to 4 show the graph of the experimental data and the model estimates for some relative densities. The coefficients of the fits are as shown in Table 1.

A second set of parameters were estimated from plots of the relative densities and the polynomial parameters. From the graph it was impossible to use linear relationships for the relative densities and the polynomial parameters, but another polynomial of order three (3) was used. The coefficients are as shown in Table 2.

The final model thus was (eqn 2.2)

$$\begin{aligned}\sigma = \; & (d_{11}\rho^3 + d_{12}\rho^2 + d_{13}\rho + d_{14})\varepsilon^7 + (d_{21}\rho^3 + d_{22}\rho^2 + d_{23}\rho + d_{24})\varepsilon^6 + (d_{31}\rho^3 + d_{32}\rho^2 + d_{33}\rho + d_{34})\varepsilon^5 + \\ & (d_{41}\rho^3 + d_{42}\rho^2 + d_{43}\rho + d_{44})\varepsilon^4 + (d_{51}\rho^3 + d_{52}\rho^2 + d_{53}\rho + d_{54})\varepsilon^3 + (d_{61}\rho^3 + d_{62}\rho^2 + d_{63}\rho + d_{64})\varepsilon^2 + \\ & (d_{71}\rho^3 + d_{72}\rho^2 + d_{73}\rho + d_{74})\varepsilon + (d_{81}\rho^3 + d_{82}\rho^2 + d_{83}\rho + d_{84})\end{aligned} \qquad (2.2)$$

where σ is the calculated stress, d is a matrix of parameters, $\rho = \frac{\rho^*}{\rho_{Al}}$ is the relative density, ρ^* is the density of the density of the Al foam, ρ_{Al} is the density of solid (non-porous) Aluminium, and ε is the strain.

2.3 Boundary Conditions

Boundary limits are placed on the relative densities to be within the range of 0.1 – 0.8 as beyond this range, the Aluminium can not technically be referred to as metallic foam because it would not manifest the properties that characterize Aluminium foams.

Assumption

The foams are assumed to have the same pore morphology; and also the pores are assumed to be uniformly distributed throughout the aluminium matrix as variations in these parameters could affect the stress-strain behaviour.

3. Results and Discussion

Table 3 presents the coefficients of correlation of the final model (eqn 2.2). It is observed that the model is highly correlated with the experimental data for strains up to 0.4. The correlation is higher for higher relative densities, attaining $R^2 = 0.9998$ for the 0.7558 relative density. Figures 5 – 9 confirm that the model perfectly captures entirely the stress-strain response of the forms with different relative densities, and the quality of the fit is very high in the three deformation regions namely, the elastic, plastic plateau-like region and densification region.

The model can be effectively utilized to study the stress-strain behaviour of the Al foams for different relative densities within 0.1 - 0.8. However, the plots show that not all foam relative densities exhibited the universal three deformation regions characterized by: an initial linear-elastic region; an extended plateau region where the stress increases slowly as the cells deform plastically; and a final densification as collapsed cells are compacted together. For example Figures 5-6 (for relative densities of 0.1217 and 0.1683 respectively) which are representative of very low relative densities does not manifest fully the three deformation stages because of its low densification stress which is less than 7 MPa for both cases. This equally indicates that foams with such low relative densities would have limited use for applications where high energy absorption (impact strength) and crashworthiness are required (Mukherjee et al., 2010).

For intermediate relative densities which are represented by the 0.3737 and 0.3811 relative densities (Figure 7 and 8), it is observed that the foams exhibits stress-strain behaviour characterized by the manifestation of the three deformation regions i.e elastic, extended plastic plateau-like, and densification regions. But for the foams with *very high relative densities* (as represented by the foam with relative density of 0.7558 in Figure 9), it is observed that the sample did not exhibit the plastic plateau-like feature but rather manifested a progressively increasing plastic flow stress after the elastic region more like that of nonporous metallic solids. Note that the higher relative density foams have a higher stiffness and plastic collapse stress (yield stress), and an earlier densification region than those of the low relative density foams. It is noted also from Figure 7 and 8 that the models predict that densification occurs at a slightly low strain in comparison with the experimental observation.

