THE LIMITATIONS OF POPOV'S STABILITY CRITERIA IN THE EVALUATION OF THE STABILITY OF NON-LINEAR SYSTEMS UNDER DYNAMIC STATES

By

U. T. Itaketo

c/o Dept. of Electrical/Electronics Engineering
Federal University of Technology, Owerri, Nigeria

Abstract

Popov's stability criteria, and their applications to practical systems, are presented. The limitations of the criteria are discussed. A more general method of determining the stability of non-linear systems under dynamics states is presented and discussed.

Keywords: *Popov criteria; stability; systems; limitations; Lyapunov's method.*

1 Introduction

The subject of the stability of non-linear systems is as controversial as it is difficult. Controversial in the sense that many scholars of control engineering have studied it without success. Also, it disobeys all the known laws that are applicable to the determination of stability of linear systems. As a result of this, as shall be shown later, it is difficult to determine whether such systems are stable or not, from the known mathematically – based theories.

Several laws, such as Nyquist's, Routh – Hurwitz, Gain-Phase, Root locus and Jury's criteria, which are commonly used to determine the stability of linear systems, all fail when applied to non-linear systems. This posed a challenge to early control engineering researchers. As a result of this, one of the early researchers, V. M. Popov[1] developed a method for determining the stability of such systems. The method later became known as Popov's stability criteria but is has its limitations.

2 The Popov's Stability Criteria

Popov's stability criteria was developed through a feedback control system arrangement as illustrated below.

The arrangement consists of a non-linear time – invariant plant represented by $\phi\{e(t)\}$ and a linear time – invariant plant represented by G(s). The input and output characteristics of the non-linear plant is illustrated graphically in fig.2.[2]

From fig. 2, it can be seen that restrictions are imposed on the non-linearity. These restrictions are excpressed by saying that the non-linearity, $\phi\{e(t)\} = U(t)$, is in the sector between the lines with slopes K_1 and K_2. An essential feature to be noted here is that the non-linearity is specified by a sector rather than a particular non-linear function. In otherwords, it is expressed geometrically which is limited in coverage, rather than algebraically which would have been generic. This concept of stability in a given sector is called absolute stability.

To define the concept of stability for the illustration of fig. 2, what was of interest to Popov was the asymptotic behaviour of the control and output variables of the combined linear and non-linear plants as described below:

2.1 The Linear Plant

The Linear time – invariant plant is described by the equations

$$x(t) = A\,x(t) + BU(t) \;......(1)$$

$$y(t) = Cx(t) \;..................(2)$$

where A is a n x n constant matrix

B is a n x 1 constant matrix

C is a 1 x n constant matrix

The condition, according to Popov, is that the linear plant must be stable as long as the following conditions are satisfied:

"The linear system must be output –stable of degree α hence the poles of G(s) are such that Re (s) $< -\alpha$." [2]

where α represents the degree of damping of the linear system and defines/characterizes the output stability of the plant.

2.2 The Non-Linear Plant

For the non-linear plant, Popov defined the non-linearity to be bounded by two straight lines with gradients K_1 & K_2 as illustrated in fig. 2. A general case is such that

$$\infty > K_2 > K_1 > 0 \;...........(3)$$

3 Application of Popov's criteria in the Evaluation of the Stability of Non-linear systems

The first step in the application of Popov's criteria is to generate the contour of G(jw) of the linear portion of the system in the G(jw) plane; Re{G(jw)} and Im {G(jw)}. Thereafter, a curve, known as Popov circle is generated in the same plane. The curve, illustrated in fig.3, is generated as follows:

(a) The points $-1/K_1$ and $-1/K_2$ are plotted in the plane, with K_1 and K_2 given.

(b) The points are joined to form a contour of any shape: straight line, circle, etc.

The plot is as illustrated in fig.3.

Now, Popov's criteria says:

"If the contour of G (jw) of the linear portion of the system does not cut or enclose the Popov circle, then the system under consideration is stable. On the other hand, if it does, this does not prove instability. The system could be stable or unstable, - the test is inconclusive." [1]

The last two sentences are what make Popov criteria ambiguous.

