DETERMINING THE COMPOSITION OF NON-FEROUS ALLOYS AT A EUTECTIC POINT BY CALCULATION

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When a number of parallel straight lines cut the x-axis, the angle these lines make with the axis is the same. Thus, parallel lines have the same gradients. A series of straight lines passing through the same points have different gradients. The equation of a straight line which cuts a reference y-axis that passes through an unknown eutectic point can be used to determine the composition of a eutectic mixture.

Two metals such as lead and tin may form a eutectic alloy. The freezing point of lead is 400°C while the freezing point of tin is 327°C. If increasing amounts of lead are added to tin, the freezing point of tin is progressively depressed (along a line) as indicated by a phase diagram. Similarly, if increasing amounts of tin are added to lead, the freezing point of lead is progressively depressed (along a line) in a phase diagram. These depression-of-freezing point lines meet at a point called the eutectic point. The eutectic point indicates the composition of the eutectic mixture. For a tin-lead alloy system, the eutectic point is 62% wt.Sn/38% wt.Pb. The metal with the eutectic composition has the lowest freezing point (Higgins R, 'Engineering Metallurgy' part 1, 5th edition).

In fig.1, WE and ZE are the solidus lines for lead and tin respectively. As the solidus lines must start, one from the melting point of lead (400°C), and the other from melting point of tin (327°C), the two lines are drawn to meet at an unknown point E (fig.1) – personal reasoning.

The eutectic temperature is 183°C . It is assumed in fig.1 that the eutectic temperature line is zero on the temperature scale. The basis for this assumption is that if 183°C is assumed to be represented by 0°c and subtracted from the melting points of lead and tin to get lower values, the result we shall get will still be the same. Consequently, W (melting point of lead) is represented by the value 400-183 = 217°C and Z (melting point of tin) by the value 327-183 = 144°C. The two solidus lines are joined from points 217°C and 144°C to meet at the assumed point E

From E, draw a reference y-axis line EY. From W, draw a line WF perpendicular to WE to cut EY at F. From Z, draw a line ZV perpendicular to ZE to cut EY at V.

The scale of the composition axis is taken to be 1cm = 1% so that it is 100cm long. Also the scale of the temperature axis is taken to be 1cm = 1^0C.

Therefore, from fig.3, if QE = r cm, ER will be (100-r) cm since QR = 100cm (representing 100%). X – axis is the composition axis.

Let gradient of WE = m = $\frac{217}{100-r}$ ….. (i)

and let gradient of ZE = n = $\frac{144}{r}$ ……(ii)

then:

$$\frac{n}{m} = \frac{144}{r} \times \frac{100-r}{217} = \frac{14400-144r}{217r}$$

$$\therefore n(217r) = m(14400-144r)$$

$$\therefore n = \frac{m(14400-144r)}{217r} \quad \text{……….. (iii)}$$

EQUATIONS:

The equation of a straight line is given by:

$$Y = mx + c$$

Where m = gradient of the line

c = its intercept on y axis

If the gradient of a straight line is m, the gradient of another straight line perpendicular to it = - (1/m)

The equation of line WE is given by:

$$Y = mx + 0, \text{……………….(iv)}$$

Since 0 is its intercept on the Y-axis EY

The equation of line WF is given by:

$$Y = \frac{-x}{m} + 217 + K \quad \text{…………(v)}$$

where K = FG.

The equation of line ZE is given by:

$$Y = \frac{m(14400-144r)x}{217r} + 0 \quad \text{…..(vi)}$$

the equation of line ZV is given by:

$$Y = \frac{-(217r)x}{m(14400-144r)} + 144 + J \quad \text{...(vii)}$$

Where J = VH

Subtracting (vii) from (v),

$$0 = \frac{-x}{m} + \frac{(217r)x}{m(14400-144r)} + 73 + K - J \quad \text{……(viii)}$$

Subtracting (vi) from (v),

$$0 = \frac{-x}{m} - \frac{m(14400-144r)x}{217r} + 217 + K \text{…(ix)}$$

Subtracting (ix) from (viii),

$$0 = \frac{(217r)x}{m(14400-144r)} + \frac{m(14400-144r)x}{217r} - 144 - J \quad \text{…(x)}$$

Subtracting (vii) from (iv),

$$0 = mx + \frac{(217r)x}{m(14400-144r)} - 144 - J \ldots\ldots(xi)$$

Subtracting (xi) from (x)

$$0 = \frac{m(14400-144r)x}{217r} - mx \ldots\ldots\ldots\ldots(xii)$$

Dividing (xii) by mx,

$$0 = \frac{(14400 - 144r)}{217r} - 1 \ldots\ldots\ldots\ldots\ldots\ldots(xiii)$$

multiplying (xiii) by 217r,

$$0 = 14400-144r - 217r \ldots\ldots\ldots\ldots\ldots\ldots(xiv)$$

$$361r = 14400$$

$$\therefore \quad r = 39.89cm$$

$\therefore$ QE = r = 39.89cm

since 1cm = 1%, 39.89cm = 39.89%

E is 39.89% away from 0% Pb

$\therefore$ eutectic composition at

E = 39.89% Pb, 60.11% Sn

The experimental value is 38% Pb, 62%Sn (Higgins R, 'Engineering Metallurgy' Part 1, 5th edition).

