APPLICATION OF THE METHODS OF INITIAL PARAMETERS AND SUBSTITUTE CANTILEVER TO DYNAMIC ANALYSIS OF A MULTI - STOREY STEEL FRAMED BUILDING

L. O. ONUNDI AND F. A ADENIJI
Faculty of Engineering
University of Maiduguri

ABSTRACT

Although Africa is largely within the earthquake-free zone of the World, recent events have shown that dangerous earth tremors were caused by earthquakes with epicenters at different locations in Africa. In this paper, a dynamic analysis of a 10-Storey building is discussed using a combination of:

1. The static method of P-delta analysis 2. Energy method of determining the minimum Eigenfrequency of the subsitute cantiliver and 3. The method of initial parameters.

It discusses the use of strucural steel, through the application of deformations obtained from the static method of P - Delta analysis with a virtual force P = 1.0 kN varied linearly from the top, to zero at the base as input or initial load. This, combined with the use of energy method for determining the minimumEigenfrequency of the substitute cantilever and the method of initial parameters was used in a dynamic analysis of a ten-Storey building.

The result of the analysis shows that the maximum deformation (top drift) caused by the siesmic load at the highest point of the building falls within conventionally acceptable limit recommended as a adequate. The minimum total equivelent flexural rigidity, the frequency of oscilliation, period and the frist normal mode of characteristic shape were also derterimined before computing the design of a seismic loading corresponding to the seventh Richter scale.

INTRODUCTION

Earth tremors with epicenters in Algeria, Zaire and a host of many other locations in Africa have caused a lot of damages to lives and property within their active radii of action. It is however important to note that similar or even stronger earthquakes occuring in many other parts in the world where buildings, bridges and other structures were delibrately designed to cope with earthquakes, cause less devastation to structures there than in Africa. With the increasing trend in development of high-rise buildings in all part of Africa, the challenges to stuctural designers within this région has intensified

Shear- walls and walls acting in parallel with frames form efficient lateral load- resisting system in tall buildings. These can be constructed of either reinforced concrete or structural **steel** frames. Many structural steel frames wth vertical **braces** serving as shear- walls common contain atleast one row of opening at every floor level, examples are corridors in administrative buildings and hotels which divide the wall into narrower wall segments interconnected by coupling or connecting beams (fig l,a). The effectivess of coupled shear walls or vertically braced shear walls in resistng horizontal loading depends strongly on the rigidity of the coupling beams and the rational management of materials along the height of the building because of the geometric regularity of reinforced concrete coupled shear- walls and frames, and the large number of storeys, lateral load analysis is often made by means of continuous medium approach. Using this approach, stiffness properties of the discrete beams are replaced by an equivalent connecting continuum. Furthermore, the connecting medium is assumed to be incompressible. For systems with constant properties along the height, the problem is reduced to the solution of a linear differential equation with constant coefficients (Rutenberg et. al.1988).

The stability of a structure can be defined as its ability to recover from displacement induced by an applied force or disturbance. Both theoretical and empirical studies have shown that any comprehensive and reliable method of considering the effect of frame instability should include; the effect of changcs in the geometry of the stucture (joint displacements) on the bonding moments and forces commonly called the P-delta effect, and the influence of axial force on member siffness, which may by accomplished by using the so-called stability function (Iffland, 1978).

The lateral deflection of tall building and other similar structures due to earthquakes or wind is usually amplified by gravity loadimgs since the latter become eccentric. This second-order or P-delta effect may become significant in slender structures. An" exact" analysis for this effect can be made by means of "general computer programmes" but appropriate analysis can be made by suitable first oder procedure (Rutenberg et. Al. 1981).

The objective of this paper is to present a method that determines the minimum Eigenfrequency of a substitute cantilever through the application of energy method and deformations obtained from the static P- delta analysis accomplished by using computer with a virtual force P= 1.0kN varied linearly from the top to zero at the base as input or initial load, this, combined with the simultaneous application of the method of initial parameters will be used to effect a dynamic anaysis on a ten-storey administrative building.

This method will principally attempt to achieve structural stability by limiting the maximum top drift caused by the design seismic loadings to values recommended as conventionally acceptable.The minimum total equivalent moment of inertia, period, frequency of oscillation, first mode of characterise shape will also be determined before computing the resulting design seismic loading corresponding to the seventh Riethter scale.

