TIDAL ANALYSIS AND PREDICTION IN A BAR BUILT COASTAL ESTUARY

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ABSTRACT

Tidal Analysis and predictions are indispensable tools for scheduling construction activities in coastal areas, prediction of changes in tidal regime due to proposed projects, prediction of sea-level rise and fixing of datum. This paper presents the results of tidal analysis and predictions using the GETYSYS software with six months water level data as input and trend analysis of monthly mean sea level data of the Bonny River estuary. The analysis shows that the forcing oceanic tide is semi-diurnal dominated by the principal lunar (M2) and solar (S2) components. Trend analysis of monthly heights of Mean Sea Levels (MSL) showed that MSL is gradually rising at the rate of 2.5mm per month. Correlation analysis of tidal data with simultaneous wave parameters indicated significant tidal influence on Tz and Hs within the Estuary than for offshore. The predicted results were in good agreement with the observed tide.

1.0 INTRODUCTION

Hydraulic engineering in coastal areas is confronted with the problems of tidal motion. When a proposed project is anticipated to have a substantial interference with the tidal regime, it is mandatory to investigate the tide and thus predict accurately the interference of the proposed offshore loading jetties on the propagation of tide motion in Bonny River estuary.

To describe tidal phenomena qualitatively, equilibrium theory of tides was first presented by Newton in 1687 and then extended by Bernoulli in 1741 (see Thomas (1885)). However, Newton's equilibrium theory was unsuitable for practical applications due to the assumptions that the water depth on the earth was uniform and the inertial force of the water mass was negligible.

In 1825 Laplace improved the equilibrium theory on combining all periodic terms into one formula, and Thomson (1885) used Laplace's method to sum the tidal constituents. Because the amplitude and the phase of each tidal constituent can be determined from measured data by the theory of harmonic analysis, the tide levels can be forecasted at any given place. After Thomson, the method of harmonic analysis was extended by Darwin (1898), especially in the techniques of numerical computation. Doodson (1958) published the least-squares method in 1928 and the admiralty method in 1957 for determination of harmonic constants. Thereafter many methods of tide prediction were developed by investigators following the fundamental work of Newton. Laplace, Thomson, Darwin and Doodson. However, tide records of approximately one months or longer are required for accurate tide level prediction by the methods proposed by these investigators.

In this study, the six months tidal data of the Estuary was used as input into the harmonic Analysis Software. The analysis include a proper selection of set of constituents where are assumed to give a proper representation of the tide. Equation 4 with the set of tidal constituents form the mathematical model of the tide.

2.0 THEORETICAL ANALYSIS

The tide generating force is given by:

$$F_s = \frac{3gM}{2K^3} \text{ Sin } 2\theta \text{ ------------------------ } (1)$$

Where g = acceleration due to gravity, K = Gravitational Constant

M = mass of the moon, and θ = Moon Inclination

Doodson (1958) decomposed Equation (1) into tidal components as:

$$F_s = \frac{3gM}{2K^3} \left[Ao + \sum_{i=1}^{n} A_i Cos(\omega_i t + \theta i) \right] \text{ ----- } (2)$$

Where A_o = constant, A_i = amplitude of component i, ω_i = angular speed of component i and θ_i = phase of component i, at t = 0. If we introduce correction factors to accounts for oscillations of the moon which takes a period of 18.6 years, Equation 2 becomes:

$$Fs = \frac{3gm}{2K^3} \left[Ao + \sum_{i=1}^{n} F_i A_i \; Cos(\omega_i t + \theta i + Ui) \right] \; (3)$$

Where Fi = correction factor for node factor, Ui = correction factor for phase angle. The analysis of the observed tide was carried out on the basis of similar equation; Delf Hydraulic Limited (DHL) (1989):

$$H(t) = Ao + \sum_{i=1}^{K} A_i.Fr_i.Cos\,(\omega_i t + (V_o + U)_i - G_i) \;\; (4)$$

Where H(t) = water level at time t, A_o = mean water level above some defined datum, K = number of relevant constituents, i = Index of the constituent, A_i = local tidal amplitude of a constituent; Fr_i = nodal amplitude factor, ω = angular velocity (Vo + U) = astronomical argument, G = local phase lag.