4. Conclusion

A multi-parameter non-linear regression model that describes the entire stress-strain behaviour of aluminium foams under quasi-static compression has been proposed. Its effectiveness was demonstrated by its remarkable high fit with the experimental data obtained from aluminium foams with different relative densities. The regression model proposed is sufficient to describe the various deformation sequence of the stress-strain response effectively. The proposed model will be invaluable for prescribing given Aluminium foam relative densities for specific applications where impact safety and crashworthiness are important.

Acknowledgments

The authors sincerely wish to acknowledge the support received from Dr. Francisco Garcia Moreno of the Department of Materials Science, Hahn – Meitner – Institut Berlin, Germany for the provision of the aluminium foams utilized for this investigation. Dr. K. E. Alowolodo's assistance in the model development is equally recognised. Also, the moral support received from Prof. S. Ranganathan is immensely appreciated.

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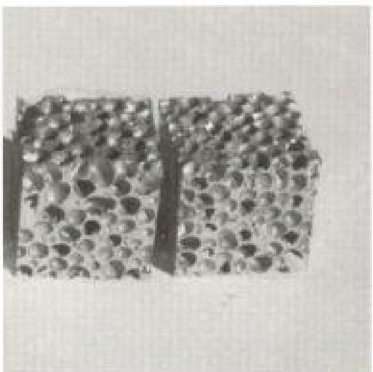
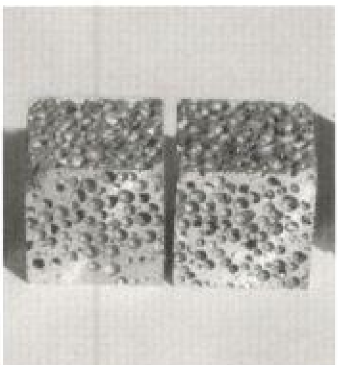

(a)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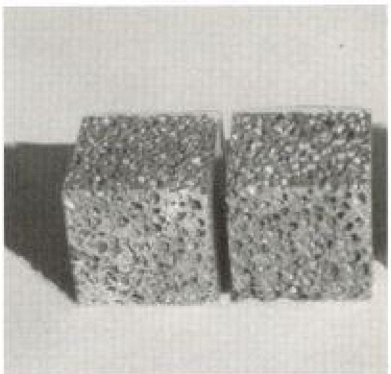
(c)

Figure 1: Al foams produced for the research (a) characteristic feature of low relative density foam revealing a higher pore concentration, (b) intermediate relative densities, and (c) higher relative density foam indicating a relatively lower pore size and concentration.

