MODELING OPTIMAL DISTRIBUTION OF URBAN WATER SUPPLY IN DEVELOPING URBAN CITIES – A CASE STUDY OF AKURE IN SOUTHWEST, NIGERIA

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ABSTRACT

Water, present in all living organisms, a major factor in the formulation of rocks and soil, with its primordial influence on public health and, indeed, all human activities, must be considered the most important of the raw materials upon which we depend. With forecasted overall per capital water consumption reaching 100 l/day in the Akure metropolis, less than one quarter of the population are adequately served with water supply, hence the public water supply system has not been able to meet the domestic and industrial requirements of the area. Increased industrialization, urbanization, population growth and demand for improved hygiene now force us to manage our resources as never before.

In this work a mathematical model for optimal allocation of available water to different zones within the metropolis was developed. The model is based on hydraulic and economic principles. The model was applied to the water distribution network of Akure urban. Results obtained from this specific applications will be useful to designers, researchers and policy makers in the public and non-governmental organizations.

INTRODUCTION

The problem of water supply is global but it is much more pronounced in the developing countries. In developing countries, cities often suffer from serious water supply shortages due primarily to uncontrolled and unanticipated growth in urban population, inadequacy in the design of the urban network water supply systems apart from management problems of operation and maintenance and natural disaster.

The design of water supply systems requires an estimate of future demands and the optimal way to meeting these demands. The choice of the design capacity is a function of factors such as population growth and demand, source of funding, the character of the water supply, the qualities and quantities of water (both raw and final product). The general urban water demands consist of residential, industrial, commercial and fire fighting demands and losses incurred in the process of transmission. Rapid growth in urban population is common to developing countries. The growing demand for water in addition to the limiting nature of the resource is the reason for exploring new trends in designing and managing water supply systems.

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Concept of Mathematical Model

The development of the model is aimed at providing an efficient tool of allocating water to individual consumers at the least cost and in a manner that maximizes the effectiveness of the entire water supply system. Under this study, water is allocated to the consumers within the same demand centre (a city) at different periods. In this case, a city can be divided into M demand centres i.e. (I.........M) and then divide the available water between them for ranges of time to be considered e.g. 4hrs, 8hrs, and 16hrs time respectively.

This rationing of the available water can also be between cities as demand centers that are getting their water supply from a source but which cannot cope with simultaneous supplies to all demand centers. Allocation cost is a function of allocation option chosen.

The main objective here then is to find:

(a) The allocation of flow to demand centers, and

(b) The allocation option, which will give minimum operating cost for pumping or energy.

Model Formulation

The main objective of model is to supply water to demand centers through allocation of flow at the least operating cost for pumping or energy.

The objective function in general is of the following form

$$\min C_T = \sum_{j=1}^{n} c(Q_j) \qquad 1$$

subject to the following constraints

$$g_{i(Q_j)} \leq b_i \; (i = 1, 2, \ldots\ldots\ldots\ldots, m). \qquad 2$$

$$f_{j(Q_j)} \geq 0 \quad (j = 1, \ldots\ldots\ldots\ldots n) \qquad 3$$

where C_T is the total cost of allocation, and c is the unit cost of supplying Qj allocation.

$$Q_j = \Sigma q_j \qquad 4$$

Besides a minimum pressure head must be maintained at each demand centre or point i.e.

$$H_s \geq H_{min} \qquad 5$$

The following assumptions are made:

(a) Since the rate of pumping remains constant and the time lag between each period is the same, then supply to all the nodes remain constant i.e.

$$q_{1t} = q_{2t} = q_{3t} = \ldots q_{nt} \qquad 6$$

(b) A day is divided into n time periods of equal intervals 1,2, 3…N

It is necessary to supply at least $q_i \geq q_{min}$ to each demand center i at minimum cost. As earlier mentioned, the mode of meeting this can be expressed as any of the following:

Total daily consumption per demand		Period of day		
	1	2	…………	n
Q_1 =	q_{11}	q_{12}	…………	q_{1n}
Q_1 =	q_{21}	q_{22}	…………	q_{2n}
.	.	.		.
.	.	.		.
.	.	.		.
Q_m =	q_{m1}	q_{m2}	…………	q_{mn} 7

The supply is on continuous basis with equal allocations in each time period.