Other wave propagation theories includes (Jasmes F.W. (1984) Airy and Helmhottz. The Airy wave theory is applicable to small amplitude, first order theory which form the basis for the probabilistic spectral description of waves and for evaluation of fluid-induced forces acting on a structure. The solution for the velocity potential (Φ) satisfying the Airy's wave theories, together with both kinematic and dynamic boundaries is

$$\Phi = \frac{-acCosh[k(y+d)]}{Sinh(kd)} \text{ Cos } (kx - \sigma\tau) \text{------- } (5)$$

Where ρ = density of water; g= acceleration due to gravity, a = wave amplitude; k = wave number, $\frac{2\pi}{L}$; L = wave length, σ = wave circular frequency, $\frac{2\pi}{T}$; T = wave period, d = water depth, t = time.

The Helmholtz differential wave equation is of the form;

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right) \psi(x, t) = \rho(x, t) \text{ --------- } (6)$$

Where c = velocity of wave propagation, ρ (x,t) = Origin or prescribed source of wave, ψ = stream function and ∇^2 = Laplacian operator.

The Helmhottz wave equation, is of great interest in physics than engineering practice. Usually Equation 6 is solved by separation of variable method, in Cartesian spherical or cylindrical coordinates.

The design of structures (Sankarayanan S. Detal 2003) must also take in account the predicted astronomincal tide levels and other disturbances on the free surface such as Windwaves, Seiches, Tsunami, Storms surges etc. These water waves have different periods or time scale, and varies with the intensity of the generating force.

3.0 STUDY AREA

The Bonny River Estuary is a bar built estuary on the West Coast of the Atlantic Ocean. The Estuary is surrounded to the East by Bonny Island, West by the mangrove forest, to the South by the Atlantic Ocean and to the North by Port Harcourt City. The tidal station is situated between Longitude 7° 19 'E, Latitude. 4°.16 'N and Longitude, 7°8.6 'E Latitude 4° 25.5' N.

Result of analysis of meteorological data revealed that 22.5 degrees is the dominant wind direction. The maximum average wind speed is 3.5m/sec while the maximum wind speed is 18.5m/sec., the mean annual precipitation is 300mm per month and a mean temperature of 27°C (Fig. 1).

4.0 MEASUREMENT AND METHODS

Sea-level measurements were carried out in the Bonny River estuary from July-December, 2003 on hourly basis using a float actuated tidal station, mounted with an OTT XX recorder having resolution and accuracy of 1cm. The construction was mounted on steel pipes and tied to sheet pile to avoid vertical drift. A manual tidal gauge was erected close to the tidal station which acts as control. Both the tidal station and manual gauge was calibrated and tied to local bench mark. The tide gauge recordings were corrected for air pressure obtained from the NLNG meteorological station. Current (Horizontal tide) measurements were also carried out using an OTT Arkansas current meter which has a built in gyro compass for direction measurement. Every 10 minutes, the current magnitude was measured at three depths: surface - 1m, mid depth, and one metre above the river bed (bottom + 1m). The current meter was situated at approximately 04^o-25.5N and 07^o-08.6' E,

5.0 TIDAL ANALYSIS

In the mathematical presentation of tide raising forces by means of the primary constituents, S_2 and M_2, compound and higher harmonic constituents are added, to account for tidal movement in shallow waters, where advection, large amplitude to depth ratio and bottom friction give rise to non-linear interactions. The general formula for the astronomical tide (DHL 1989) is:

$$H(t)=Ao+\sum_{i=1}^{K} A_i*Fr_i*Cos(\in_i\text{-}t+(Vo+U)_i-G_i) \quad (4)$$

Where H(t) = water level at time (t), Ao = mean water level over a certain period,
K = number of relevant constituents, i = index of the constituent, A_i = local tidal amplitude of a constituent; Fr_i = nodal amplitude factor, ω = angular velocity, $(V_o + U)$ = astronomical argument, G = improved kappa number (local phase lag).