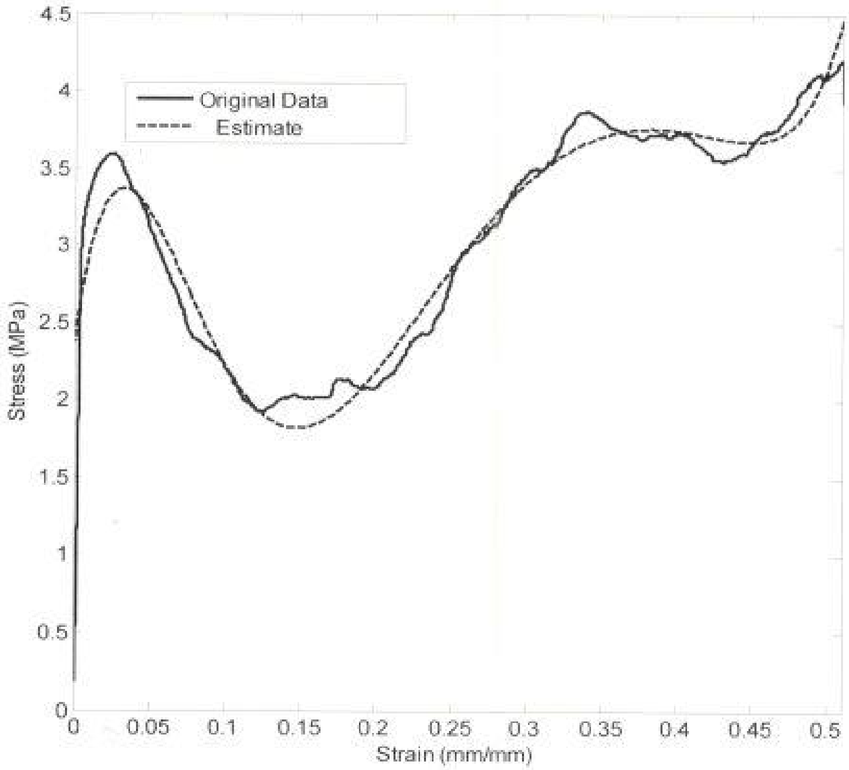

Figure 2. Graph of Stress vs Strain for Relative Density of 0.1217

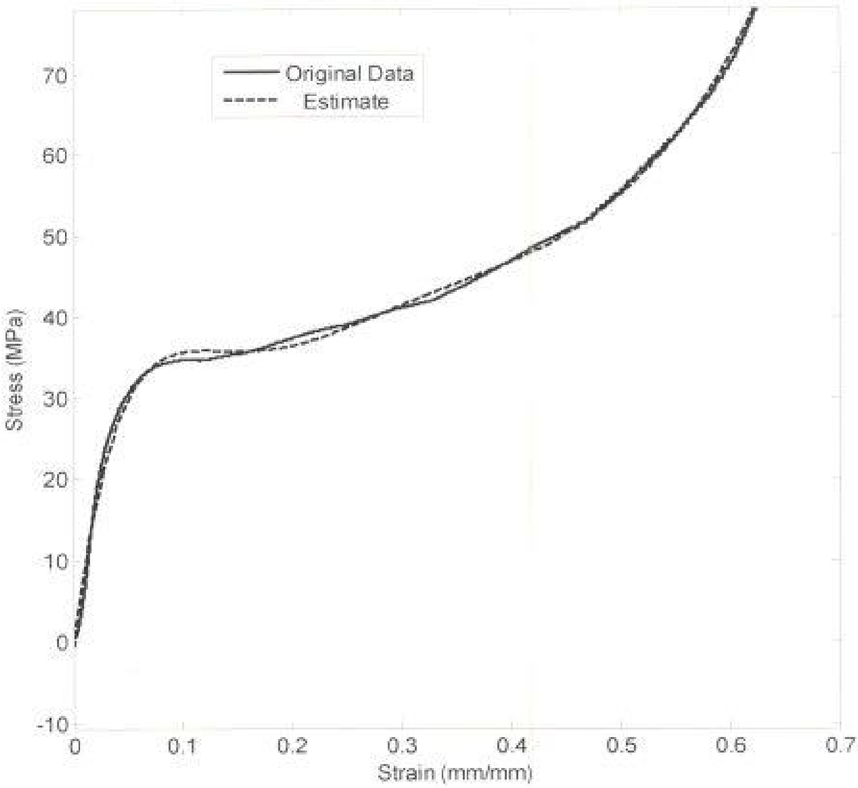

Figure 3. Graph of Stress vs Strain for Relative Density of 0.3737

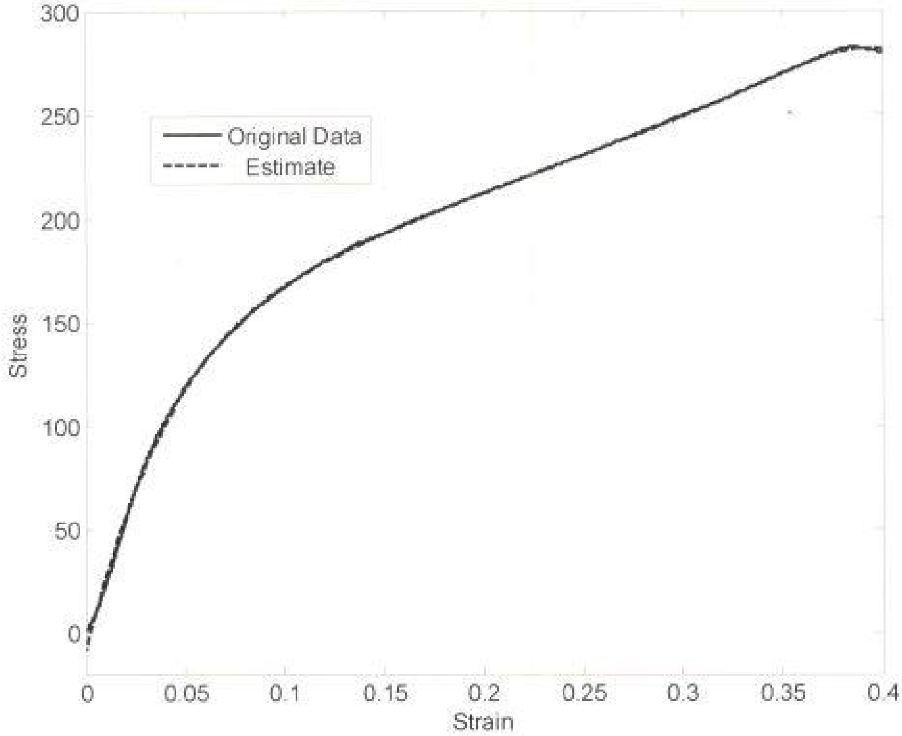

Figure 4. Graph of Stress vs Strain for Relative Density of 0.7558

Table 1. Polynomial fit coefficients for different relative densities

Relative Density	a	b	c	d	e	f	g	h
0.1217	7.83E+04	-1.59E+05	1.34E+05	-5.94E+04	1.43E+04	-1.69E+03	7.25E+01	2.37E+00
0.1337	3.69E+05	-7.34E+05	5.85E+05	-2.38E+05	5.17E+04	-5.72E+03	2.72E+02	-1.32E-01
0.1471	-2.49E+05	3.77E+05	-2.20E+05	5.93E+04	-6.24E+03	-1.67E+02	7.36E+01	-9.89E-02
0.1488	1.63E+05	-3.77E+05	3.48E+05	-1.63E+05	4.09E+04	-5.27E+03	3.06E+02	-1.12E+00
0.1683	2.32E+05	-5.24E+05	4.73E+05	-2.17E+05	5.30E+04	-6.57E+03	3.54E+02	-6.31E-01
0.1779	8.53E+04	-2.23E+05	2.36E+05	-1.29E+05	3.83E+04	-5.88E+03	4.14E+02	-3.81E+00
