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DYNAMIC ANALYSIS OF GUYED MASTS USING THE STIFFNESS METHOD

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ABSTRACT:

The Global System of Mobile Communication GSM revolution in Nigeria has resulted in an increased usage of communication towers and masts. However, observation has shown that most of the towers being used in the industry presently are the free-standing type. It will be necessary to investigate the suitability of guyed masts in the mounting of antennae for the GSM services in Nigeria.

This paper presents an investigation into the dynamic analysis of multi-level guyed masts using the stiffness method. The displacements and reactions of the masts are obtained from a linear dynamic analysis of the structure based on the stiffness method. The mast shaft and guy rope problems are solved separately, forcing compatibility of displacements at the points of guy attachment. The model is then used to analyse a 60m two-level guyed mast at wind speeds of 80km/h, 160km/h and 240km/h in the positive X-direction.

Results show that the bending moment values for the mast shaft at the third node points decrease gradually between zero and 3 seconds elapsed time, thereafter increase sharply.

The axial forces on the shaft also decrease gradually from a peak value at about 0.2 seconds elapsed time to a minimum value at about 3.0 seconds, increasing sharply thereafter.

1.0 INTRODUCTION:

A **Mast** belongs to a group of tall structures having relatively small cross section and a large ratio between the height and the maximum width. They are constructed in a vertical position, and can be self-supporting or guyed. This category of structures comprises mostly of towers and masts.

Kolousek (1947) determined the first dynamic solution for massive guy cables. This solution was based on the parabolic approximation and assumed small vibrations. The method of solution was to represent the deflected mode shape by a Fourier series of sinusoidal components and to solve for the amplitude of each component by substitution into the governing differential equations. The guy modulus could then be expressed in terms of these coefficients.

Other researchers have proposed the parabolic representation of guy cables as well. **Davenport (1959)** simplified Kolousek's approach and proposed a linear model to describe vibration of guys and obtained an expression for guy modulus. **Cohen and Perrin (1957)** and, **Cohen and Perrin (1957A)** basing their analysis on the parabolic approximation of the guy cable, presented a procedure for calculating the maximum moments due to wind load by treating 25% of the wind load as a live load which was placed in different spans to cause maximum moments (patch loading) on the guyed mast. **Schott et al (1954)** also presented an analysis of the structural behaviour of guyed masts based on the parabolic approximation of the guy cables.

Dean (1961) criticized the assumption of the parabolic geometry for guy cables and proposed a method for dynamic analysis assuming the deflected shape of the cables to be **catenary.**

Stottrup-Anderson (1991) discussed the project to build new 12no. 300m high masts and the rebuilding of 4no. 150m high masts for the Danish Government in 1986 to carry the UHF antennae for the second national television channel. He discussed the structural outline of the new masts, the very stringent precautions to corrosion protection and the recommendations on the use of the world's best guy ropes to build masts with a maintenance free period of at least 50 years. He also presented the fabrication and erection techniques adopted for the project under very tight delivery schedule.

The **IASS (1981)** guidelines recommend that wherever possible, a full dynamic analysis may be carried out to check the design of guyed towers. The assumption of linear vibrational behaviour is more doubtful especially when leeward guys lose most of their tension. Due to lack of methodologies for a complete nonlinear dynamic analysis, it is recommended that a quasi-static analysis or a deterministic linear dynamic analysis about static equilibrium position., as seen in some of the references above be carried out for dynamic response of guyed towers. There is therefore the need for full nonlinear analysis of guyed towers including probabilistic approach to account for wind variations. A few researchers have attempted this.

Argyris et al (1972) presented an application of their method to guyed towers apart from other structures. **Wilson et al (1973), Knudson and Suryakumar (1978) and Adeli et al (1978)** have all attempted solutions for the nonlinear model of the dynamic response of structures to wind loading. **Belytschko et al (1976)** developed an algorithm for nonlinear transient analysis by finite elements. **Jayaraman et al (1980)** developed a catenary cable element and used it for static and dynamic analyses of a guyed tower. The nonlinear dynamic analysis (deterministic) was performed using an explicit method (central difference). The tower was idealized using truss elements. **Suryakumar and Raman (1986)** made a comparison of responses obtained using explicit and implicit methods for nonlinear dynamic analysis. The tower was idealized using beam-column elements and the guys were idealized as assemblage of truss element. **Lannuzzi (1985)** presented a method to determine the aerodynamic response of masts using a deterministic approach based on the simulation of wind speed at discrete points in space, and using direct time integration of the equations of motion. The cables were divided into a number of elements and were also idealized as simple spring- mass models for comparative study. This simple model resulted in considerable reduction in computer time. **Raman et al (1988)** presented a deterministic nonlinear dynamic analysis of a guyed mast using an integrated finite element approach. The dynamic response was obtained using Wilson's direct time integration procedure. They presented a procedure similar to substructuring in which guys and mast are solved alternately and forcing compatibility of displacements, velocities and accelerations at the guy support points until the final equilibrium state is obtained.

Not too long ago, **Ashmawy et al (1997)** proposed a numerical method based on the minimization of the total potential energy of the structure and has been extended to include the eccentricity of stay attachments to the mast. The minimization was achieved by utilizing the conjugate gradient method in which the stability and bowing functions are included and updated using Newton-Raphson iteration. To verify the proposed method as well as the computer program developed for the investigation, a numerical analysis of a selected number of solved problems was conducted and the solution obtained compared with those published by other investigators.

In this study, a linear dynamic model is developed for the analysis of multi-level guyed towers. The analysis of displacements and reactions is based on the stiffness method. The mast shaft and the guys are solved separately, forcing compatibility of displacements at the guy attachment nodes.

2.0 CONFIGURATION OF MASTS:

A mast is a tall, vertical structure whose height is several times greater than its maximum lateral dimension. It derives its stability from guy cables attached to at least one level along its height and anchored to the ground. Most modern masts have triangular cross-section with an arrangement of three guys at a level. The configuration of a typical mast is shown in **Fig. 1**

3.0 STRUCTURAL BEHAVOUR:

A mast or guyed tower exhibits one of the most complex structural behaviours of all engineering structures. Structural theory is yet to sufficiently account for all the effects of dynamic loading on a mast. When a mast is subjected to dynamic loading, the mast as well as the guy cables vibrate. The response of the guys differs from that of the mast. Also, the Leeward guys will respond differently from the windward guys. Each element is characterized by frequency, mode shape, displacement and stresses. Fig.2 shows a 2-level guyed mast vibrating under the action of wind load.

As the wind load is applied to the mast, the shaft of the mast will tend to yield in the direction of the wind. This causes the tension in the windward guys to increase and that of the lee ward guys to reduce. This situation is repeated according to the period of the gust, thus setting up a vibration sequence.

4.0 MAST LOADING:

The principal loads on a mast are due to wind and the vertical component of guy tensioning forces. However, other factors like temperature, ice and earthquake are sometimes taken into consideration when evaluating the loading on masts. When considering dynamic analysis, the gust wind-which is the portion of wind load above the mean wind velocity is estimated and added to the mean wind spectrum.