The difference between the values got from calculation and experimental values should be attributed to experimental errors.

Using this method for other eutectic alloy systems, the results show that the method is correct.

From equation (i), $m = \frac{217}{100-r}$

$$\therefore m = \frac{217}{100-39.89} = \frac{217}{60.11} = 3.61$$

substituting m = 3.61 in equation (xii),

$$0 = 3.61 \frac{[14400 - (144 \times 39.89)]x}{217 \times 39.89} - 3.61x$$

$\therefore$ x = 0

substituting x = 0 in equation (ix),

$$0 = 0-0 + 217 + K$$

$\therefore$ K = - 217

substituting x = 0 and K = -217 in equation (vii),

$$0 = -0+0 + 73 - 217 - J$$

$\therefore$ J = -144

Since the intercept of the line WF (perpendicular to the solidus line WE) on the Y-axis is 217 + K (equation V), its intercept on the Y-axis is therefore 217-217 = 0

Also, since the intercept of the line ZV (perpendicular to the solidus line ZE) on the Y-axis is 144 + J (equation vii), its intercept on the Y-axis is 144-144 = 0

Therefore, fig.1 can be redrawn more accurately as shown in fig.2. In. fig.2, QER is the X-axis.

EQ is the line perpendicular to the solidus line WE, and it cuts the Y-axis at E (y=0) while EP is the line perpendicular to the solidus line ZE, and it cuts the y-axis also at E (y=0).

Other eutectic alloy systems considered are bismuth – cadmium and silicon – aluminium alloy systems. For the bismuth-cadmium alloy system, the freezing points of bismuth and cadmium are 217°C and 321°C respectively. The eutectic temperature is 140°C and the experimental composition of the eutectic mixture is 60% Bi, 40% cd while for the silicon – aluminium alloy system, the freezing point of aluminium is 660°C, the eutectic temperature is 577°C and the experimental composition of the eutectic mixture is 11.6% Si, 88.4%Al, (Higgins R, 'Engineering Metallurgy', part 1, 5th edition). The freezing point of silicon is 1410°C (Hill G.C. and Holman J.S, 'Chemistry in Context', 3rd edition).

The result of all Alloys systems mentioned in this discussion are given in table 1.

USES: One of the uses of this method of calculation is that it gives the exact composition of the metal with the lowest freezing point, including eutectics that do not appear under the microscope in a well-defined laminated form.

CONCLUSION: From table 1, the calculated values of eutectic compositions are very close to the experimental values. The percentage variation for each alloy system is very small. It is clear that the calculated values are more accurate than the experimental values since they are not affected by experimental errors. The method of calculation should therefore be used to determine the composition of alloys at eutectic points. This method cannot be used for ferrous alloys due to the polymorphic changes occurring in iron at 1400°C and 910°C.

REFERENCES

1. Higgins R., Engineering Metallurgy', Part 1, 5th edition.

2. Hill G.C. and Holman J.S., 'Chemistry in Context', 3rd edition.

Fig. 1: **Equilibrium diagram of Sn-Pb alloy system showing the reference y-axis and the intercepting straight lines.**