Fr and (Vo + U) are time- dependent factors which, together with ω, can easily be calculated and are generally tabulated in the various tidal year books. Fr and U are slowly varying amplitude and phase corrections and are also frequency dependent. A_o, A_i and G_i are position – dependent, that is, if for a specific location A_o, A_i and G_i are known, equation (4) can be used to predict the local water level H(t) at any time (t). Conversely, if at a location, a series of tidal observations w(tj) is known, Equation (4) can be used by means of least squared analysis to estimate the values of A_o, A_i and G_i. The Getysys software has a total of 226 components. However, in this project, only M_2, S_2 and 32 other constituents were used (Table 1). The available tidal records have been analyzed to obtain the amplitude and phase of the tidal components (Table 1) which in turn, were used to predict the astronomical tide in advance (Yen P. *et al.*, (1996) and Wiyeratne E.M.S *et al.*, (1995)

The ANALYSIS subroutine of the GETYSYS software was used to determine the A_o, A_i and G_i of equation (4). A major component of the analysis is the generation of a set of constituents (Table 1) which are assumed to give a proper representation of the tide. Information on local character of the tide, sampling rate and duration of the observations were essential inputs in the software.

The analysis of the tidal data include:

- An estimate of the tidal harmonics constants (34 components) on a monthly and half yearly basis (see Table 1)
- Trend analysis of the monthly heights of mean sea level (MSL)
- Time series plot of observed and hind coasted tidal elevation, reference to mean sea level.
- Listing of daily times and heights of high and low water, extreme values of high and low water and monthly heights of MSL.

The hydraulic analysis uses K_1, O_1, M2 and S2 constituents to characterize the tide as semidiurnal, mixed and diurnal. The type of tide that occurs at a particular place depends on the form number (F), expressed as the ratio $F = (K_1 + O_1)/(M_2 + S_2)$ (May, 2000).

If $F < 0.2$, the tide is classified semidiurnal; diurnal if $F > 3.0$ and mixed otherwise. The result for the Bonny river estuary yielded the form number 0.11, therefore the tide is classified as semi-diurnal.

6.0 TIDAL PREDICTION

The parameters; A_o, A_i and G_i determine the characteristic of the tide at a given location. Thus, once they have been generated by the ANALYSIS subroutine, the PREDICT subroutine was then used to make tidal water levels predications using Equation 4; at time Interval of 60 minutes. Constituents (Column 2 of Table 1) with very close frequencies were astronomically coupled in order to have a proper resolution of the two constituents. The graphical plots of the predicted water levels, observed tide and residues are shown in Fig. 2.

The predicted tide showed a good agreement with the observed tide, which implies that the applied tidal constituents gave a good fitting of equation 4.

7.0 Time Series Analysis of Mean Sea Level (MSL)

Time series analysis of monthly values of MSL data consist of time series plots and trend analysis. Trend estimation of monthly values of MSL was performed using the method of 4 year moving averages to establish the trend i.e. downward or upward movement. The trend line is shown in Fig. 3. Trend calculations show that the MSL is rising at the rate of 2.5mm per month, also the Time Series Plot (Fig. 3) follows a seasonal pattern, which is attributable to climatic influences.

8.0 Horizontal Tide

The tidal currents was measured at three depths; one metre above the Estuary bed (bottom +1), mid depth and one metre below the water surface (Surf -1) (Fig. 4). Current measurement were made at each spring and neap tide period for a duration of at least 26 hours. Analysis of horizontal tide shows the tidal current climate in the estuary are related to the season as well as the vertical tide. The wavy top (Fig. 4) shows the influence of vegetation on the current pattern. This occurs when the tidal range reaches about 65 centimetres above the mean tidal level (MTL + 65) and as it begins to flood the backswamps. The effect resulted in about fifty percent (50%) reduction in surface currents. The maximum velocities are shown in Table 2.

The maximum velocity occurred 2 -3 hours before low waters for about two hours ranging between 140cm/sec to 160cm/sec, while flood flow occurred 1 – 5 hours with velocities ranging between 60cm/sec and 100cm/sec. During the rise of the tide, the change in velocity per minute was 54cm while for the fall the change was 72cm.