5.0 DETERMINATION OF WIND LOADING:

The most influential load in the dynamic analysis of masts is wind load. Other loads on the structure which include the weight of the structural elements and accessories are easily determined. But the load due to wind depends on the height of the mast, its orientation to wind, the ratio between effective and total area of a face and the shape of the cross-section of the elements of the mast and the place of location. The wind generates both static and dynamic effects on the mast.

5.1 Static Wind Analysis:

The static wind load is the pressure applied on the structure by that component of the wind composed of the basic wind speed. To determine the wind load, the velocity pressure is first determined. This is obtained by calculating the velocity profile of the wind which increases with height and depends on the basic wind speed at the site of the structures, among other factors. The details of Static Wind analysis of guyed Towers are presented in **Dean (1961).**

5.2 Dynamic Wind Analysis

The dynamic wind component may be obtained deterministically or non-deterministically. The deterministic approach involves obtaining the amplitude of the gust as a percentage of the basic or mean wind velocity and assuming that it follows the sine wave.

The non-deterministic approach involves statistical studies or observations of wind occurrences over the years. Using this data, a power spectral density curve is established which gives the fluctuation of wind velocity with time.

From the deterministic approach, the dynamic wind velocity at a height 10m above ground level is given **by (Suryakumar and Raman, 1986),**

$$V_{10} = V_B + V_{AMP}. \text{Sin}(2\pi t / T) \quad (1)$$

where

V_{10} = Velocity at any time, t at 10m height

V_B = Basic (mean) wind velocity at 10m height

V_{AMP} = Amplitude of gust

T = Time period of gust

The velocity variation is then computed using the power law given as:

$$V_Z = V_{10} (^Z/_{10})^{\alpha} \quad (2)$$

Where

α= Power law coefficient taken as $^1/_7$

V_Z= mean wind velocity at height Z

Z= height above ground level in metres.

6.1 DYNAMIC ANALYSIS FORMULATION

In many structural dynamic problems, the solution technique commences by finding the vibration mode shapes and natural frequencies of the structure. Thereafter, the transient response analysis will involve the solution of large, second-order differential equations of equilibrium. The equations are the equations of motion for the structural system.

Determination of Vibration Mode Shapes:

The free undamped equation of motion of a MDOF system vibrating with mode ' i ' is expressed as:

$$[K - \omega_i^2 M] \Delta^i = 0 \quad (3)$$

where

K = the Structure Stiffness Matrix

ω_i = the i^{th} vibration frequency

Δ^i = the i^{th} vibration mode shape

M = the Structure Mass Matrix

Once the vibration frequencies have been found then the matrix, $E^i = K - \omega_i^2 M$, can be evaluated for ω_i and equation (3) becomes

$$E^i \Delta^i = 0 \qquad (4)$$

E^i is singular and therefore has no unique, non-trivial solution. That is, the mode shape can be found, but not its amplitude. To find the mode shape, one element of Δ^i is set to an arbitrary value say, unity. Therefore (4) can be rewritten as

$$\begin{bmatrix} e^i_{11} & e^i_{12} & e^i_{13} & \cdots & \cdots & e^i_{1n} \\ e^i_{21} & e^i_{22} & e^i_{23} & \cdots & \cdots & e^i_{2n} \\ \vdots & \vdots & \vdots & & & \vdots \\ e^i_{n1} & e^i_{n2} & e^i_{n3} & \cdots & \cdots & e^i_{nn} \end{bmatrix} \begin{bmatrix} 1 \\ \Delta^i_2 \\ \vdots \\ \Delta^i_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad (5)$$

In submatrix form, the above can be written as:

$$\begin{bmatrix} e^i_{11} & E^i_{IO} \\ E^i_{IO} & E^i_{IO} \end{bmatrix} \begin{bmatrix} 1 \\ \Delta^i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (6)$$

giving

$$E^i_{II} \Delta^i_{OI} = - E^i_{OI} \qquad (7)$$

The above system of equations can now be solved simultaneously for the remaining displacement amplitudes, Δ^i_{OI}. The displacement amplitudes are usually normalized by dividing each component of the vector by the largest component to produce the mode shape vector, ϕ_i.

$$\phi_i = \begin{bmatrix} \phi_i \\ \phi_i \\ \cdot\cdot \\ \phi_i \end{bmatrix} = \frac{1}{\Delta^i_{max}} \begin{bmatrix} 1 \\ \Delta^i_2 \\ \cdot\cdot \\ \Delta^i_n \end{bmatrix} \qquad (8)$$

where Δ^i_{max} is the largest element of Δ^i .

7.0 EXAMPLES, ANALYSIS AND DISCUSSION

Based on the methodics briefly described above, we present the analysis of a 60m two level guyed mast at wind speeds of 80km/h, 160km/h and 240km/h, all in the positive X-direction.

Fig 3 shows the details of the 60m high mast guyed at two levels. It carries an antenna 3m long. The structure was subjected to mean wind velocities of 80km/h, 160 km/h and 240 km/h, each having a 10% gust with sine variation.

Fig. 4 shows the graph of the displacement of the mast against the elapsed time. This is for the third node (mid-point) of the mast. The graph indicates the behaviour of the mast at all the 80 km/h 160km/h and 240 km/h mean velocities. It can be observed from the figure that the curves follow a sine wave character with the amplitude increasing as the wind velocities is increased, with an approximate frequency of 4s.

Fig 5. on the other hand, depicts the bending moment curves of the mast, also at the 3rd node point at all the indicated wind velocities. It could be noted from the graphs that the bending moment reduces gradually between O and 3sec; thereafter it rises sharply. This same trend is also observed in the case for the Axial force variation of the mast at the 3rd node point. This is depicted in Fig. 6, which also shows this phenomenon for the wind velocities of 80 km/h, 160 km/h and 240 km/h. Analysis of the graphs shows that the axial force decreases gradually from a peak value at about 0.2 seconds to a minimum value at about 3.0 seconds, thereafter it rises sharpy.

In Fig. 7 we show the displacement curves of the midpoint of the Guy No.4 (windward) while Fig. 8 shows the curves at the mid point of the Guy No.6 (Leeward) both against the elapsed time in seconds. The curves are shown for all the three wind velocities. Analysis of the curves shows that the midpoint of the windward guys generally follow a sin wave phenomenon, however the amplitude of oscillation is smaller than that of the leeward guys. This is to be expected, since the windward guys increase in tension while the leeward guys slacken. (see Fig 8.) Figs. 9 and 10 depict the Tension variation in the guys respectively for the windward and the leeward guys against the elapsed time for the various mean wind velocities. Analysis of the graphs shows that the tension in the windward guys experience high increase at zero elapsed time, but quickly drop to their lowest values at t = 0.2 sec, however the tension rises gradually thereafter to another peak value at approximately t = 4.0 seconds. At t = 0, the windward guys and the leeward guys experience the lowest tension, increasing suddenly to a maximum value at t = 0.2 sec. At about 4.0 seconds, another peak value was observed as depicted in fig. 10.

