PERFORMANCE ANALYSIS OF A SINGLE-PHASE LINEAR INDUCTION MOTOR USING SYMMETRICAL COMPONENTS

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Abstract. – **The paper presents the circuital approach and performance calculation for a shaded-pole single-sided linear induction motor using symmetrical components for a two-phase system. The physical model has been built. Results obtained from the circuital approach and experimental tests are compared and discussed.**

Key words: linear induction motor, shaded-pole motors, symmetrical components.

I. Shaded-pole Motor

Shaded-pole motors are recognized to be among one of the most robust and simple-to-design machines but difficult to analyze when compared to other induction motors [1], [2]. The single-phase single-sided shaded-pole linear induction motor with a rotating disc [3], [4] and a stationary primary stack has salient poles with a main multi-turn winding with concentrated parameters and slots accommodating auxiliary single-turn shorted coils (auxiliary winding). Constructional details of this prototype machine are outlined in [3], [4].

Since the currents in the main and auxiliary windings are shifted by an angle less than 90° and the space angle between the two windings is also less than 90°, an elliptical travelling magnetic field is produced in the airgap. The normal component of the magnetic flux density distribution in the airgap can be described by the following equation [5]:

$$b(x,t)=\sum_{\nu=1}^{\infty}\left[B_{m\nu}^{+}e^{j(\omega_{s\nu}^{+}t-\beta_\nu x)}+B_{m\nu}^{-}e^{j(\omega_{s\nu}^{-}t+\beta_\nu x)}\right] \quad (1)$$

where $B_{m\nu}^{+}$ and $B_{m\nu}^{-}$ are the peak values of the ν^{th} space harmonic waves travelling in the x-direction (along the pole pitch), $\nu = 1, 3, 5$ ... are the higher space harmonics, $\omega_{s\nu}^{+}$ is the angular frequency for the forward travelling field, $\omega_{s\nu}^{-}$ is the angular frequency for the backward travelling field, $\beta_\nu = \nu\pi/\tau$, and τ is the pole pitch. The peak values of magnetic flux densities are:

$$B_{m\nu}^{+}=0.5\left[B_a b_{\nu a}+B_b b_{\nu b}e^{-j(\beta-\nu\alpha)}\right]$$
$$B_{m\nu}^{-}=0.5\left[B_a b_{\nu a}+B_b b_{\nu b}e^{-j(\beta+\nu\alpha)}\right]$$

where B_a, B_b are the normal components of the magnetic flux density (rectangular distribution) in the symmetry axis of the phase a and b, respectively, $b_{\nu a}$, $b_{\nu b}$ are Fourier's coefficients, $\beta<90°$ is the phase angle between the currents in phase a and b, α is the angle between symmetry axes of phase a and b.

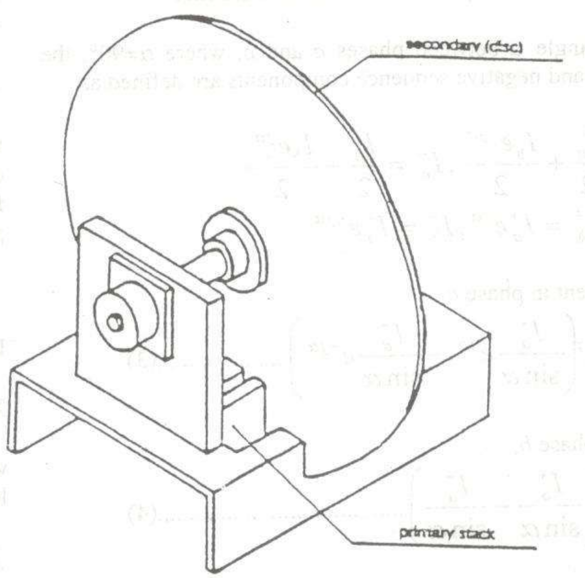

Fig.1. General Assembly of Shaded-pole LIM

For $\nu=1$ the angular frequencies are: $\omega_{s\nu=1}^{+}=2\pi fs$, $\omega_{s\nu=1}^{-}=2\pi f(2-s)$, where s is the slip. The slip relative to the $+ve$ and $-ve$ sequence field is;

$$s^{+}=\frac{\omega_1-\omega_2}{\omega_1},\quad s^{-}=2-s^{+} \quad (2)$$

where ω_1 - synchronous speed (stator supply frequency) for fundamental harmonic, ω_2 - rotor speed.