0.2875	1.67E+05	-4.00E+05	3.89E+05	-1.96E+05	5.50E+04	-8.36E+03	6.60E+02	-7.40E-02
0.3334	1.89E+05	-4.49E+05	4.35E+05	-2.18E+05	6.06E+04	-9.11E+03	7.17E+02	2.87E-01
0.3737	1.26E+05	-3.38E+05	3.72E+05	-2.14E+05	6.89E+04	-1.21E+04	1.06E+03	-6.33E-01
0.3811	2.85E+05	-6.93E+05	6.92E+05	-3.64E+05	1.08E+05	-1.77E+04	1.48E+03	4.92E-01
0.4704	5.79E+05	-1.32E+06	1.24E+06	-6.07E+05	1.69E+05	-2.64E+04	2.17E+03	-2.43E+00
0.4913	5.40E+05	-1.23E+06	1.16E+06	-5.73E+05	1.63E+05	-2.63E+04	2.32E+03	-1.13E+01
0.4929	4.77E+05	-1.12E+06	1.07E+06	-5.33E+05	1.50E+05	-2.34E+04	1.96E+03	6.92E+00
0.6301	9.19E+05	-1.98E+06	1.75E+06	-8.21E+05	2.23E+05	-3.51E+04	3.20E+03	-9.00E+00
0.7558	-2.03E+06	1.91E+06	-3.04E+05	-2.78E+05	1.52E+05	-3.29E+04	3.81E+03	-8.38E+00

Table 2: Coefficients of relative density vs Strain

Strain(ε)	d_1	d_2	d_3	d_4
ε^7	-7.26E+07	8.24E+07	-2.64E+07	2.44E+06
ε^6	1.06E+08	-1.23E+08	3.93E+07	-3.72E+06
ε^5	-6.54E+07	7.72E+07	-2.44E+07	2.37E+06
ε^4	2.17E+07	-2.62E+07	8.03E+06	-8.01E+05