In order to make operational definition of pumping cost a variable Q_{ijt} can be introduced to indicate the rate of flow in a pipe link ij during time period t.

Pumping cost includes cost due to elevation head and cost due to frictional head loss. Any increase in Q_{ijt} will lead to increase in pumping cost. From here it can be deduced that pumping cost of each link is a convex function of the discharge Q_{ijt}.

This kind of problem can be solved using piecewise linearization (Loucks et al, 1981). Here there are three basic methods. Usually for concave functions, it is maximization while for convex function, it is minimization. The case under consideration is a convex function. Fig.1 can be used to explain this fact.

The problem can be formulated as

$$\text{Min } C_T = \sum_i \sum_j \sum_t c_{ijt} \, Q_{ijt} \qquad 8$$

Where C_T is the total cost and c is the unit cost of supplying Q_j allocation. $Q_j = \Sigma q_i$

$$Q_{ijt} = a_1 w_1 + a_2 w_2 + a_3 w_3 + \ldots\ldots + a_n w_n \qquad 9$$

With the weighing factor,

$$w_1 + w_2 + w_3 + \ldots\ldots\ldots\ldots\ldots w_n = 1$$

$$\text{or } \Sigma w_i \leq 1 \qquad 10$$

it can otherwise be written as Minimize C_T =

$$\sum_i \sum_j \sum_t c_{ijt} \left[(a_1)w_1 + (a_2)w_2 + (a_3)w_3 + \ldots\ldots.(a_n)w_n\right] \qquad 11$$

Where w_i will be the variable to be determined and a represent a flow at a_i particular period at a particular node for a stated period of time.

Scope of the Mathematical Model

It is expected that this model will provide the quantity of water to be supplied to each demand centre for each period with minimum pumping or energy cost. For the purpose of this work, the following variables have to be considered:

(i) the quantity of water to be supplied to each demand centre Qj is equals to the sum of small discharges (q_n) i.e.

$$R_j = \sum_i^n q_{it} \qquad 12$$

(ii) the schedule for water allocation or rationing system;

(iii) the number of loops in the entire network;

(iv) the minimum target pressure for effective supply;

(v) the pumping head for each demand centre or reservoir height.

Characteristics and Constraints in The Linear Equations

Flow Rate Constraints

This constraint requires that the sum of all flows to a given demand center j for any 24 hour period must not be less than total consumption (demand) at that centre (node), that is total flow to node j in period t is designated by Q_{jt} so

$$\sum_{t=1}^{24} Qjt \geq Q_j \qquad \forall(j = 1, \ldots\ldots..J) \qquad 13$$

where Q_j is the expected consumption at node j

$$\sum_{t=1}^{24} Q_{jt} = \left[Q_{j1} + Q_{j2} + Q_{j3} + \ldots\ldots\ldots + Q_{jn} +\right] \geq Q_j \qquad 14$$

for any J-node network .the sets of linear equations below describe the physical characteristics of the flow systems

Node 1 $Q_1 \geq Q_{11} + Q_{12} + Q_{13} + \ldots\ldots\ldots + Q_{1N}$ 15

Node 2 $Q_2 \geq Q_{21} + Q_{22} + Q_{23} + \ldots\ldots. + Q_{2N}$ 16

: :

: :

Node j $Q_j \geq Q_{j1} + Q_{j2} + Q_{j3} + \ldots\ldots + Q_{JN}$ 17

According to the law of conservation of mass

$Q_1 + Q_2 + Q_3 + \ldots\ldots\ldots\ldots + Q_j \geq Q_j$ 18

Continuity constraint

Flow received in a node j_{th} will be carried to another node j+1 through a link (pipe) and continuity equations will be written for each time period for which there is flow the link entering or leaving node j. The sum of water flows entering a node and the sum of flows leaving a node including the withdrawals during any time period are equal.