The tested shaded-pole LIM has a rotor (secondary) which is a double-layer disc made of aluminum and back-iron plates. The motor is shown in Fig.1 and the design data is presented in Table 1. The construction has been described in paper [4]. In the circuital approach to single-phase or two-phase motors, symmetrical components for a 2-phase system are usually used.

Table 1. LIM Design Data

Quantity	Value	Unit
Length of primary stack	$L_\tau = 0.192$	m
Width of primary stack	$L_i = 0.09$	m
Number of pole pairs	$p = 2$	
Number of turns per main phase	$N_a = 520$	
Resistance of main winding for dc current	$R_{dc} = 12.813$	Ω
Primary winding factor	$k_{wa} = 1$	
Pole pitch	$\tau = 0.048$	m
Air gap	$g = 0.0015$	m
Height of pole	$h_P = 0.048$	m
Width of pole	$w_p = 0.016$	m
Diameter of wire with insulation	$d_w = 0.00125$	m
C.S.A of copper wire	1.227×10^{-6}	m^2
Height of shading ring slot	$h_s = 0.005$	m
Width of shading pole slot	$w_s = 0.005$	m
Conductivity of aluminum disc at 20°C	36×10^{-6}	S/m
Conductivity of back iron at 20°C	5×10^{-6}	S/m

II. Equations for Phase Currents

For an angle α between phases a and b, where $\alpha \neq 90°$, the positive and negative sequence components are defined as;

$$I_a^+ = \frac{I_a}{2} + \frac{I_b e^{-j\alpha}}{2}, I_a^- = \frac{I_a}{2} + \frac{I_b e^{j\alpha}}{2}$$

where $I_b^+ = I_a^+ e^{j\alpha}, I_b^- = I_a^- e^{-j\alpha}$

The current in phase a,

$$I_a = -j\left(\frac{I_a^+}{\sin\alpha}e^{j\alpha} - \frac{I_a^-}{\sin\alpha}e^{-j\alpha}\right) \quad (3)$$

For the phase b,

$$I_b = j\left(\frac{I_a^+}{\sin\alpha} - \frac{I_a^-}{\sin\alpha}\right) \quad (4)$$

III. Voltage Equations

The input voltage across the main phase terminals is

$$V_a = I_a Z_{1a} + I_a^+ Z^+ + I_a^- Z^- + (I_a + I_b) Z_{ab} \quad (5)$$

Similar equations can be written for the auxiliary phase

$$V_b = 0 = I_b Z_{1b} + I_b^+ Z^+ + I_b^- Z^- + (I_a + I_b) Z_{ab} \quad (6)$$

The impedances Z'_{1a}, Z_{1b}, Z^+ and Z^- are explained in Sections IV and VIII.

IV. Stator Winding Impedances

The winding impedances for the main and auxiliary phases are

$$Z_{1a} = R_{1a} + jX_{1a}, Z_{1b} = R_{1b} + jX_{1b} \quad (7)$$

where R_{1a}, X_{1a} - resistance and leakage reactance of main stator winding, and R_{1b}, X_{1b} - resistance and leakage reactance of auxiliary winding.

The transformation factor, k_{tr}, i.e. turns ratio between the primary and secondary system for the resistance and leakage reactances, is

$$k_{tr} = \frac{4(N_a k_{w1a})^2}{p} \quad (8)$$

where N_a is the number of turns of the main phase, k_{w1a} is the winding factor of the main phase a for fundamental space harmonic and p is the number of pole pairs.

The primary winding resistance of the main phase is

$$R_{1a} = \frac{N_a l_{av}}{\sigma_{cu} A_{cu}} \quad (9)$$

where the electric conductivity of copper σ_{cu} should be calculated for 75°C. The average length of turn l_{av} can be found as in the case of any other winding with concentrated parameters.

V. Stator Leakage Reactance

For main stator phase a the leakage reactance is [6],

$$X_{1a} \approx \gamma(\sigma_1 - 1) X_m \quad (10)$$

where linkage factor, $\gamma = 0.6 \ldots 0.9$, and $\sigma_1 = 1.1 \ldots 1.16$, is leakage factor of main winding. For auxiliary phase b,

$$X_{ib} = 0.5\mu_0 f \lambda_{1b} L_i \quad (11)$$

where λ_{1b} is the specific leakage permeance.