REASONS FOR THE TRANSITION FROM STRUCTURAL STATICS TO STRUCTURAL DYNAMICS

The fundamental concept on which structural statics is based, assume that external static loads are applied at specific positions.This means that although the acceleration of the natural inertia force which develops as a result of deformation causes additional loads, such loads are usually very small and can be negelected. Nevertheless, most loads acting on building structures are dynamic in character. Furthermore, steel is elastic and since dynamic forces are time-dependent and cause vibrations and deformations of the strucures, it is necessary to take into account the forces produced by theinertia of the accelerating masses.

Sometimes when attempting to analyse the effects of dynamic loads on structures, some engineers simply increase the factor of safety of the structural system, and apply the statical approaches rather than applying the methods of structural dynamics (Varvanov, Part2, chap. 1, 1975). This approach may result in either a very costly design or underdesign which will be very dangerous to the service life span of the structure. Since the actual magnitude of the forces, stresses and infact deformation cannot be ascertained, the application of the method of structural statics becomes inadquate for most practical realities. Hence the method of structural dynamics was developed as a more rational solution to some related engineering problems (Christev, 1974). Although, structural dynamics is based essentially on the same fundamental concept as structural statics, the former differs from the latter in that, the load applied may not be physically seen on the structure and are usually time-dependent causing vibration of the structural system (Varvanov, Part2, chap. 1, 1975).

COORDINATE, LUMPED MASSES AND VIBRATION EQUATIONS

In a vibrating structure, the product of the mass of all the members and their acceleration represents distributed forces. This causes some difficulties. The difficulties involved in dynamic analysis of a specific elastic system depends first of all on the degrees of freedom of the system. This means the number of independent geometric parameters that describes the position of all masses for all possible displacements of the system at any moment in time. In statics, the degrees of freedom are counted for absolutely rigid disks and bars, whose strains are thus neglected. In dynamics however, it is the elastic or elasto-plastic strains that are taken into account when counting the degrees of freedom (Darkov et al. Chap. 15, 1975). For instance, a cantilevered beam with evenly distributed mass, an infinite number of coodinates are required to define the displacement configuration. If we imagine that the beam is lumped into two bodies with the force-displacement properties of the cantilever unchanged and if we assume that the external forces causing the motion are applied at these two masses, then the deflected shape of the beam at any time can be completely determined by 12 coordinates, 6 at each masses. The idealised model which has 12 degrees of freedom may therefore conveniently be considered in dynamic problems in place of the actual structure (Ghali et al, chapter 23, 1978).

A more accurate representation can be obtained by using a larger number of masses at a closer interval and consequently requiring a greater number of coordinates. To demonstrate the use of lumped mass idealisation on a multi storey frame subjected to free vibration in its plane, the well known differential equation of undamped free vibration becomes

$$\frac{\partial^2(x,t)}{\partial t^2} + \frac{EI}{m}\frac{\partial^4(x,t)}{\partial t^4} = 0 \quad \text{.................(1)}$$

y(x,t) represents the displacement as a function of x and t; EI, the flexural rigidity and m an equivalent unit mass of the structural system .(e,g. m =mass/ unit length)

the solutions of this linear partial differential equation which describes the " standing wave" i.e Time-dependent shape of bending, constitute the basic mode of vibration and can be given in the form of a product of two functions, each depending on a single variable according to(Varvanov, Part 2 chap. iv, No.23, 1975), namely

$$Y(x,t)=X(x)T(t) \quad \text{..............................(2)}$$

$$T(t) = A \sin(\omega t + \varphi_o) \quad \text{.......................(3)}$$

$$X(x) = B_1 \sin kx + B_2 \cos kx + B_3 \sinh kx + B_4 \cosh kx \text{.........(4)}$$

$$K^4 = \frac{m\omega^2}{EI} \quad \text{.........................(5)}$$

T(t) is a harmonic function of time and ω represents the angular speed or cyclic frequency

A - chracterizes the range of oscillations and is described as amplitude of oscillations.

φ_o - is the initial phase of oscillations.