For a simple network as shown in Figure 2, the equation can be written as

$Q_{ijt} - Q_{jkt} - D_j = O$ 19

where D_j is the consumption at node j

or generally,

$$\sum_{i=1}^{m} Q_{ijk} - \sum_{k=1}^{m} Q_{jkt} - D_{jt} = 0 \quad \forall\, j = 1 \ldots\ldots m. \qquad 20$$

$$t = 1 \ldots\ldots\ldots 24$$

with the non-negativity constraint

$$Q_{ijt} \geq 0; Q_{jkt} \geq O \qquad 21$$

However any of the flows can be zero e.g. $Q_{ijk} = O$, if there is no flow in the link during the time period t.

Each of the components of Equation 19 except D_{jt} can be written as follows

$$\sum_{i=1}^{m} Q_{ijt} = \begin{bmatrix} Q_{1j1} + Q_{1j2} + Q_{1j3} + \ldots\ldots\ldots + Q_{1j24} \\ + \\ Q_{2j1} + Q_{2j2} + Q_{3j3} + \ldots\ldots\ldots + Q_{2j24} \\ + \\ Q_{3j1} + Q_{3j2} + Q_{3j3} + \ldots\ldots\ldots + Q_{3j24} \\ : \\ : \\ + \\ Q_{mj1} + Q_{mj2} + Q_{mj3} + \ldots\ldots\ldots Q_{mj24} \end{bmatrix}$$

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also for $\sum_{k=1}^{m} Q_{jkt}$ as in Eq. 5.21 above.

The nodal demand, D_{jt}, is now given by

$D_{ji}, D_{j2}, D_{j3} \ldots D_{j24}$ 23

In general, the problem under consideration can be expressed as shown below

Minimize $\sum_j \sum_e \sum_t C_{ej_t}\ (a_1) w_1 + C_{ejt}(a_2)\, w_2 + C_{ejt}$

$(a_3)\, w_3 + \ldots$ 24

Subject to

$$\underbrace{\sum_{t=1}^{24} Q_{jkt}}\ \geq \underbrace{\ Q_j\ } \qquad \forall j\,;\, j = 1;\, \ldots . m$$

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where Q_{jkt} is inflow into node j in time t.

Consumption at node j is given as

$$\sum_{t=1}^{m} Q_{ejt} - \sum_{k=1}^{m} Q_{jkt} - D_{jt} = 0 \quad \forall j\, t \qquad 26$$

$Q_{ijt} = a_1 w_1 + a_2 w_2 \ a_3 w_3 \quad \forall$ ijt 27

$w_i \leq 1$ 28

$Q_{ijt} \geq 0$ 29

The objective of this model is to find the allocation of flows to demand centers and the best allocation option that will minimize the operating cost for pumping or energy.

To achieve this, the following data will be required:

(i) Location of supply sources and the demand centers;

(ii) Routes from the sources to the demand center;

(iii) Allocations to each demand center;

(iv) The distance and height from the centre.

In addition, other data, such as the friction factor (f) and the efficiency of the pumping system (i.e. motors and pumps) η_o as well as the unit cost of electricity (in ₦/KWH) must be known.

Cost Functions

Pumping cost: Pumping cost of water in distribution network is a function of the age of the pipe, the extent of corrosion damage in the pipes, quantity of water pumped, the length of pipe network, the diameters of the pipes, the head losses due to friction in the pipes and the head against which the water has to travel.

Cost of Operation

This can be obtained by equating the pumping power required for a link i.e. the total dynamic head (H_T) to that of an equivalent number of kilowatt-hours and then multiplying that by the corresponding unit cost

$$H_T = h_{f\ (i-j)} + h_{e} \qquad 30$$

where h_e is the static or elevation that is required to lift the water form one node to another.