8.0 CONCLUSION:

A model for analyzing multilevel guyed mast using the stiffness method has been proposed. The model is then used to analyse a 60m high two level guyed mast at wind speeds of 80km/h, 160km/h and 240km/h, all in the positive direction of x-axis.

The results for the displacement, Axial force and bending moment characteristics against the elapsed time at third node points of the mast were presented and analysed. We also presented the results for the tension variation of the guy cables at the windward and leeward sides. It was observed that there was a gradual decrease in the axial force developed in the mast from a peak value at an elapsed time of about 0.2 seconds to a minimum value at an elapsed time of 3 seconds, thereafter the axial force value rose sharply.

Another interesting results obtained were that the displacement of the midpoint of the windward guy cables followed a sine wave character, however the amplitude of oscillation was found to be smaller than that of the leeward guys. This was to be expected, since the tension in the windward guys increased while that of the leeward guys decreased.

REFERENCES

1. Achu, E. T. (2001): "Linear Dynamic Analysis of Multi-Level Guyed Masts by the Stiffness Method". Unpublished M.Sc. Thesis of the Department of Civil Engineering, University of Lagos, Nigeria.

2. Adeli, H.; Gere, J.M. and Weaver, W. (1978): "Algorithms for Non-linear Structural Dynamics" Journal of Structural Division, ASCE, Vol. 104, ST2, February 1978, pp. 263-280.

3. Argyris J.H. Dunne, P.C. and Angelopoulos, T. (1972): "Non-linear Oscillation using the Finite Element Technique" ISD-Report No.136, University of Stuttgart, West Germany, 1972.

4. Ashmawy M. A., Bucholdt H.A. and Stefanou, G.D. (1997): "A Numerical Method for the Non-linear Static Analysis of Space Cable Skeletal Structures". Journal of International Association of Shell and Spatial Structures, Vol. 38, No. 123, April 1997, pp. 3-18.

5. Belyschko, T.; Chiapetta, R.L and Bortel, H.D. (1976): "Efficient Large Scale Non-linear Transient Analysis by Finte Elements". International Journal for Numerical Methods in Engineering, Vol.10, 1976, pp. 579-596.

6. Cohen, Edward and Perrin, Henri (1957): "Design of Multi-level Guyed Towers under Wind Loading". Journal of the Structural Division, ASCE. Vol. 83, No. ST5, Proc. 1355, September, 1957.

7. Cohen, Edward and Perrin, Henri (1957A): "Design of Multi-level Guyed Towers". Journal of the Structural Division, ASCE, Vol. 83, No. ST5, Proc. Paper 1356, September, 1957.

8. Davenport, A. G. (1959): "The Wind Induced Vibration of Guyed and Self-supporting Cyclindrical Columns". Transactions, Engineering Institute of Canada, Vol.3 pp. 119-141.

9. Dean, Donald, L. (1961): "Static and Dynamic Analysis of Guy Cables". Journal of Structural Division, ASCE. Vol.87, ST1, January, 1961, pp.1-21.

10. Iannuzzi, A. (1985): "Aerodynamic Response of Guyed Masts - A Deterministic Approach". Bulletin of IASS, No. 89, December, 1985, pp.47-59.

11. Iannuzzi, Attilio and Spinelli, Paolo (1990): "Response of a Guyed Mast to Real and Simulated Wind". Journal of International Association of Shell of Shell and Spatial Structures Madrid, Spain Vol.2 pp.38-45.

12. IASS Recommendation for Guyed masts, Madrid, Spain, 1981.

13. Jayaraman, H. B., Knudson, W. C., Schneider, S. and Herman, P. (1980): "Small Strain Curved Cable Element for the Analysis of Cable Structures (with special Reference to Guyed Towers)". ISD-Report No. 276, University of Stuttgart, West Germany, 1980.

14. Raman, N. V.; Suryakumar, G. V. and Sreedhara Rao, V. V. (1988): "Large Displacement Analysis of Guyed Towers". Computers and Structures, Vol.28, No.1, pp. 93-104.

15. Schott, G; Thurston, F. and Pocook, P. (1954): "Analysis of Structural Behaviour of Guyed Antenna masts". Division of Mechanical Engineering Report, MM-238, National Research Council of Canada, November, 1954.

16. Stottrup-Andersen, Ulrik (1991): "The 12 New 360 Metres Masts for the Danish Channel". Bulletin of International Association of Shell and Spatial Structures, Madrid, Spain, Vol.1, pp171-178.

17. Suryakumar, G. V. and Raman, N. V. (1986): "Non-linear Dynamic Analysis of Guyed Towers". Proceedings of international Conference on Computational Mechanics, Tokyo, Springer-Verlag, May, 1986, Vol-1 pp III - 118.

18. Wilson, E. L.; Farhoomand, I. and Bathe, J. (1973): "Non-linear Dynamic Analysis of Complex Structures". Earthquake Engineering and Structural Dynamic, Vol.1, 1973, pp.241-252.