Equation (3) yields the same values of T(t)for $t=t_1$, $t=t_1+2\pi/\omega$; ...., $t=t_1+2\pi n/\omega$ and therefore called the period(T),which is the duration of one full cycle of oscillations(in seconds.). this is expressed as;

$$T=2\pi/\omega \quad \text{...........................(6)}$$

$$\text{whereas, } \lambda = I/T = \omega/2\pi \quad \text{.......................(7)}$$

is the frequency of oscillations per unit time usully measured in Hertz (Hz), k is the dynamic characteristic of an elastic structural system and X(x) is a function of x which describes the elastic line of the deformed structure. This is usually assumed to be the functions that describes the standing wave. Since it is not dependent on time, B1,B2,B3 and B4 are all constants of intergration which can only be determined through the end conditions of the structural systems under consideration through the end conditions of the structural systems under consideration through the method of initial parameters.

THE METHOD OF INITIAL PARAMETERS FOR INVESTIGATING FREE VIBRATIONS ON AN ELASTIC STRUCTURAL SYSTEM.

The standing wave X(x) is the differential equations(1).A similar equation can be written as y(x) for a structure having its deformation perpendicular to system with the wave X(x) considering the following equation (Varvanov, part 2, chap. iv, N0.26, 1975

$$y^{iv} - K^4 y = 0 \quad \text{......................................(8)}$$

Its intergral will be

$$y(x)=B_1 \sin Kx+B_2 \cos Kx+B_3 \sinh Kx+B_4 \cosh kx \quad \text{..................(9)}$$

To be determined are four linearly independent functions at each row of the differential equation with each an integral of equation (8). They must all form fundamental system of integrals which are multiplied by logical constants which should sum up to be the general integral of equation (8), hence, it becomes logical to ask if sin kx, cos kx, cos kx, sinh kx and cosh kx are the only funtions required to form the fundamental system of integrals of the differential equation(8).

Is it not possible to create additional four linear independent funtions which will give the same result of this differential equation (8). Krilov(1933) and Bezuhov(1939) gave the following positive answers

Because of the linearity of differential equation(8), and their system of fundamental integral which consists of four linear independent funtions Ax, Bx, Cx, and Dx can be created and the equation becomes;

$$y(x) = C_1A_x + C_2B_x + C_3C_x + C_4D_x \quad \text{.......................(10)}$$

Where, C_1, C_2 C_3 and C_4 are arbitrary integral constants. The meaning of this transformation is that, we are creating A_x, B_x, C_x and D_x additional constants that give the solution of equation......(10) with a more convenient practical benefit when compared to that provided by equation........(9) will be determined in the form of the first fundamental system Krilov (1933),to ensure the linear independence of the funtions:

$A_x = ½ (\text{Cosh } kx + \text{Cos } kx)$ ....(11a)

$B_x = ½ (\text{Sinh } kx + \text{Sin } kx)$ ....(11b)

$C_x = ½ (\text{Cosh } kx - \text{Cos } kx)$ ....(11c)

$D_x = ½ (\text{Sinh } kx - \text{Sin } kx)$ ....(11d)

Substituted...(11) into ..(10) and obtained the generalised amplitude displacement (y(x)), rotation (α(x)),bending moment (M(x)), and shear force (Q(X)),from

$EI\,y(x)' = \alpha(x)$

$EIy(x)'' = M(x)$

$EI\,y(x)''' = Q(x)$ ....(12)

as follows

$y(x) = C_1A_x + C_2B_x + C_3C_x + C_4D_x$

$\alpha(x) = K(C_1D_x + C_2A_3 + C_3B_x + C_4C_x)$

$M(x) = EIK^2(C_1C_x + C_2D_x + C_3A_x + C_4B_x)$

and

$Q(x) = EIK^3(C_1B_x + C_2Cx + C_3D_x + C_4A_x)$...........(13)

For the initial conditions:

$y(0) = y;\ \alpha(0) = \alpha_0;\ M(0) = M_0;\ Q(0) = Q_0$............(14)

and from equation ..(13), the following constants are obtained,

$$C1 = y_0;\ C_2 = \frac{\alpha_0}{k};\ C_3 = \frac{M^0}{EIK^2};\ C_4 = \frac{Qo}{EIK^3}$$ ....(15)