$h_{f\ (i-j)}$ is the loss due to friction between two nodes (i - j).

If the unit cost of overcoming friction between node i and j is C_{ij}, then Ci_j, $h_{f,(i-j)}$ is the total cost of friction head loss in link i-j assuming that $C_{ij}\ h_{f(e-j)}$ is continuous and monotone increasing as shown in Fig. 3 below.

The implication of these assumptions is that the objective function is differentiable and convex in nature. Energy dissipation in a pipeline during flows is a function of the pipe roughness, diameter and discharge (Q). This can be expressed using Darcy-Weisbach equation

$$h_{f(i-j)} = \frac{16 fLQ^2}{19.6D^5\pi^2} = \frac{0.083 fLQ^2}{D^5} \qquad 31$$

Where

Q_{ij} is the flow between nodes i and j

f is the friction factor

L and D are the length and diameter of pipe between node i and j.

For $h_{f,(i-j)}$ the horse power or energy for conveying a flow Q is given by

$$\Delta p = \frac{\gamma HQ}{19.6.D^5\pi^2 1000\eta} \qquad 32$$

Where γ the specific weight of water kg/m^3

η - The efficiency factor of the pumping station

Putting the value of h_f into equation 31, we have

$$\Delta p = \frac{\gamma f\, LQ^3}{19.6.D^5\pi^2.1000\eta} \qquad 33$$

Daily pumping cost along link L

$$C_P = \frac{16\gamma f L Q^3 t.C_{en}}{19.6.D^5\pi^2 1000\eta} \qquad 34$$

where

C_{en} is the unit cost of electricity (tariff) in ₦/K WH. This value can be obtained from NEPA and it is N6/KWH (2002), t is the hours of pumping per day.

The overall pumping cost for all the links in the network can be written as

$$C_T = \sum_{ij} C_p = \sum \frac{16\gamma f LQ^3.t.C_{en}}{19.6D^5\pi^2.1000.\eta} \qquad 35$$

where γ is the specific weight of water (γ= 9810N/m^3).

The option that was earlier mentioned can now be rewritten with 5.34, assuming the supply and consumption in each case is constant initially, that is,

$$A_1 = A_2 = A_3 = \ldots\ldots\ldots\ldots = A_n = A \qquad 36$$

Where $A_1, A_2, \ldots\ldots A_n$ are the constants

To satisfy the consumption at each demand centre continuously over 24 hours per day, the equations can be written as consumption.

	t_1	t_2	t_3	t_{n-1}	t_n		
Node 1	q_{111}	q_{112}	q_{113}	$q_{11(n-1)}$	q_{11n}	$\geq$ A	
Node 2	q_{121}	q_{122}	q_{123}	$q_{12(n-1)}$	q_{12n}	$\geq$ A	
	:	:	: :	:	:	:	37
Node m	q_{1m1}	q_{1m2}	q_{1m3}	$q_{1m(n-1)}$	q_{1mn}	$\geq$ A	

where q_{1mn} indicates flow q for option 1, node m, time n. The flow constraints at network nodes for n-time periods can be written as follows:

First the flow in the links is:

	In period 1(t_1)	In period 2(t_2)	In period n(t_n)
Along link S-1	$q_{111}+q_{121}+\ldots q_{1m1}$	$q_{121}+q_{122}..+\ldots+q_{1m2}$	$q_{11n}+q_{12n}+\ \ldots+..\ q_{1mn}$

Further combination here will be a function of the network layout.