The impedance of the auxiliary phase referred to the main phase is

$$X'_{1b} = \left(\frac{k_{w1a}}{k_{w1b}} N_{1a}\right)^2 X_{1b} \quad (12)$$

where $k_{w1a} = 1$, $k_{w1b} = \sin\left(\frac{b_{sh}}{bp}\frac{\pi}{2}\right)$ and b_{sh} is width of shading ring edge and b_p is the width of pole.

MMFs of both two phases balance each other, i.e. $I_b N_b k_{w1b} = I_a N_a k_{w1a}$ where N is number of turns of the respective winding and k_w is winding factor. Refer shading ring winding to main stator winding side by similar transformation; $I_b' = I_b / k_{ab}$, where $k_{ab} = N_a k_{w1a} / k_{w1b}$.

VI. Impedance of Vertical Branch

The mutual reactance referred to the main phase a is [5]

$$X_m = \frac{8}{\pi} f \frac{\mu_0 (N_a k_{wa})^2}{p(g + d_{Al}) k_{sat}} \tau L_i \quad (13)$$

where g is the airgap (mechanical clearance) d_{Al} is the thickness of the aluminum layer, k_{sat} is the saturation factor of magnetic circuit and L_i is the effective width of the primary stack. It is convenient to replace the parallel connection of R_{Fe} and X_m by series connection

$$Z_0 = R_0 + jX_0 = \frac{R_{Fe} X_m}{R_{Fe} + jX_m} \quad (14)$$

where R_{Fe} is the resistance representing core losses.

VII. Mutual Reactance Between Main and Auxiliary Phases

The mutual reactance is

$$Z_{ab} = jX_{ab} = j\omega M_{ab} \quad (15)$$

where X_{ab} is mutual reactance between main phase a and auxiliary phase b[7]. In practical calculations,

$$X_{ab} \approx \frac{\alpha_b}{\alpha_a} \frac{X_0}{2p} \quad (16)$$

where α_b is the angle representing the width of shading ring and α_a is the angle representing the width of the main pole, and X_0 is according to equation (14).

VIII. Rotor (Secondary) Impedance

The impedances of aluminum cap $Z_{Al}'(s)$ and solid back iron $Z_{Fe}'(s)$ for the fundamental space harmonic $\nu = 1$ have been found on the basis of [5]. The secondary circuit impedances referred to the primary system for the fundamental harmonic is,

$$Z_2'^{+} = Z_2'(s) = \frac{Z_{Al}'(s) Z_{Fe}'(s)}{Z_{Al}'(s) + Z_{Fe}'(s)} \frac{1}{s} \quad (17)$$

$$Z_2'^{-} = Z_2'(2-s) = \frac{Z_{Al}'(2-s) Z_{Fe}'(2-s)}{Z_{Al}'(2-s) + Z_{Fe}'(2-s)} \frac{1}{2-s} \quad (18)$$

IX. Total Impedance of LIM

Total impedance as seen from the input terminals of the equivalent circuit,

$$Z_t^{+} = -j \frac{Z_{1a} e^{j\alpha}}{\sin\alpha} + j \frac{Z_{ab}(1 - e^{j\alpha})}{\sin\alpha} + Z^{+} \quad (19)$$

$$Z_t^{-} = j \frac{Z_{1a} e^{-j\alpha}}{\sin\alpha} - j \frac{Z_{ab}(1 - e^{-j\alpha})}{\sin\alpha} + Z^{-} \quad (20)$$

The impedances of the vertical and secondary branch are

$$Z^{+} = \frac{Z_0 Z_2'^{+}}{Z_0 + Z_2'^{+}}, Z^{-} = \frac{Z_0 Z_2'^{-}}{Z_0 + Z_2'^{-}} \quad (21)$$

X. Electromagnetic Torque

The phase currents I_a^{+}, I_a^{-} can be found on the basis of equations (5) and (6). The symmetrical components of the secondary currents referred to the main winding as obtained from the equivalent circuits are;

$$I_2'^{+} = I_a^{+} \frac{|Z_0|}{|Z_0 + Z_2'^{+}|}, I_2'^{-} = I_a^{-} \frac{|Z_0|}{|Z_0 + Z_2'^{-}|} \quad (22)$$

where, $Z_2'^{+}$ and $Z_2'^{-}$ are forward and backward impedances of the secondary referred to the main stator winding turns.