If equation (15) is susbstituted in ...(13),

$$y(x) = y_0A_x + \frac{\alpha B_x}{K} + \frac{Mo\,Cx}{EIK^2} + \frac{Qo\,Dx}{EIK^3}$$ ....(16a)

$$\alpha(x) = Ky_0D_x + \alpha A_x + \frac{M_0B_x}{EIK} + \frac{Q_0C_x}{EIK^2}$$ ....(16b)

$$M(x) = EIK^2y_0C_x + EIK\alpha_0D_x + M_0A_x + \frac{Qo\,B_x}{K}$$ ......(16c)

and

$Q(x) = EIK^3y_0B_x + EIK^2\alpha C_x + KM_0D_x + Q_0A_x$.............(16d)

Equations (16a,b,c,d) constitutes the set of equations used in the method of initial parameters.

DETERMINATION OF THE GEOMETRIC AND DYNAMIC PARAMETERS FOR THE SUBSTITUTE CANTILEVER

Fig 1a,b shows the cross - section and substitute cantilever of the 10- storey building under consideration. By assuming a substitute cantilever as the main structural model, from equation ... (5),

$$\omega = K^2\sqrt{\frac{EI}{m}}$$ ....(17a)

and

$$\omega_i = \frac{U_i}{r^2}\sqrt{\frac{EI}{m}}$$ ....(17b)

where $U_i = K\iota$ I known as dynamic parameter and K is the spring constant.

This frequency spectrum indeed contains an infinite number of frequencies related to one another as squares of intergers of the natural numbers. Each frequency represents the corresponding mode of vibrations i.e. the shape of a standing wave. Whereas, the total number of possible modes of free oscillation in an elastic system is equal to the degree of freedom in the system. In practical situations, it is often sufficient to determine the minimum frequecy of the spectrum, as constituting maximum danger in the sense of coming into resonance with the oscillating load. The fact is, first of all the resonance at the lowest frequency results in maximum dynamic effect. Therefore the lowest frequency is often referred to as the fundamental tone of vibrations, (Darkov et al. 1979, pp. 569).

$$\omega 1 = \omega_{min} = \frac{U^2}{L^2}\sqrt{\frac{EI}{m}}$$ ....(17c)

Where m is the intensity applied laad i.e. mass/unit length.This dynamic parameteer(U) is determined by considering the initial paramters of the substitute cantilever from the following conditions.

x=0, displacement and rotation are both equal to zero, hence $y_0 = 0;\ \alpha_0 = 0,$

similarly at $x = \iota$

shear force and bending moment are also equal to zero, and from equation..........................(16)

$$M(\iota) = M_0A_{kx} + \frac{Q_0B_{kx}}{K} = 0$$

$$Q(\iota) = KM_0D_{kx} + Q_0A_{kx} = 0$$ ....(18)

These equations will only produce a non - zero amplitudes if the determinant constituting the coefficents of the amplitudes are equal to zero, that is if

$$\begin{vmatrix} A_{kx} & B_{kx} \\ KD_{kx} & A_{kx} \end{vmatrix} \qquad \text{.........................(19a)}$$

$$A\text{-}kx^2\ B\text{-}kxD\ \text{-}\ kx = 0 \qquad \text{.........................(19b)}$$

Since X = i, and Ki = U

$$A^2_{ki} = B_{ki} D_{ki} = 0 \qquad \text{.........................(19c)}$$

$$\text{CoshU Cos U} = -1 \qquad \text{.........................(19d)}$$

$U_{min} = U_1 = 1.875$, $U_2 = 4.69$ and $U_3 = 7.863$ are the dynamic parameters corresponding only to a cantilever. Their present values are in radians and or in degrees are equal to 107.43, 284.187 and 450.8(90.8)

Darkov et al., 1979, pp. 578, As it is shown under the law of energy conservation, the sum of potential (P) and Kinetic (V) energies of and oscillating system is constant.

$$P + V = \text{Const.} \qquad \text{.........................(20)}$$

Within each cycle of oscillation, energy of one kind is transferred into the other. The potential energy reaches its maximum and velocity and kinetic energy diminishes to zero, at the time of maximum deflection of the mass.