The optimization problem is to get the least total daily cost of pumping C_T

Recalling Eq. 5.35, this can be written as

$$C_T = \sum \frac{16\gamma f LQ^3.t.Cen}{19.6D^5\pi^2.1000.\eta} = \sum Y\frac{(LQ^3)}{D^5} \qquad 38$$

where $Y = \dfrac{16\gamma ft.Cen}{19.6.\pi^2.1000.\eta}$

Now for policy 1,

$$C_T{}^1 = Y_1\left[L_{5-1}\frac{(q_{111}+q_{121}+....+q_{1mn})^3}{D^5{}_{s-1}}+\frac{L_{12}(q_{121})^3+}{D^5{}_{1-2}}.....+\frac{L_{(m-1)}-m(q_{1mn})^3}{D^5{}_{(m-1)-m}}\right]$$
$$+\left[L_{5-1}\frac{(q_{112}+q_{122}+....+q_{1mn})^3}{D_{1-2}{}^5}+\frac{L_{1-2}(q_{112})^3+}{D^5{}_{1-2}}.......+\frac{L_{(1-3)}(q_{132})^3+..+}{D^5{}_{1-3}}\frac{L(m-1)-m(q_{1mn})^3}{D^5{}_{(m-1)-m}}\right]$$
$$+\left[L_{5-1}\frac{(q_{11n}+q_{12n}+....+q_{1mn})^3}{D^5{}_{s-1}}+\frac{L_{1-2}(q_{12n})^3+}{D^5{}_{1-2}}.......+\frac{L_{(m-1)-m}(q_{1mn})^3}{D^5{}_{(m-1)-m}}\right]$$

It should however be noted that this arrangement can change depending on the network configuration.

The constraints are

$q_{111} + q_{112} + \ldots + q_{11n} \geq A$

$q_{121} + q_{122} + \ldots + q_{12n} \geq A$ 40

: : : :

$q_{1m1} + q_{1m2} + \ldots + q_{1mn} \geq A$

It is observed that there are some terms in the objective functions that cannot be subjected to piecewise linearization such as $q_{111} + q_{121} + \ldots + q_{1mn}$. Therefore, some transformation is needed, thus:

$q_A = q_{111} + q_{121} + q_{131} + \ldots + q_{11n}$

$q_B = q_{121} + q_{122} + q_{132} + \ldots + q_{12n}$

: : : 41

: : :

$q_x = q_{1m1} + q_{1m2} + {}_{1im3} + \ldots + q_{1mn}$

The above are the necessary transformations that are also constraints.

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The above transformations are to be used in conjunction with piecewise linearization, method (Loucks, 1981) which was mentioned earlier.

A computer programme was written to solve this model and it has been tested. The details of this are given in the results section.

Application of Allocation Model

Example 1 – Owena–Igbara-Oke Scheme.

Criteria

(a) Allocation of water to demand centres is controlled using valves bearing in mind all constrains associated with pumping capacity and duration.

(b) Water is allocated to all demand centres at minimum cost.

To achieve this, the following were considered

i) Selection of flow rate of each demand centre;

ii) Grouping of the demand centres into zones first by contiguous arrangement and non-contiguous arrangement. Selection of the demand centers' discharges were selected close to the daily mean flow of the main pipelines. Thus,

$$\sum qi \le Q \qquad 42$$

where qi per zone and Q is the total flow expected from the source.

Assumptions

To simplify the mathematics of the model, the following assumptions were made:

i) The water distribution system is on the same elevation, neglecting the topography and therefore only pumping is required.

ii) The discharges to the demand centres are equal;

Input Data

To apply the model, the following data are required:

q_i = 500-750m^3/period - allocation to each demand centre

L_p = ₦6, 000 - pipe material /m^3

K_p = 0.1 - pipe thickness

C_{EN} = ₦6.00/kW h - Energy cost/kW h

C_{HW} = 90 –150 - Hazen-Williams Coefficient.

c_u = 0.278 - unit coefficient

The overall pumping cost as expressed in Equation 43 is

$$C_T = \sum_{ij} C_p = \sum \frac{16\gamma f L Q^3 . t . C_{en}}{19.6 D^5 \pi^2 . 1000 . \eta} \qquad 43$$

Results

Owena-Igbara-oke and Owena–Ondo Water Supply Scheme

Owena- Igbara Oke and Owena Ondo schemes were selected for this analysis because of the relative availability of operational records.