ε^3	-4.21E+06	5.19E+06	-1.51E+06	-1.51E+06
ε^2	4.65E+05	-5.91E+05	1.58E+05	-1.58E+04
ε	-2.39E+04	3.44E+04	-8.26E+03	7.68E+02
ε^0	4.53E+01	-9.14E+01	3.57E+01	-3.72E+00

Table 3: Coefficient of Correlation of model and original data for different relative densities

Relative Density	Correlation
0.1217	0.7606
0.1337	0.8721
0.1471	0.8893
0.1488	0.8252
0.1683	0.9202
0.1779	0.9008
0.2875	0.9714
0.3334	0.9712
0.3737	0.9942
0.3811	0.9894
0.4704	0.9972
0.4913	0.9973
0.4929	0.9988
0.6301	0.9989
0.7558	0.9998

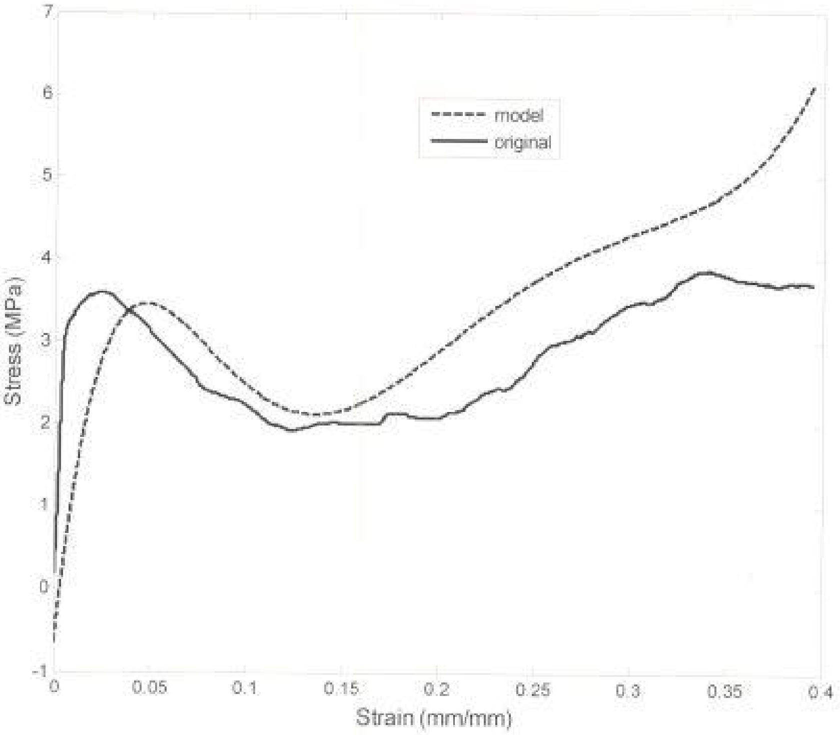

Figure 5. Graph of Experimental and modeled Stress vs Strain for Relative Density of 0.1217