When the mass passes through the point of equilibrium, the potential energy of strain (more precisely, the increment of potential energy as compared to its value at static equilibrium position) is zero, while the kinetic energy reaches its maximum level, V_{max}. Since minimum values of both potential and kinetic energies equal zero, equation (20) signifies that

$$P_{max} = V_{max} \qquad \text{.........................(21)}$$

Substituting the expressions for energy into this equation leads to the determination of frequencies of vibrations. If we know the mode of vibrations, i.e. the shape of the elastic string (line) in advance, equation..(21) would yield the strict solution of the problem. An appropriate solution of a problem can be obtained by choosing a dynamic elastic string, i.e. a standing wave curve, with an arbitrary equation y = f (x) satisfying boundary conditions. For example, this could be the equation of the bent axis of the substitute cantilever subjected to static load corresponding to the included concentrated masses. This techinque is hereby applied to deternine the lowest frequency. The closer the shape of the elastic string to that realised in vibrations with the frequency of interest, the higher the accuracy of the sought frequency. Potential energy of bending strain of the substitute cantilever with concentrated masses can be expressed in terms of the work done by external forces $Q_i = m_i g$ on displacement y_i(Darkov et., 1979 pp. 577-8)

$$P_{max} = {}^1/_2 \sum_0^n Q_i y_i = \frac{g}{2} \sum_0^n m_i y_i \qquad \text{...............(22)}$$

Where n is the number of masses. In order to find the maximum value of kinetic ennergy, it is necessary to deternine the maximum velocity. Assuming that, a mass at an arbitrary moment of time t has the velocit

$$v_i = y_i \omega \cos(\omega t + \varphi_o) \qquad \text{.........................(23a)}$$

the maximum velocity corresponding to $\cos(\omega t + \varphi_o) = 1$ is

$$V_{max} = y_i \omega \qquad \text{.................................(23b)}$$

The sought kinetic energy is then

$$V_{max} = {}^1/_2 \sum_0^n m_i V^2_{i\,max} = \omega^2 \sum_0^n m_i y_i^2 \qquad \text{.............(24)}$$

By equating P max and V max we derive the formula for calculating the frequency (ω) of interest

From

$$\omega = \sqrt{\frac{g \sum_{i=1}^{n} m_i y_i}{\sum_{i=1}^{n} m_i y_i^2}} \qquad \text{...............(25)}$$

For the lumped or concentrated masses along the height of the building. These values have been provided by the static analysis (using the computer obtained P - delta effect) of virtual forces P = 1.0 kN at the highest point of the building and varied lineraly to zero at the base as shown in fig 1a.

Malook et al. (1983), the dynamic factor (β) is given by

$$\beta = 0.7/T \qquad \text{.............................(26)}$$

Period (T) is to be taken as the mean of the the theoretical and empirical (or 0.1n) values (Bowles, 1981).

For most pratical cases, $0.8 \leq \beta \leq 2.4$ is recommended (Malook et al. 1983). The seismic loading or forces (F_{ik}) corresponding to this frequency at any floor or normal mode of characteristic shape is given by

$$F_{ik} = Q_k K_c i_o \beta i \eta_{ik} \qquad \text{.......................(27)}$$

$$M_0 \quad \frac{y(\imath)\ EI\ K^2}{} = 392270kNM$$

$$C_{kl} - \frac{A_{kl} D_{kl}}{B_{kl}}$$

$$Q_0 = \frac{KA_{ks} M_0}{B_{ks}} = -14984.72kN$$

$y(x) = 0.01006\ C_{sk} = 0.00737' D_{ks}$

$\eta_{ik} = AX_{IK} = 149.7\ X_{IK}$

Since the period T = 0.1692 sec or 0.1n = 0.1 x 10 = 1.0

$$T_m = \frac{0.169.1}{2} = 0.58Sec$$

$$\beta_1 = \frac{0.7}{0.585} = 1.20, K_c = 0.033, \iota_0 = 1.37$$

Therefore, $F_{IK} = 149.742\ \eta_{ik}$

$$m = 76.67\ kN\ \frac{S^2}{m^2}, \quad Q_k = 2760.12kN\ \frac{S^2}{m}$$

Table 1 also shows the computed values of η_{ik} and consequently the value of F_{ik} and their corresponding displacements. The maximum allowable displacement 5.792 x10^{-2} meters which is less than 6.0 x10^{-2} meters the maximum allowable displacement