9.0 Multiple Regression Analysis

In order to evaluate the influence of wave climate on tidal levels in the Estuary, multiple regression analysis was used to fit equations to the available inshore and offshore wave data. The form of the equations selected for the multiple regression analysis was;

$$Y = a_o + a_1x_1 + a_2x_2 \text{ --------------- } (7)$$

Where Y = inshore or offshore wave parameter, a_o = constant, a_1, a_2 are coefficients of correlation, x_1 and x_2 = parameters.

The outputs provided by the analysis using Equation 7 are summarized in Table 3.

Where H_{si} = Inshore significant wave height, H_{so} = Offshore significant wave height, T_{zi} = Inshore zero mean crossing wave period, T_{zo} = Offshore zero mean crossing wave period, T_{pi} = Inshore peak wave period, and T_{po} = Offshore peak wave period.

In order to quantify the "goodness" of fit, the coefficient of determination given by (Chapra S. C. *et al.* 1998) was computed as

$$R^2 = \frac{S_t - S_r}{S_t} \text{ ---------------- } (6)$$

Where S_t = total sum of the squares around the mean or the residual error associated with the tidal levels prior to regression.

S_r = sum of the squares of the residuals around the regression line.

r^2 = Coefficient of determination and r = Coefficient of correlation.

Thus $S_t - S_r$ quantifies the improvement or error reduction due to describing the data in terms of a straight line rather than as an average value.

For a perfect fit, $S_r = 0$ and $r = r^2 = 1$, signifying that the line explains 100 percent of the variability of the data. For $r = r^2 = 0$, $S_r = S_t$ and the fit represents no improvement.

The coefficient of determination (Table 3) shows weak correlation between the inshore and offshore wave climates. The reason for this lack of correlation was probably due to the presence of the Bar which absorbs the offshore wave energies.

10.0 DISCUSSION/CONCLUSION

The magnitude of the velocities at the surface and midwaters are of the same nature, while at the change of the tide the velocities at midwater are in general higher and it takes approximately one hour before velocity at the surface can cope with the one at midwaters. Apart from the usual procedure of analysing the hourly water level data into harmonic constants, standard deviation and residuals, special attention has been paid to the trend of the monthly mean sea level heights over the period and a trend analysis has been applied to the monthly MSL heights, extrapolation of the linear model leads gradually to higher values. A rise of 2.5mm per year was estimated in the mean sea level. The mean sea level response to meteorological and hydrological changes. The characteristic movement of the time series show cyclical and seasonal movements; the water level rise between September and November, and peaks around October.

The meteorological influence on the tide, clearly affects the harmonic components in the monthly analysis. For example, if a storm passed the Nigeria Coast in the first and third week of a particular month, this will affect the values of the harmonic components with periods of around two weeks. In the analysis for a longer period of half a year, this effect will be insignificant.

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Table 1: Tidal Constituents

S/No	Name constituent	Amplitude	Phase (G)	Frequency	(Vo + U)	F
1.	SSA	5.891	36.2	0.082137	287.7	1.000
2.	MM	2.433	1.0	0.544374	109.7	0.961
3.	MF	2.059	340.6	1.098033	83.8	1.167
4.	SN	1.298	93.5	1.560270	242.1	0.989
5.	25M	0.290	249.3	2.031791	266.2	0.979
6.	2Q1	0.359	129.8	12.85428	108.5	1.075
7.	Q1	0.622	115.3	13.39866	217.5	1.075
8.	01	2.244	345.0	1394303	326.6	1.217
9.	MI	0.346	322.5	14.49669	139.7	1.047
10.	KI	12.678	16.8	15.04106	226.9	1.000
11.	PI	4.011	13.4	14.95854	111.1	1.075
12.	JI	1.316	13.4	15.58593	339.4	1.267
13.	OO1	0.496	85.5	16.13910	314.2	0.968
14.	3MS2	0.233	200.4	26.95231	290.8	0.979
15.	MN2	0.604	154.2	27.42383	314.8	0.989
16.	MUZ	1.879	187.2	27.96820	61.8	0.989
17.	N2	3.458	142.0	28.43972	87.9	0.989
18.	NU	2.611	142.0	28.51258	170.8	0.989
19.	M2	70.151	147.7	28.98410	196.9	1.219
20.	L2	2.488	161.8	29.52847	136.2	1.00
21.	S2	23.053	181.5	30.00000	330.0	1.102
22.	K2	6.547	181.5	30.08213	274.3	0.979
23.	MSN2	0.693	317.8	30.54437	79.0	0.989
24.	2SM2	1.293	15.5	31.01589	103.1	0.063
25.	M03	0.887	18.7	42.92713	163.5	0.984
26.	M3	9.863	267.1	43.47615	295.4	0.968
27.	2MNS4	0.225	178.4	56.40793	151.7	0.979
28.	MN4	1.770	77.8	57.42383	284.8	0.979
29.	M4	5.789	101.2	57.96820	33.8	0.979
30.	SN4	0.463	158.0	58.43972	57.9	0.989
31.	MS4	3.289	153.2	58.98410	166.9	0.989
32.	3SM4	0.284	54.0	61.01589	73.1	0.989
33.	3MO5	0.540	187.0	73.00927	264.2	1.041
34.	M6	1.131	17.5	86.95231	230.0	0.968