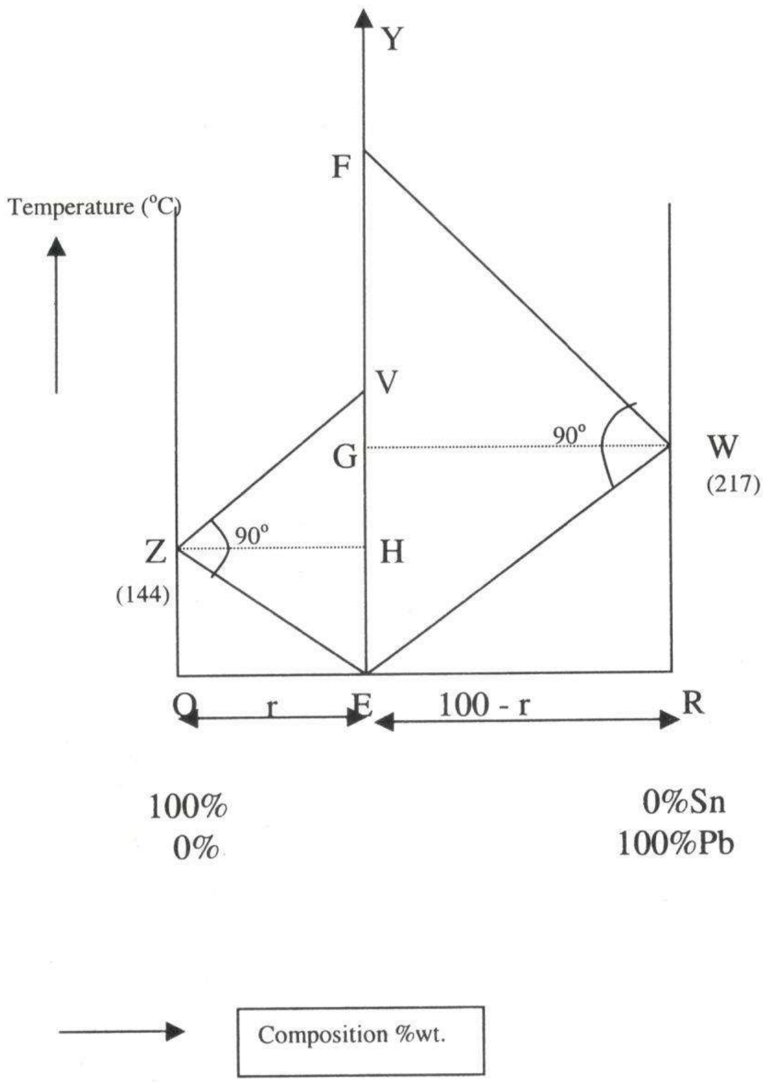
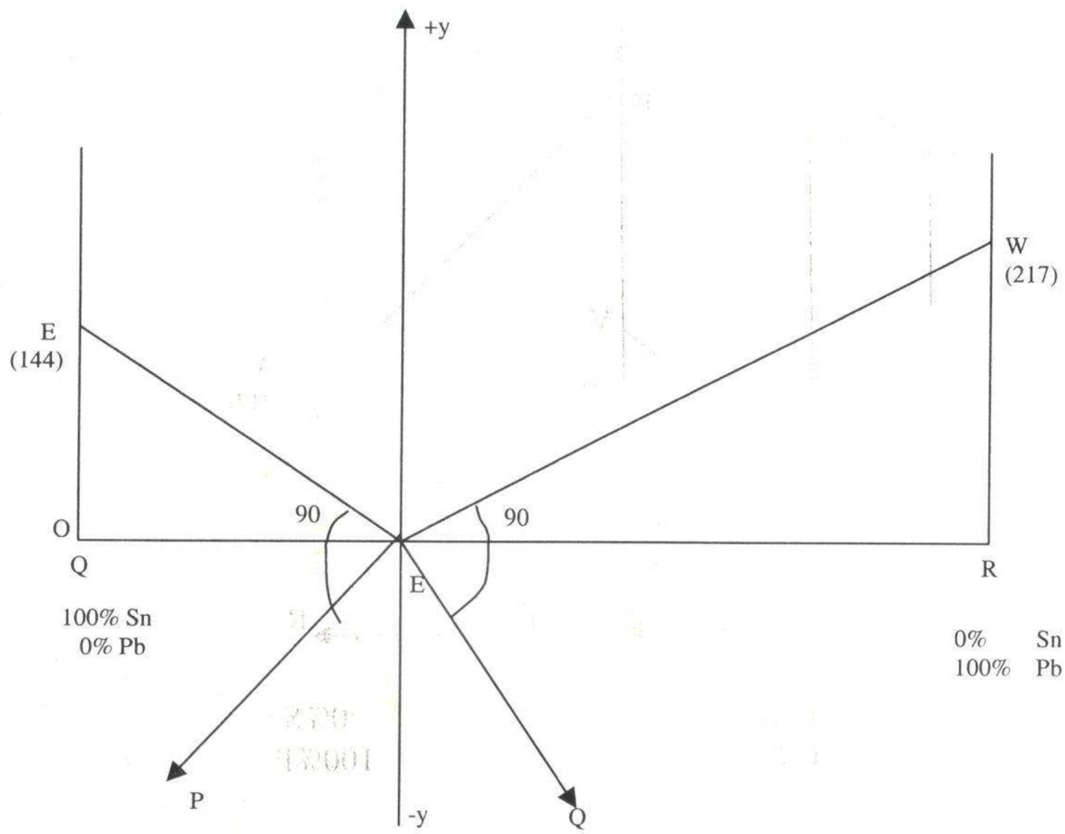


Fig. 2: Modification of Fig.1. Diagram



Alloy Systems	Experimental values	Calculated values	% Variation $\frac{\text{(exp.val-cal val)}}{100}$
Sn-Pb	62% Sn 38% Pb	60.11% Sn 39.89% Pb	Sn: $\frac{62-60.11}{62}$ = 0.03% Pb: $\frac{38-39.89}{38}$ = - 0.05%
Bi-Cd	60% Bi 40.% Cd	58.01% Bi 41.99% Cd	Bi: $\frac{60-58.01}{60}$ = 0.033% Cd: $\frac{40-41.99}{40}$ = - 0.05%
Si-Al	11.6% Si 88.4% AL	9.06% Si 90.94% AL	Si: $\frac{11.6-9.06}{11.6}$ = 0.22% AL: $\frac{88.4-90.94}{88.4}$ = -0.029%

Table 1: Experimental and calculated values of composition of alloys at eutectic points and their percentage variations.