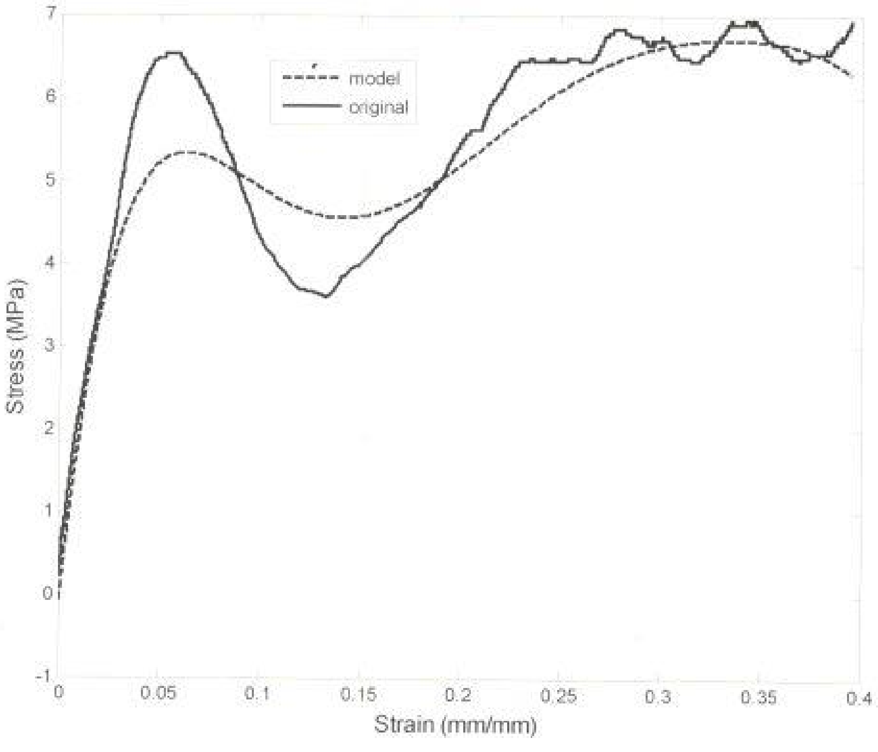

Figure 6. Graph of Experimental and modeled Stress vs Strain for Relative Density of 0.1683

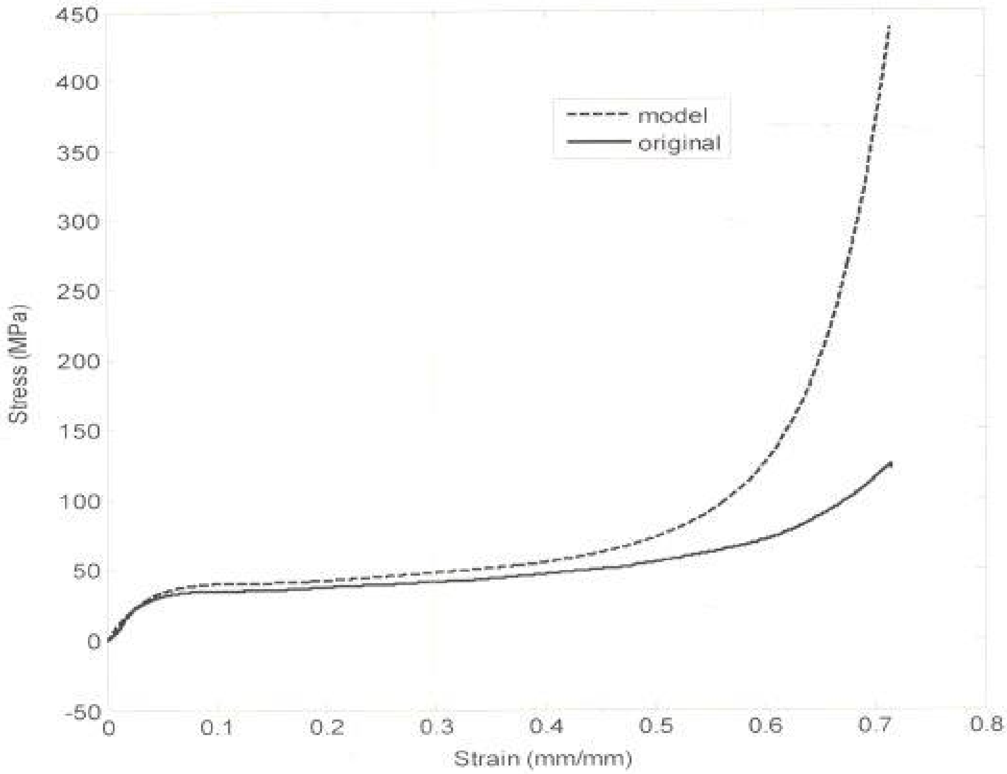

Figure 7. Graph of Experimental and modeled Stress vs Strain for Relative Density of 0.3737