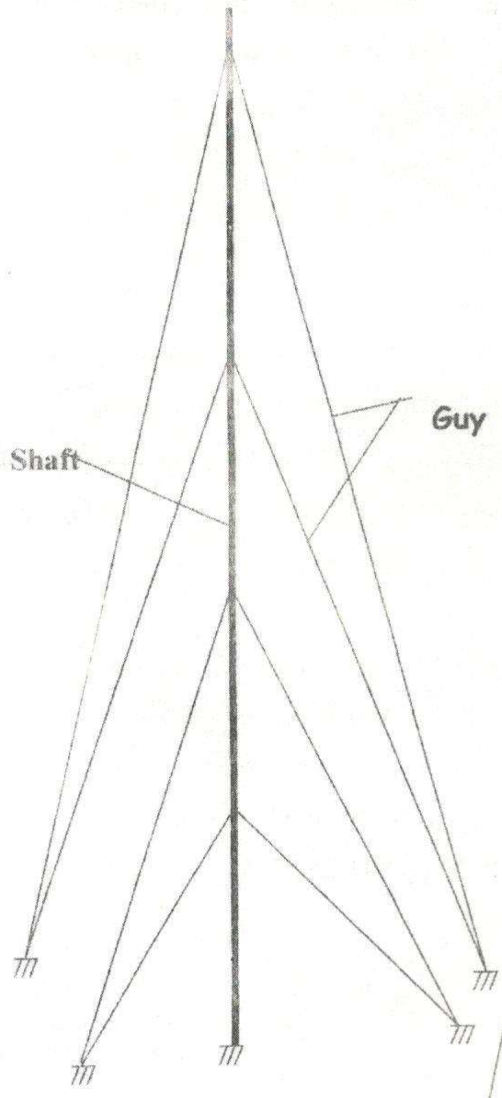


Fig. 1 : Typical Guyed Mast

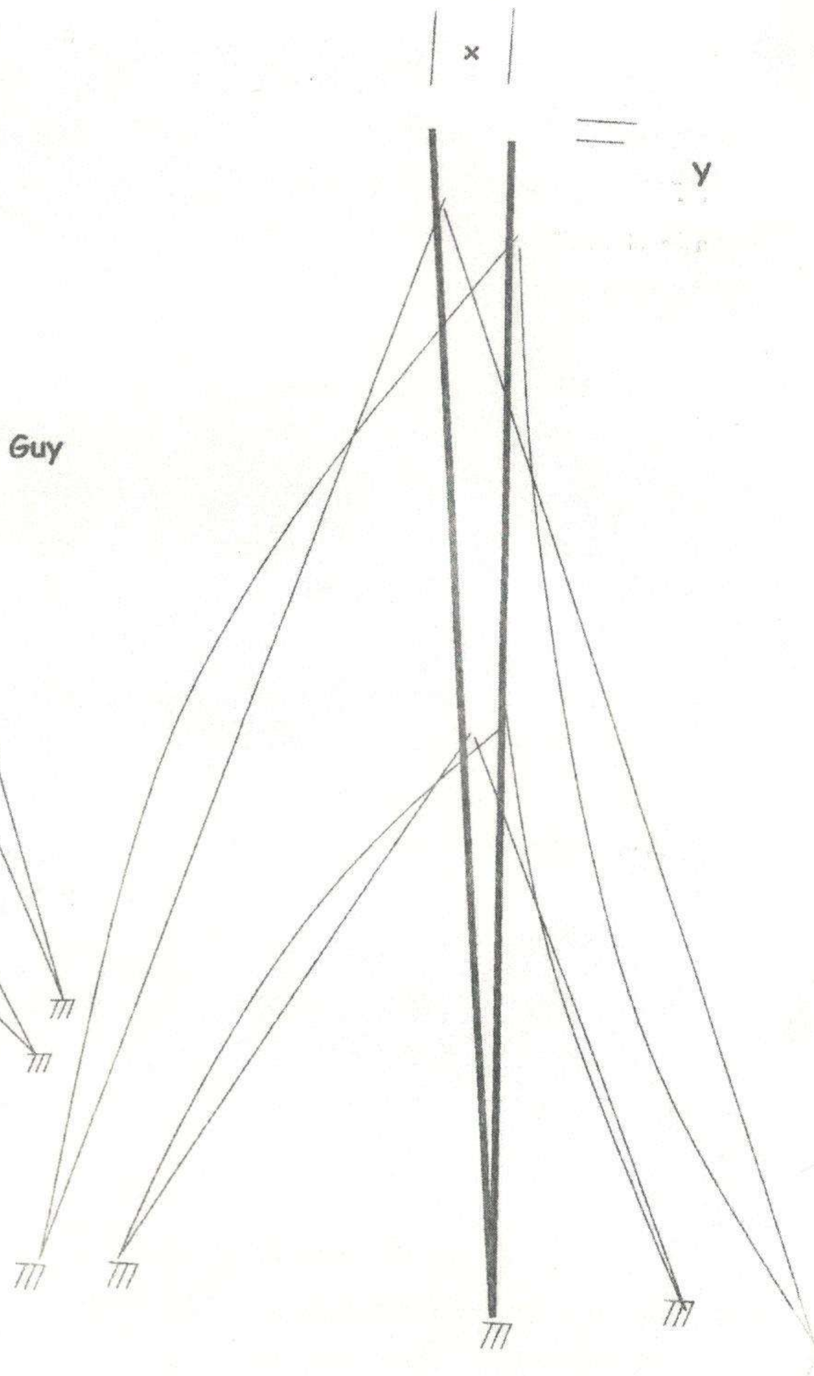


Fig. 2. Structural Behaviour of a 2-level Guyed Mast

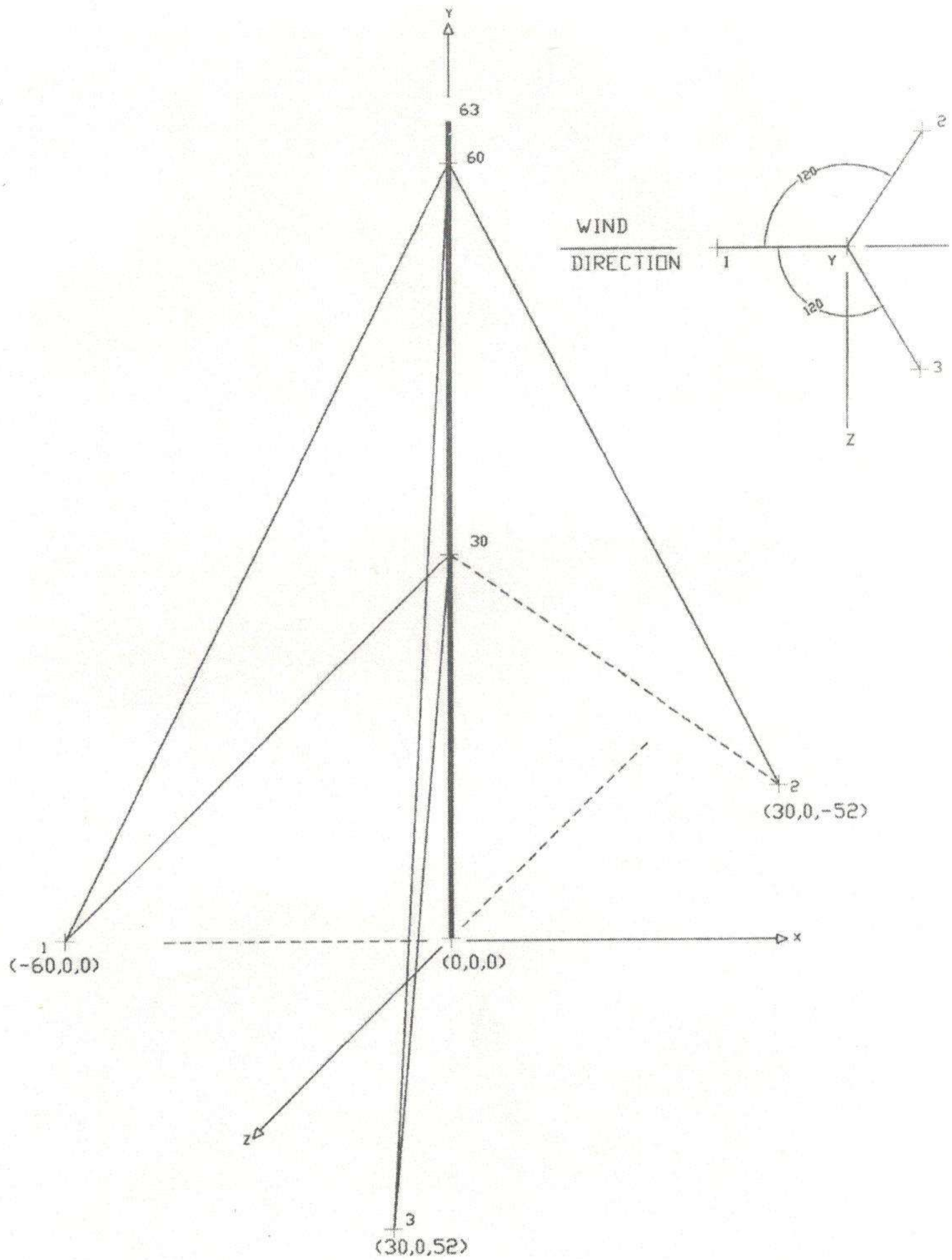


Fig. 3: Example Problem - A 60m, 2-Level Guyed Most

Fig. 4: Variation of Mast Displacement with Time and Wind Velocity (Node 3)