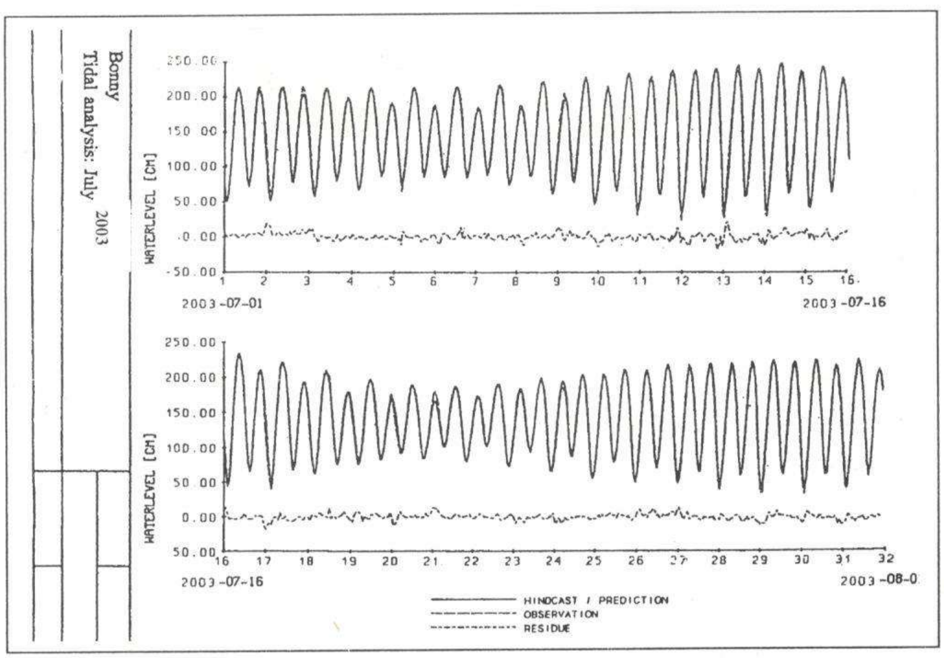


Fig. 2: Tidal Prediction

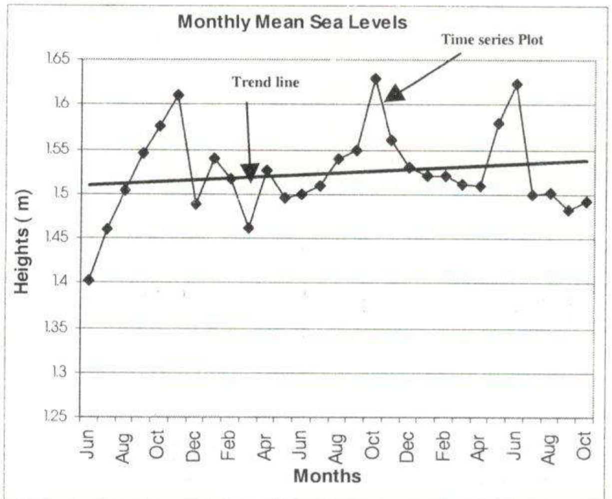


Fig. 3: Trend and Time Series Plot

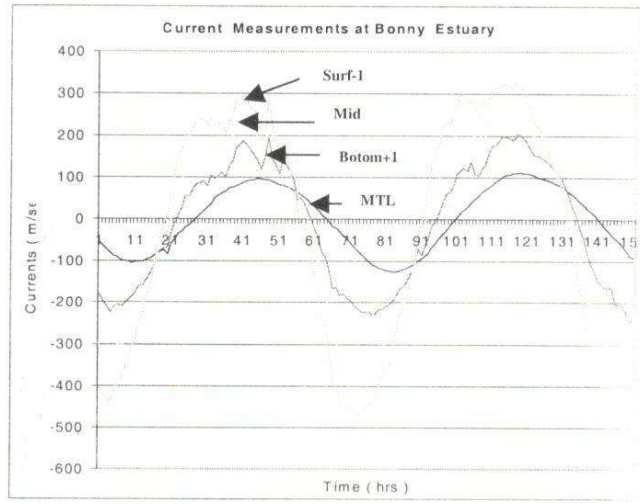


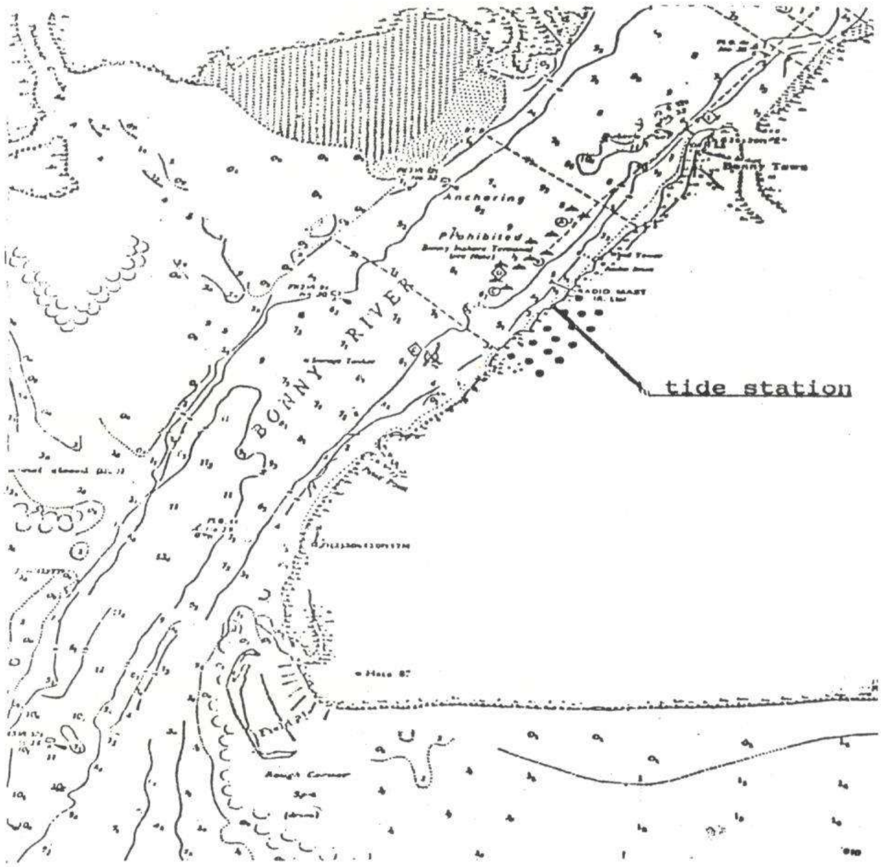
Fig. 4: Tidal Current

Table 2: Maximum velocities

Depth	Flood (cm/sec)	Ebb cm/sec
Surface – 1m	105	159
Midwaters	107	150
Estuary bed + 1m	92	131

Table 3: Summary of Statistics

S/No	Regression models	Coefficient of determination $(R)^2$
1	$H_{si} = 0.321 + 0.192\ H_{So} - 0.120$ tide	0.56
2	$T_{zi} = 3.58 + 0.17\ T_{zo} - 0.521$ tide	0.28
3	$T_{pi} = 5.345 + 0.0231\ T_{po} - 0.231$ tide	0.023


Anchoring
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tide station