Allocation of water from Owena –Igbara Oke scheme is allocated to 7 demand centers namely Owena, Igbara Oke, Ilara Mokin, Ijare, Shagari Housing Estate and Akure. From the results obtained, it appears there is a linear relationship between the friction head and pumping costs. A similar operation was carried out on Owena –Ondo scheme supplying basically Owena, Ondo, Ile-Oluji, and Akure using the current allocation ratio. The result is as shown in Figure 1.

Akure Zone

This model also used the existing nine zones in Akure to carry out the allocation analysis bearing in mind that Owena-Ondo which supplying the zone is only operating at approximately 60% of its full capacity. In this arrangement, equal volume is allocated to each zone to maintain equity except Alagbaka which is allocated 500m^3 and must flow on continuous basis. Table1 shows a typical allocation arrangement.

Since the major objective this model is to get water to every zone and at minimum pumping cost, three different arrangements were used to the programme. Summary of these arrangements is shown in Table 2. The best allocation arrangement is the one that gives the least pumping/energy cost.

Conclusion

The allocation model has shown that a single source of water supply with limited production can serve several zones of demand which can be intra-city, or based on town to town. Efficient allocation is indicated by grouping of the zones in such a way to obtain minimum pumping costs, and hence minimum energy costs. The model can be used to specify the number and combinations of zones to be supplied at different periods of daily operations. Specific application of the model to Akure township water supply shows that the minimum costs of water allocation actually depend on the arrangement of zones at different period.

Reference:

1. Abdel-Latif, M.Y., 2001: Water Distribution Modelling with Intermittent Supply: Sensitivity analysis and performance evaluation for Bani Suhila City – Palestine Unpublished MSc Thesis. South Bank University, London.

2. Abebe, H. and Solomatine, W. F. 1998: Water Supply in Developing Urbans. Publication of CWRM. Madrid.

3. Alausa, S.B., 1997: Privatization of Urban Infrastructure: Issues, Myths and Realities Proceedings. Annual Conference of Nigeria Institute of Estate Surveyors and Valuers (NIESV), Ibadan, pp. 18 – 27.

4. Alavi, N.A.,1984: Proceedings of the International Seminar on Water Resources Management Practices. Ilorin, Nigeria. pp109.

5. Australian Academy of Technological Sciences and Engineering, (1988). Urban Water Supplies. Australian Science And Technology Heritage Center.

6. Babatola, J. O., 2004: Optimal Allocation and Distribution of Water Supply: Case Study of Ekiti and Ondo States. Unpublished PhD Thesis. Federal University of Technology, Akure. Njgeria.

7 Bhave, P.R., 1992: A Critical Study the Linear Programming Gradient Method of Optimal Design of Water Supply Networks. Water Resources Research 28(6), 1577-1584.

8. Bhave, P.R., 1985: Optimal Expansion of Water Distribution Systems. Journal of Environmental Engineering Division. American Society of Civil Engineers, Vol.111, No 2.

9. Bogale M.M., 1994: Design and Implementation of a modeling system for water Distribution Network: An object oriented Approach. (MSc Thesis).

10. Looucks, D.P. and Falkson, L.M., 1981: Water Resources Analysis and Management. Water Resources Centre, University of California, California.