4 Parameters of the Popov circle

The parameters of Popov criteria are as follows:

$$\text{Centre: } \frac{-1/K_1 - (-1/K_2)}{2} = \frac{-1/K_1 + 1/K_2}{2}$$

$$= \quad K_1 - K_2 \quad \ldots(4)$$

$$\text{Radius: } \quad R = \left\| \frac{-K_2 - K_1}{2\,K_1K_2} \right\|$$

$$2\,K_1K_2$$

An interesting feature of Popov's Criteria is that if $K_1 = K_2 = 1$, then $R = -1$ and the Popov circle degenerates to a straight line through the point $-1 + j0$ in G (jw) plane. With this development, and under these conditions, the Popov stability criteria becomes Nyquist's stability criteria for linear systems. Nyquist's stability criteria is recapitulated below:

"A closed – loop system is stable if the contour GH of the open-loop transfer function G(s) H(s) does not encircle the -1 + j 0 point in the complex S – plane, i.e. the net encirclement is zero." [3]

5 Comments On Popov's Stability Criteria

As earlier mentioned, if the locus of G (jw) of the linear portion of the given system does not cut or enclose the Popov circle, then the system is stable. This is just as if the Popov circle were (- 1 + j0) point of G(s) H(s) contour in the Nyquist Stability criteria for linear systems. If the non-linearity lies within the sector characterized by the lines whose slopes are K_1 and K_2, and the G(jw) contour cuts or encloses the Popov circle (that is, if only one of Popov's conditions for stability is met), the test is inconclusive, the system could be stable or unstable.[3] The Popov stability criteria is hence not comprehensive enough for the evaluation of stability of non-linear systems.

More on these criteria, there is a classified restriction imposed on the non-linear system. This is recalled as follows:

"An essential feature to be noted in the Popov stability criteria is that the non-linearity is specified by a sector rather than a particular non-linear function." [3]

The restriction is demonstrated in fig. 2, as the region of absolute stability defined by Popov.

The question that immediately comes to mind is: suppose the non-linearity is not within the specified region, or covers more region than specified, then what happens to this criteria? What this implies is that Popov's criteria becomes inapplicable in such situations. Moreover, why not specify the non-linear system by its function rather than by a geometric sector? More to this, if the locus of G(jw) of the system cuts or encloses the Popov circle, the system is not necessarily unstable. It could be stable or unstable, the test is inclusive. More still, even if the non-linearity lies within the given sector, and the contour of G(jw) cuts the Popov's circle, the test is still inconclusive, the system could be stable or unstable. [1]

6 Discussion

From the study so far, it can be seen that the Popov's stability criteria is not comprehensive enough for the analysis of non-linear systems. Due to these lapses, alternate methods have been sought for, for the evaluation of the stability of non-linear systems. One of these methods is Lyapuuov's second method.[4] Application of this method in the evaluation of the stability on non-linear system is demonstrated in the author's Ph.D. thesis. [5]

Other methods for determining the stability of non-linear systems, such as the describing functions, phase–plane techniques and the method of isoclines, are all based extensively on approximation and linearization techniques. Hence by applying them, most of the primary information are always lost. The phase-plane techniques and the method of isoclines, in particular, are mostly limited to second-order systems. They are therefore not the best approaches for determining the stability of non-linear systems.

7 Conclusion

The studies conducted so far have revealed the limitations of Popov's criteria in determining the stability of non-linear systems under dynamic states. As a result of these limitations, Lyapunov's method has been acclaimed as the best method for the evaluation of the stability of non-linear systems under dynamic states. That method is now internationally accepted in the analysis and evaluation of the stability of non-linear systems both time – varying and time – invariant.[6]

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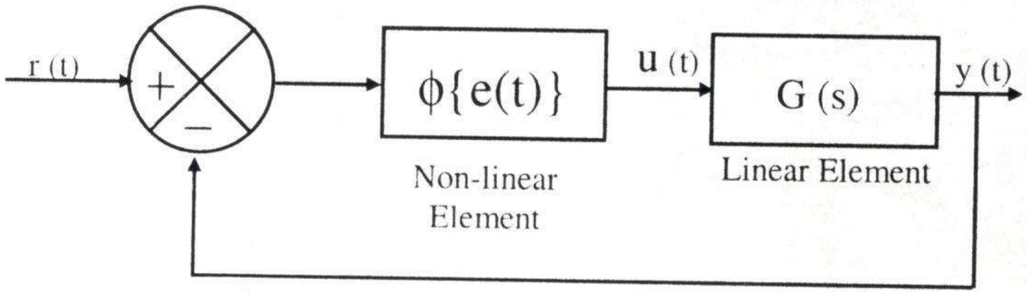