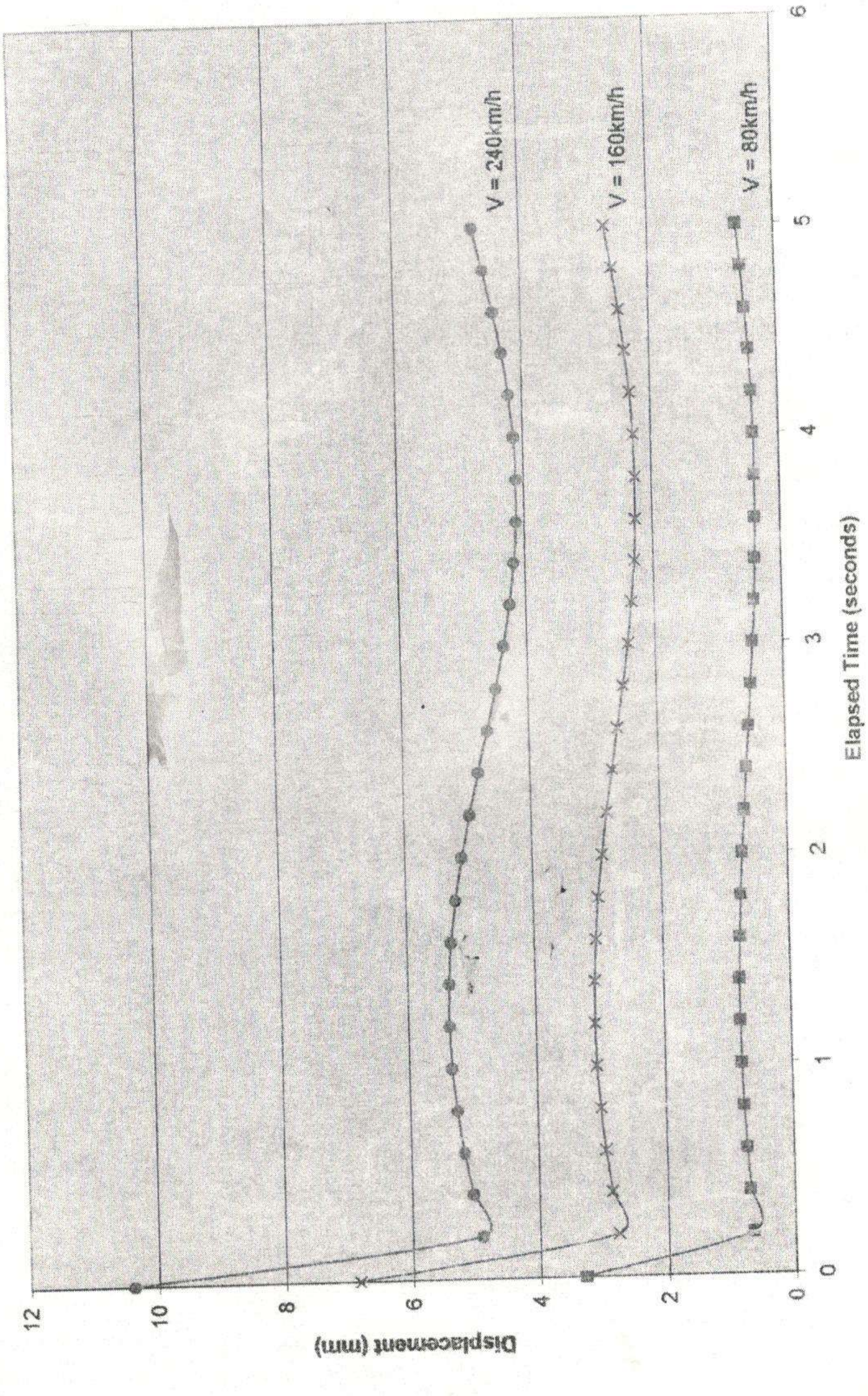


Fig. 5: Variation of Bending Moments of Mast Node 3 with Time and Wind Speed

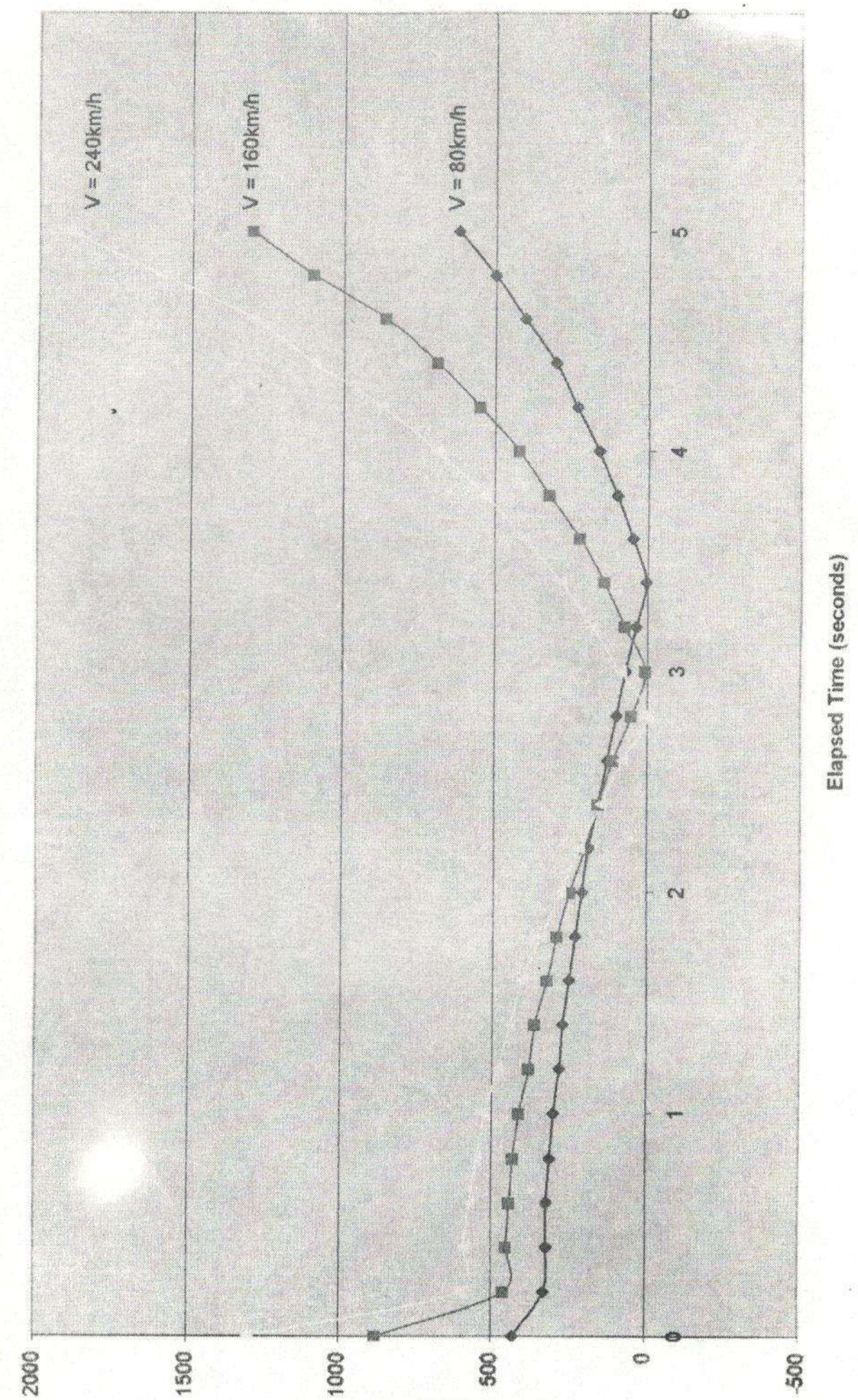