The electromagnetic torque components for the forward and backward sequences are,

$$T^{+} = \frac{2(I_2'^{+})^2 R_2'(s)}{\omega_1 s}, T^{-} = \frac{2(I_2'^{-})^2 R_2'(2-s)}{\omega_1 (2-s)} \quad (23)$$

where $I_2'^{+}$, $I_2'^{-}$ are the secondary currents referred to the stator main winding, and $R_2'^{+}$, $R_2'^{-}$ are the referred rotor resistances. These components are referred to the secondary side by the transformation factor between the stator and rotor, $k = 2m(N_a k_{w,q})^2 / p$.

The difference between the positive and negative sequence torques is the resultant torque $T = T^{+} - T^{-}$.

XI. Comparison of Results

Figs 2 and 3 show the shaft torque $T = F_x r$ against velocity at constant frequency. The agreement between computation and measurements is satisfactory. The measured efficiency curves are shown in Fig.4.

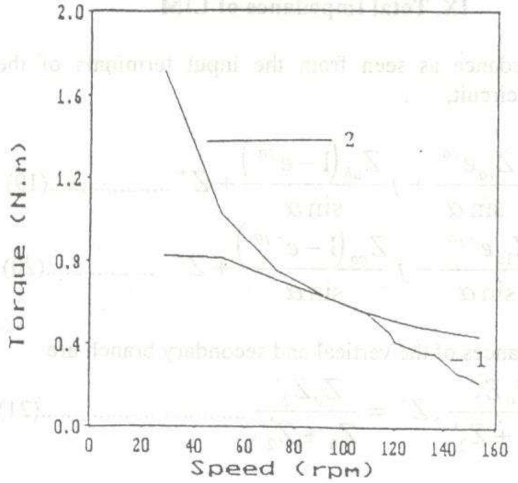

Fig.2 Torque against linear velocity at $f = 75Hz$:, 1 - measurements, 2 - symmetrical components.

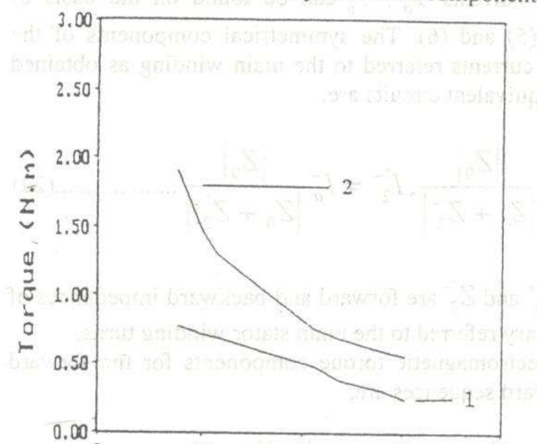

Fig.3. Torque against linear velocity at $f=40Hz$: 1 - measurements, 2 - symmetrical components.

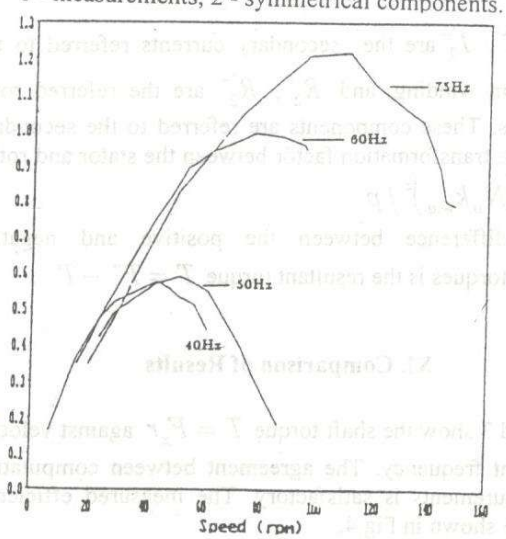

Fig.4. Efficiency vs Speed (f = 40 to 75Hz)

XII. Conclusion

The performance calculation of the single-phase single-sided shaded-pole LIM using the method of symmetrical components of two-phase systems has been presented. The results compared with experimental tests are satisfactory. With a large air gap of 1.5 mm, the magnetizing current in the LIM increases significantly and consequently the power loss I^2R in the shading coil (auxiliary winding), thereby reducing the efficiency and power factor cos θ. The efficiency of the shaded-pole LIM is very low at power frequency. However, it has been found that the efficiency increases with the input frequency as shown in Fig.4.

The shaded-pole LIM is a robust and simple-to-design machine, though difficult to analyze when compared to other induction motors. It can find applications in turntables used in industry. It also finds ready use in small mechanisms where a three-phase power supply is not available or the price and simplicity of drive is important. In countries like Nigeria, India, South Africa and Australia, these motors can be used in rural areas with simple single-phase reticulation systems.

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