Fig.1 Popov's Feedback Control System Arrangement

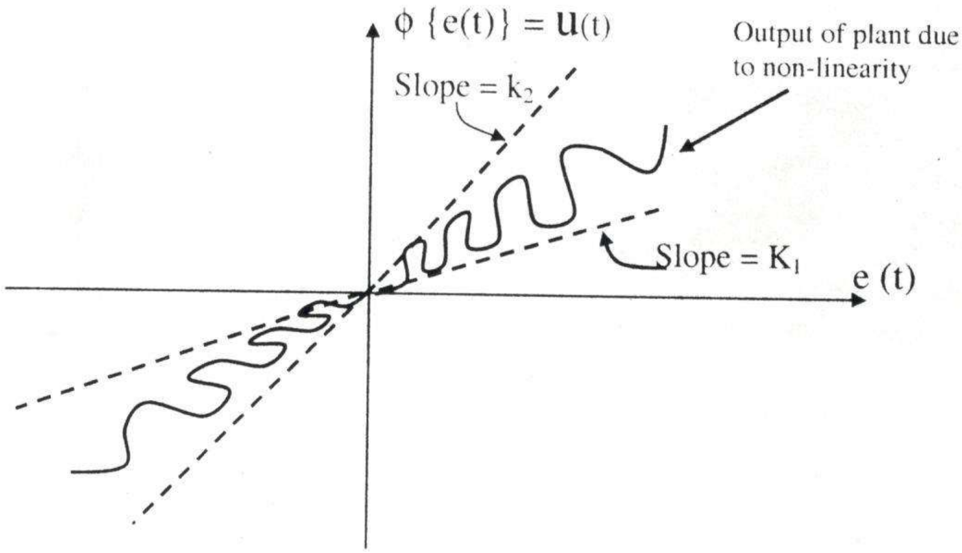

Fig. 2 Graphical Illustration of Popov's Feedback control system For Stability Evaluation of Non-Linear systems

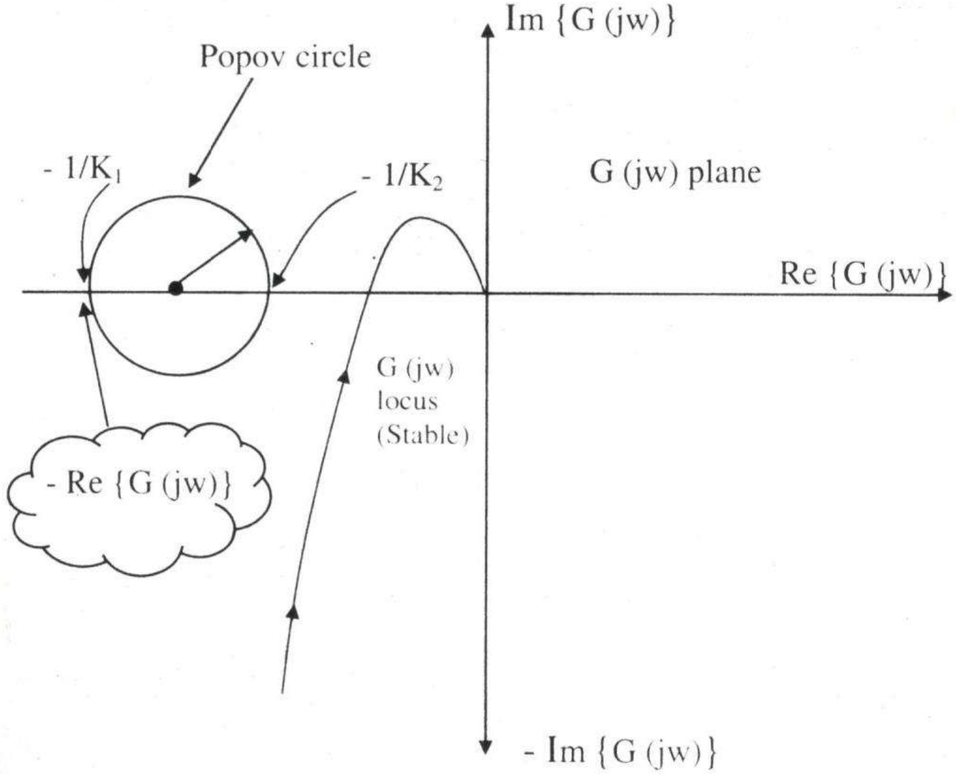

Fig. 3: A general case plot of G (jw) with Popov circle for $\infty > K_2 > K_1 > O$