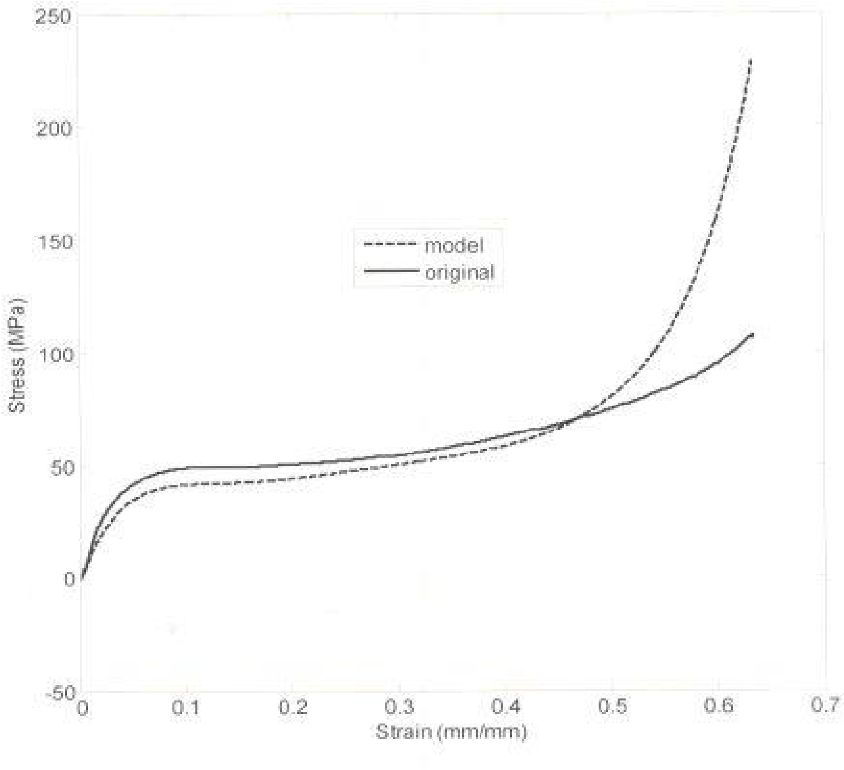

Figure 8. Graph of Experimental and modeled Strain vs Stress for Relative Density of 0.3811

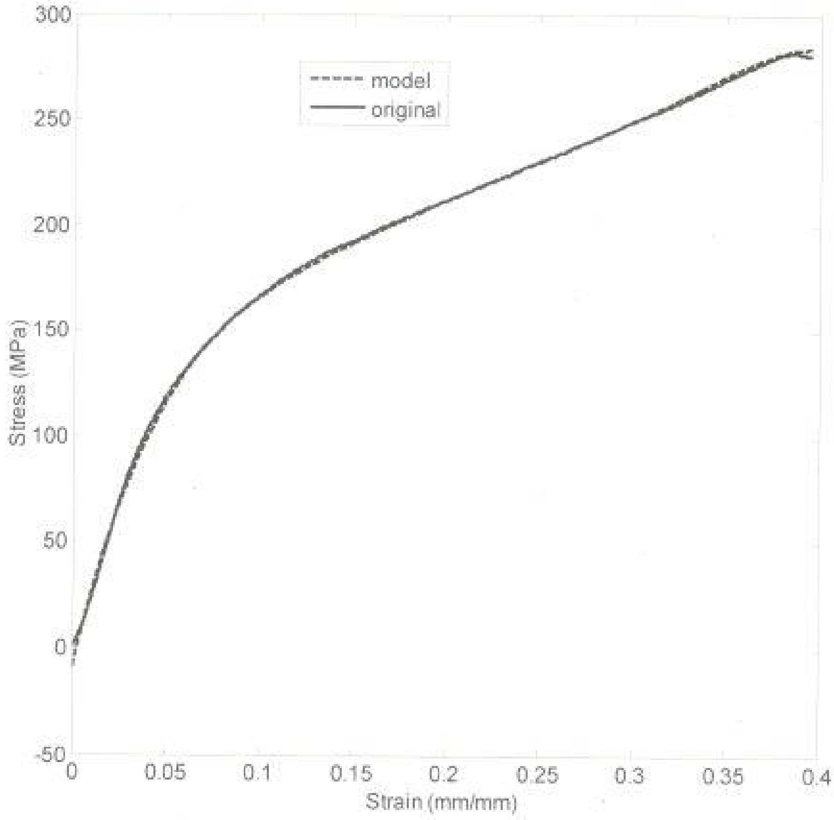

Figure 9. Graph of Experimental and modeled Stress vs Strain for Relative Density of 0.7558