Fig. 6: Variation of Axial Force of Mast Node 3 with Time and Wind Speed

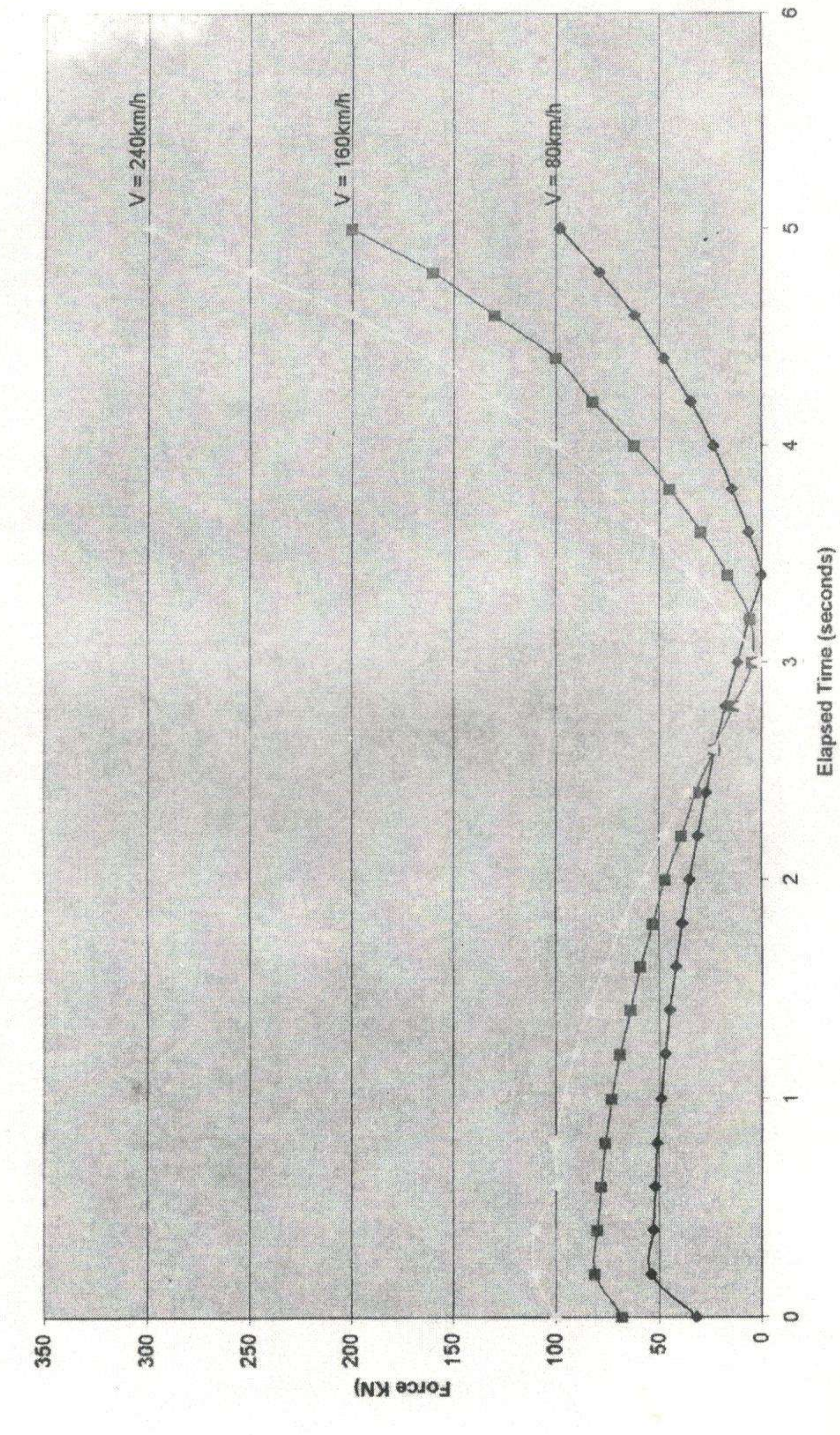
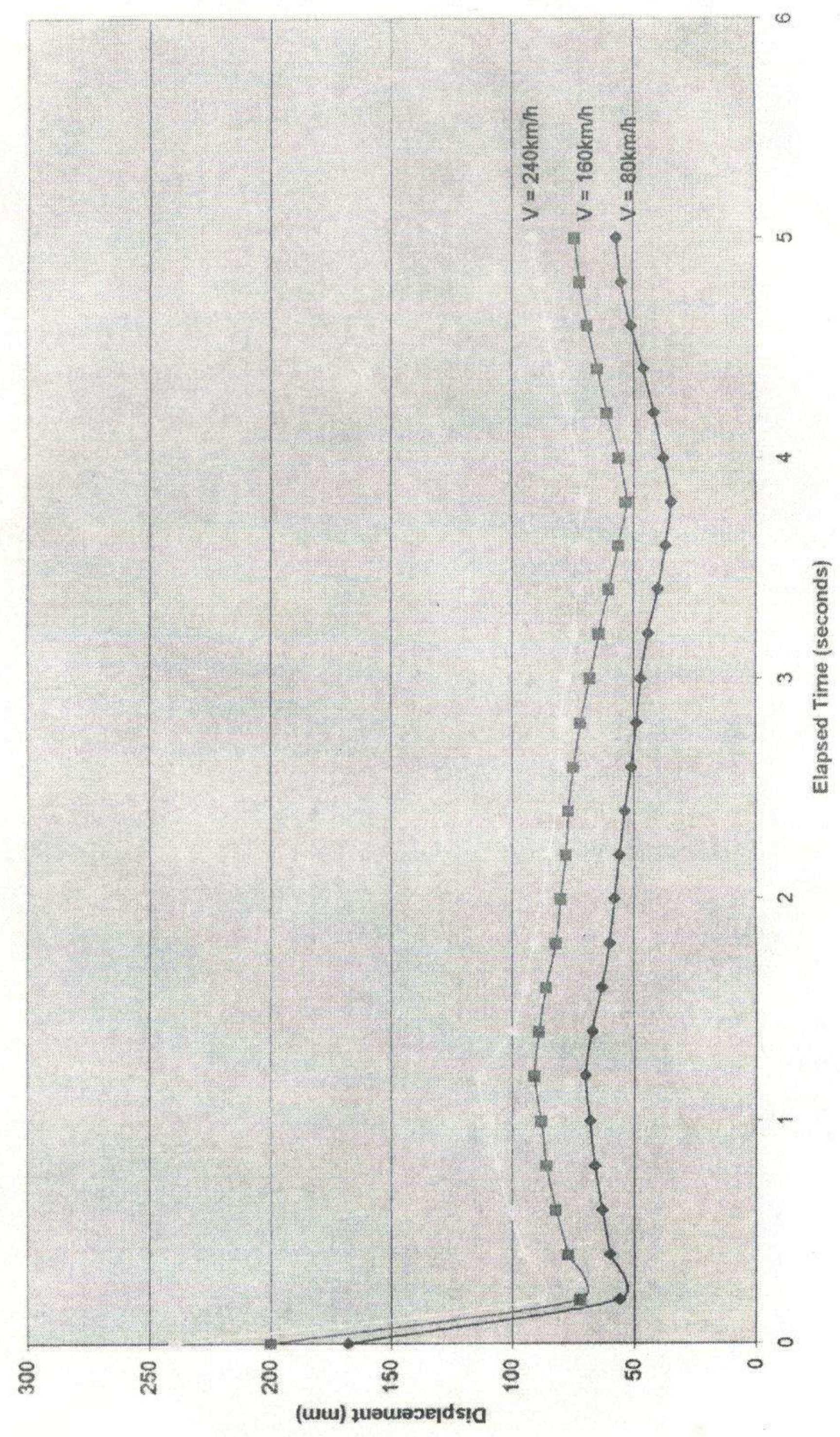