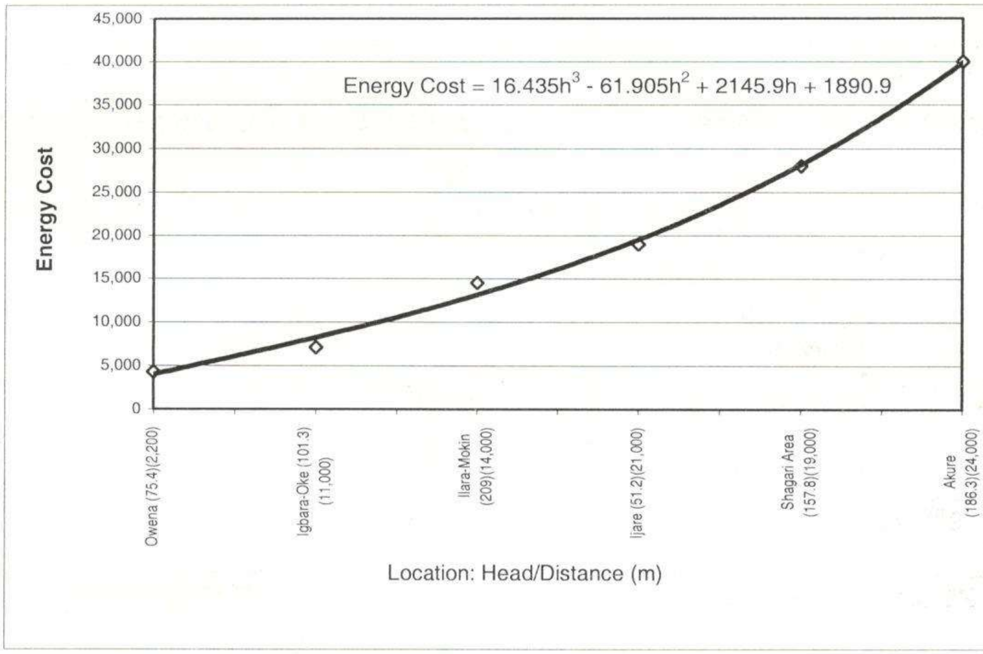

Fig. 1: Results of Implementation

Table 1 Typical Arrangement of Water Allocation

	PERIOD OF THE DAY				ALLOCATED	PUMPING COST (₦)
	t_i	t_2	t_3	t_4	$Q_I(M^3)$	
Zone 1	√				750	16,550
Zone 2	√				750	
Zone 3		√			750	21,270
Zone 4		√			750	
Zone 5			√		750	23,500
Zone 6			√		750	
Zone 7				√	750	26,800
Zone 8				√	750	
Zone 9	√	√	√	√	500	12,400

Table 2 Summary of Possible Allocation Arrangement

1st Arrangement			
ZONE/LOCATION		**TOTAL FLOW M^3**	**PUMPING COST ₦**
1	Zones 1 + 2	1500	16,550
2	Zones 3 + 4	1500	21,270
3	Zones 5 + 6	1500	23,600
4	Zones 7 + 8	1500	26,800
5	Zones 9	500	9,400
2nd Arrangement			
ZONE/LOCATION		**TOTAL FLOW M^3**	**PUMPING COST ₦**
1	Zones 1 + 2 + 3	22,500	13,600
2	Zones 4 + 5 + 6	22,500	16,400
3	Zones 7 + 8	1,500	17,400
4	Zones 9	500	9,400
3rd Arrangement			
ZONE/LOCATION		**TOTAL FLOW M^3**	**PUMPING COST ₦**
1	Zones 1 + 3	1,500	25,100
2	Zones 2 + 4	1,500	16,600
3	Zones 5 + 7	1,500	17,500
4	Zones 6 + 8	1,500	18,100
5	Zone 9	500	9,400