TABLE 1

Node Nos. i	Nodal Displacements caused by virtual force yi(m)	Standing wave $X_{ik} = y(x)$	First mode of characteristic shape η ik	Seismic force at floor level Fik (kN)	Nodal displacements caused by seismic load δi (m)	Limiting values of displacement as recommened by the code δi (m)
1	0	0	0	0	0	0
5	6.074×10^{-4}	1.59×10^{-4}	2.38×10^{-2}	3.56	2.34×10^{-3}	
9	1.4284×10^{-3}	6.506×10^{-4}	9.74×10^{-2}	14.60	15.86×10^{-3}	
13	2.3708×10^{-3}	1.39×10^{-3}	2.08×10^{-1}	31.16	1.03×10^{-2}	
17	3.4019×10^{-3}	2.31×10^{-3}	3.46×10^{-1}	51.81	1.561×10^{-2}	
21	4.5034×10^{-3}	3.44×10^{-3}	5.15×10^{-2}	77.12	2.17×10^{-2}	
25	5.6544×10^{-3}	4.65×10^{-3}	6.96×10^{-2}	104.24	2.84×10^{-2}	
29	6.8122×10^{-3}	5.96×10^{-3}	8.922×10^{-1}	133.6	3.56×10^{-2}	
33	7.9956×10^{-3}	7.22×10^{-3}	1.081	161.87	4.33×10^{-2}	
37	9.1214×10^{-3}	7.823×10^{-3}	1.171	175.25	5.07×10^{-1}	
41	1.01852×10^{-2}	1.01×10^{-2}	1.512	226.00	5.792×10^{-2}	6.0×10^{-2}

CONCLUSION

In this paper, data obtained from a static analysis provided by a software capable of solving P - delta or finitc element related problems were used as initial parameters for dynamic analysis of a high rise (10 - storey) building. The initial load that was applied on the structural system was p = 1.0kN at the top, varied lineraly to zero at the base. This load model actually served as virtual load used to simulate the effect of earthquakes on the chosen structural sections at various levels and locations along the structural system. From this mode of loading, it was discovered that using more materials may not necessarily mean a more stable structure and vice - versa. Rational management and arrangement of structural elements actually influence greater stability, hence, the choice of assuming varied structural sections along the heiht of the building. The deternination of the cyclic or fundamental tone frequency of vibration was realised through the enegy method for determining the eigenfrequencey for the substitute cantilever. Form this result, the equivalent minimum total moment of inertia that ensures stability of the model was deternined. Similarly, the frequency of oscillation and the corresponding seismic loading were computed and used to test whether or not structural sections provided for the frame and diagonal braces ensured the code requirements for drft limit, interstorey drift and relative lateral displacements at storey levels are not exceeded.

The illustrative example solved with this method on the shear-wall vertically braced frame fig. 1a, shows that the beam connecting the two rows serves as the core of the structural frame for both vertical and lateral loads, hence, it is recommended to be adequately rigid to reduce excessive deflection and to enable the structure satisfy the fundamental stability requirements of the codes. In addition, because of the difficulties associated with correct predication of the epicenter and its proximity to the building, a coefficient io = 1.37 has been proposed for high rise buildings. The comparison of the results obtained by this method for the determination of th period etc. with those from RISA-2D developed by SUNOMA Engineers shows a very good compliance.

With a satisfactory testing of this procedure, a programme algorithm can now be developed to make it a complete software for solving a number os enigneering problems.

ACKNOWLEDGEMENTS

The authors wish to thank the Authorities of the University of Maiduguri for all the resources made available for this study. The writer also wish to thank Engr. Mustapha Bulama of Sani Mustapha and Associates for the collaborative reserarch opportunities extended to the Department of Civil and Water Resources Engineering of University of Maidguri. Our profound appraciations are also extended to Yusuf Abubakar of the University Computer Centre and Mrs ONUNDI of Femo Computer Service a division of FEMO Consultnat for their numerous assistance while solving this problem.

REFERENCES

Barvanov, C.P. (1975). "Ystoichivost I Dynamika Systemi" 2nd edutuib, National Press, " Technika" , Sofia.