Fig. 7: Variation of Displacement of Mid-point of Guy No.4 (Windward) with Time and Wind Speed



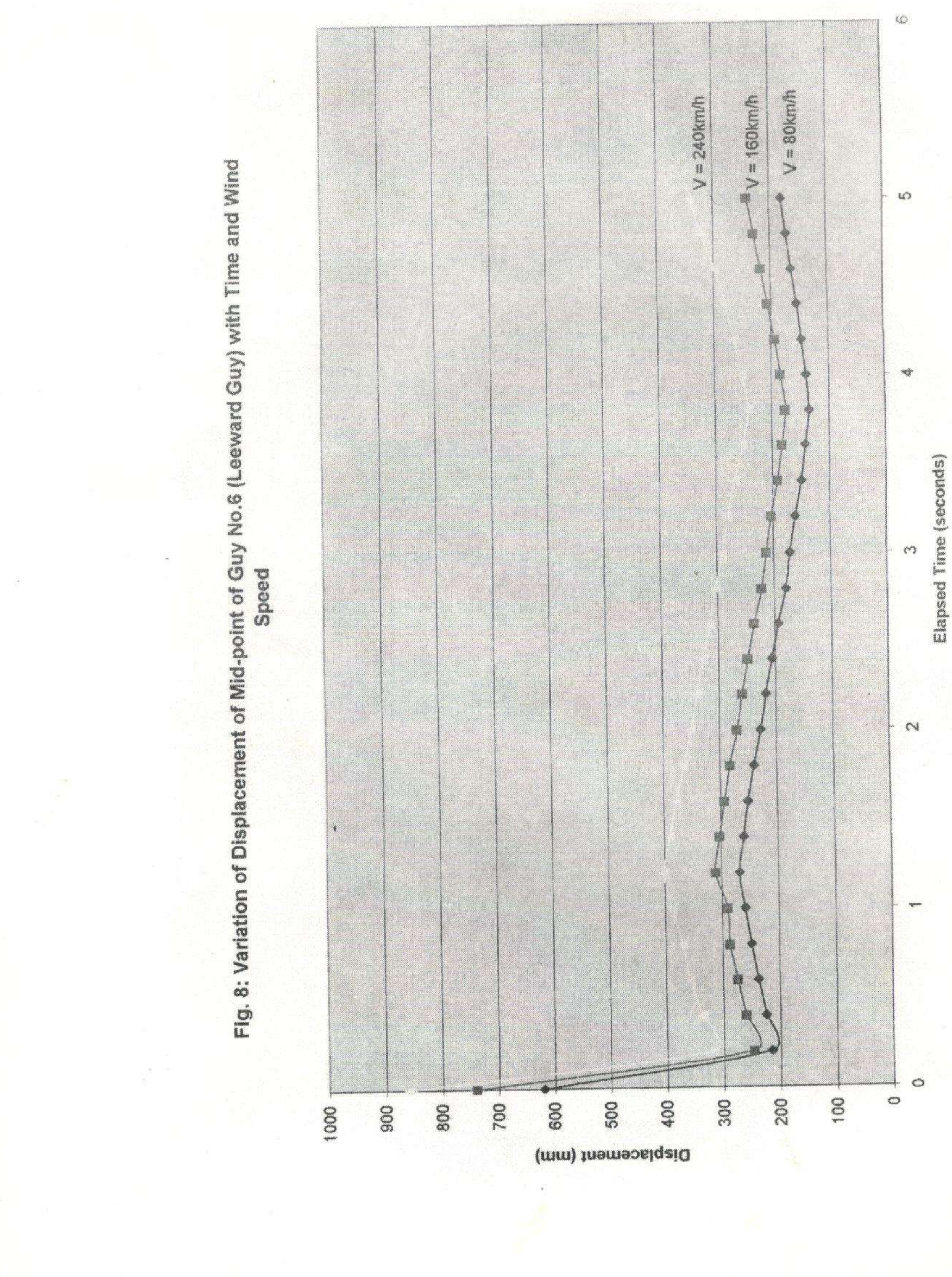
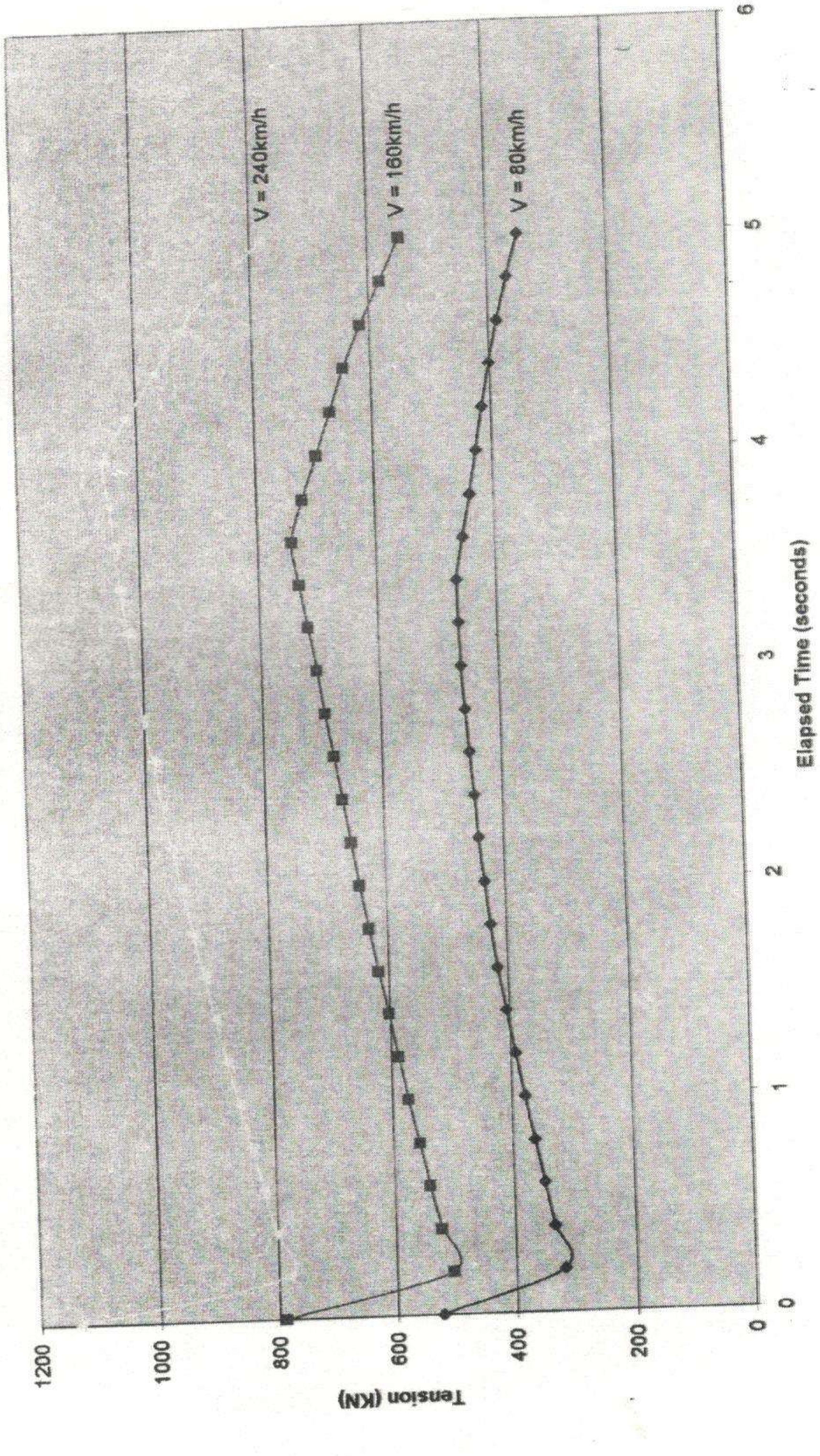