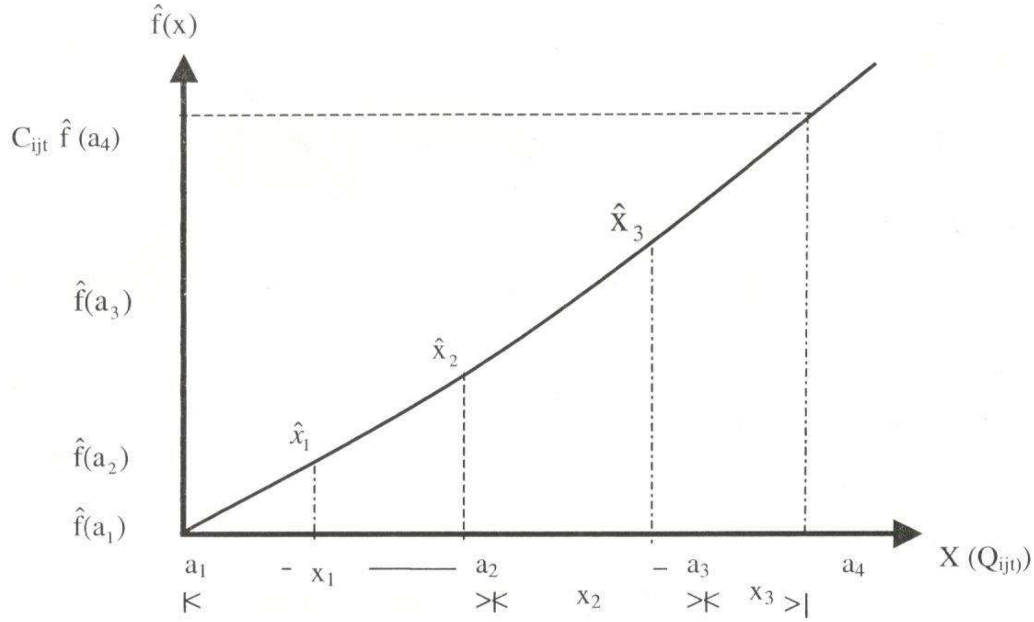

Figure 1: Graph of Convex Function

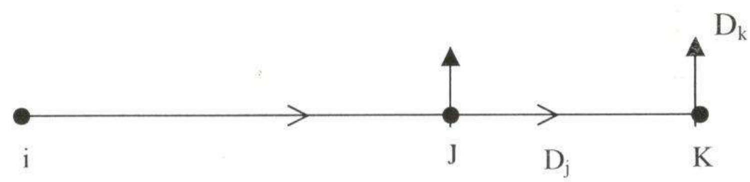

Figure 2: A Simple Flow Network

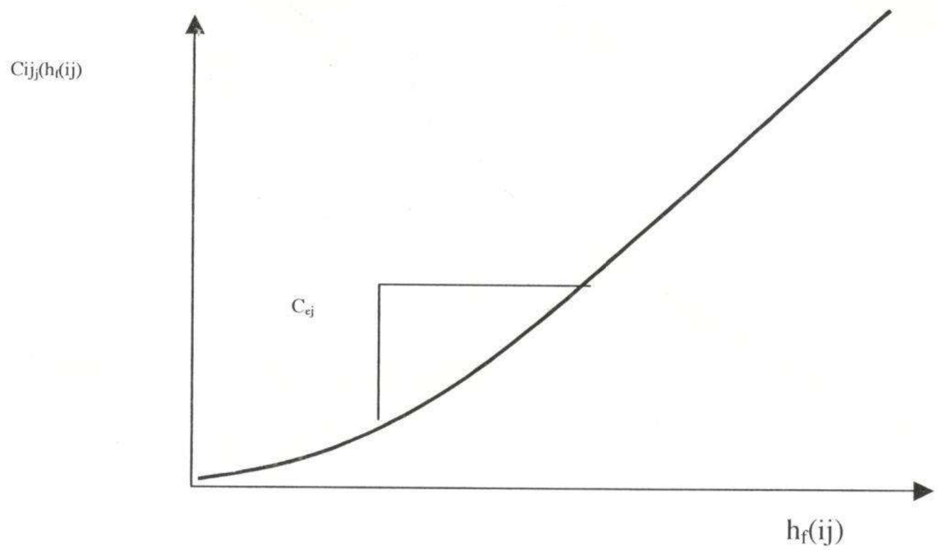

Figure 3: Graph of Cost Versus Head Loss