Bowles, J. E. (1981). "Structural Steel Design". International Student Edition, McGraw Hill International Book Company, London.

Brankov, G. V. (1978). "Mácivni Konstruktii" 4th Edition National Press " Technika", Sofia.

Christev, T. N. Karamanski, T. D. and Rangelov R.P. (1974). "Rekovodstvo za Resavana Nazadachi Po Teuria Na Elastichnost, Yostoichivost I Dynammika Na Elastichinite Systemi, "National Press, "Technika", Sofia.

Darkov, A., Klein, G., Kuznetsov, V., Luzhin O., Rakach, V., Sinelnikov, V., Kisin, V. And Shpiro, G. (1979). " Structural Mechanics" 3rd Edition, Mir Publishers, Moscow.

Ghali, A. Neville, A. M. and Cheung, Y. K. (1978). " Structural Analysis, A Unified classical and Matrix approach" Second Edition, Chapman and Hall, London.

Iffland, J. S. (1978), "High Rise Building Column Design'" Proceedings of the American Society of Civil Engineers. Journal of the structural Division, Vol. 104, No. ST9, pp. 1469 - 1483.

Rutenberg, A. (1981). " A direct P - delta analysis using standard plan frame programmes". Computers and Struc., 14, 97 - 102.

Rutenberg, A. (1988). "Stability of shear - wall structure", Journal structural engineering, Vol. 114. No. 3, pp. 707 - 716.

Malook,C., Mladezhnov, P. V., Dimitrova, M. A. Reshenov,H. A. and Ymmboirski,P. A. (1983), " Narachnik po Metalni Konstruktii". 1st edition, National Press, "Technika". Sofia.

SUNOMA Engineers (1993). "Rapid Interactive Strutural Analysis - 2 Dimensional" Demostration version, Risa Technologies, Irvine, California 92714.

S/No	STRUCTURAL ELEMENTS	SYMBOL	NODES DESCRIBING THE ELEMENTS	SECTION TYPE
1	COLUMN	C1	1-17, 2-18, 3-19, 4-20	W24 x 131
2	COLUMN	C2	17-29, 18-30, 19-31, 20-32	W24 x 62
3	COLUMN	C3	29-41, 30-42, 31-43, 32-44	W21 x 50
4	BEAM	B1	5,6 - 41,42; 7,8-43,44	W14 x 30
5	BEAM [illegible]	B2	6,7 - 42, 43	W24 x 117
6	DIAGONAL BRACING	DB1	1,6;2,5-21,26;22,25 & 3,8;4,7-23,28;24,27	11.5 x 5 x 14
7	DIAGONAL BRACING	DB2	25,30;26,29-37,42;38,41 & 28,44;40,43	0.5 x 5 x 8

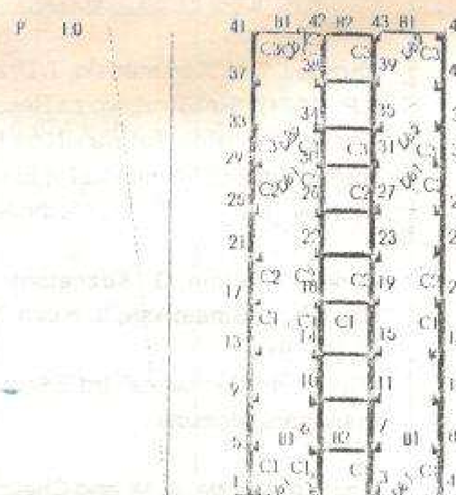
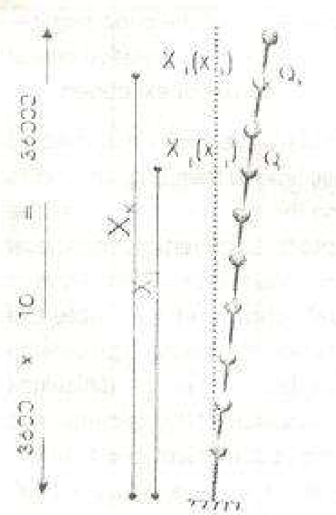


Figure 1.0a



SUBSTITUTE CANTILEVER
(First Mode of Characteristic Shape)

Figure 1.0b