Fig. 8: Variation of Displacement of Mid-point of Guy No.6 (Leeward Guy) with Time and Wind Speed

Fig. 9: Variation of Tension in Guy No.4 (Windward Guy) with Time and Wind Speed



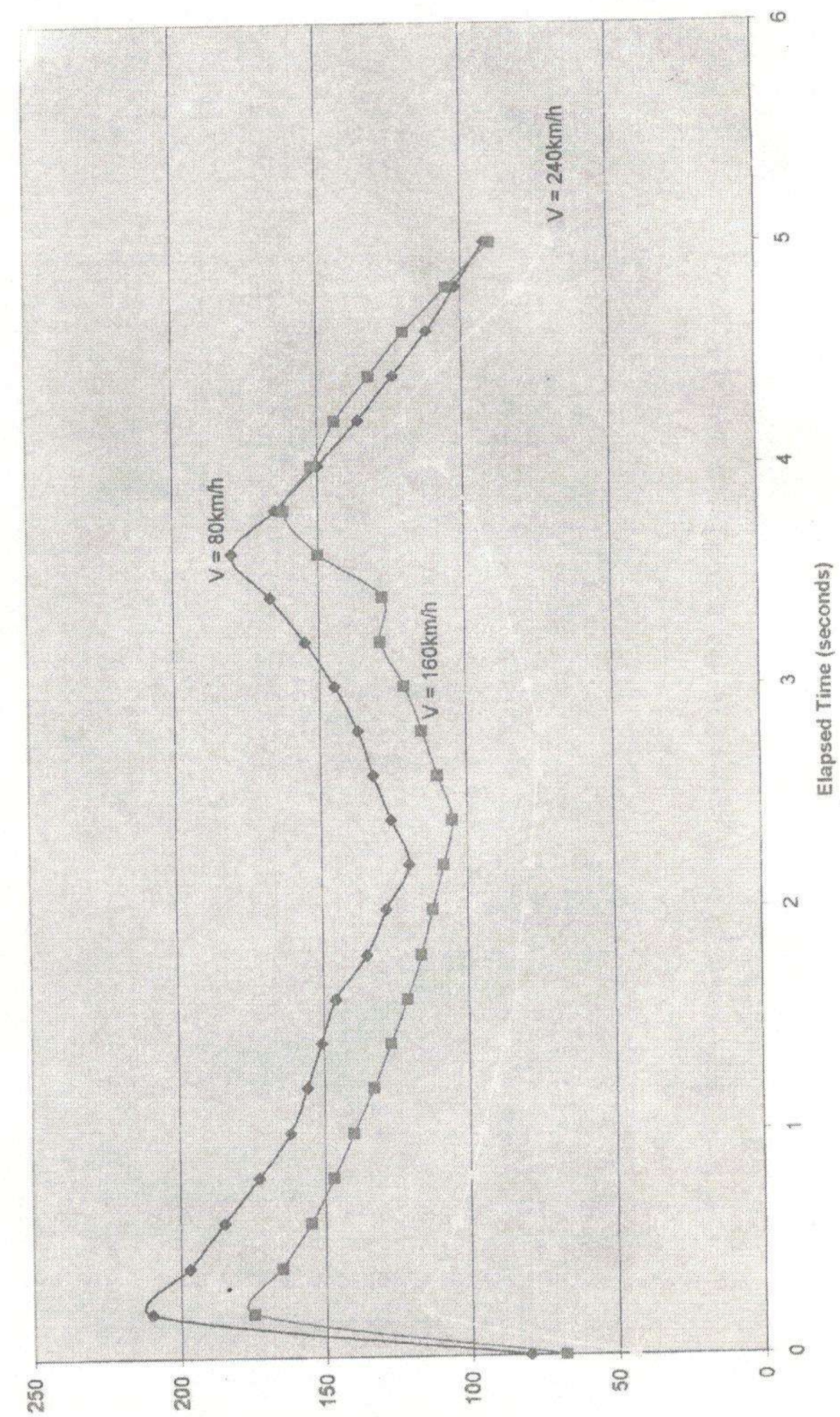


Fig. 10: Variation of Tension in Guy No.6 (Leeward Guy) with Time and Wind Speed