ACCURACY OF SPLINE FINITE STRIP INTEGRAL SCHEMES IN BRIDGE DECK ANALYSIS

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ABSTRACT

In the application of the spline finite strip procedure to the analysis of non-uniform slab structures, numerical integration schemes were applied to be able to incorporate the variable rigidities in the stiffness matrix expressions. This made possible the analysis of strips with variable thickness in the longitudinal direction. Earlier formulations of the analysis have used four Gauss integration points between spline section knots. This paper compares the performance of spline finite strip integral schemes for stiffness matrix evaluation using two, three and four Gauss integration points between spline section knots. The results of using the different schemes in analysing a slab bridge structure are compared with the ones obtained by using the finite element procedure. The comparison shows that the two point Gauss integral schemes provide satisfactory results for design purposes.

INTRODUCTION

The spline finite strip method has been used by a number of researchers for the analysis of prismatic structures. Cheung and Fan[1983] have applied the procedure to prismatic box-girder bridge decks while Loo et al[1985] have presented formulations based on the auxiliary nodal line [ANL] technique for the analysis of curved and prismatic structures having a variety of support conditions including deformability. In the above cases, it was found that a small number of sections were enough for accurate results to be obtained. It was also possible to use exact integrals in the evaluation of the stiffness matrix expressions since strip rigidities were constant for each strip. For structures with variable thickness in the longitudinal direction, Uko and Cusens[1988] made use of numerical integration for the evaluation of the stiffness matrix expressions.

The bridges that have variable thickness along their longitudinal direction are concrete bridges. These could be reinforced concrete or prestressed concrete using high performance concrete [HPC] which can be economically produced by using the particle parking approach. The HPCs, according to Senthil and Santhanam [2004], are capable of achieving compressive strengths of 70 to 80 MPa with less than 300kg/m^3 of cement. They provide viable structural forms for integral bridges which are becoming very popular [Bhowmick, 2005].

In this paper, Gauss integration schemes with two and three points between spline section knots have been utilised and the results of the analysis are compared with that using the earlier formulations by Uko and Cusens[1988] which used four Gauss integration points between section knots. The schemes are applied in the analysis of a variable thickness slab bridge deck which has been analysed using the finite element method. The comparisons are made in terms of accuracy and computational efficiency.

CUBIC B-SPLINE FUNCTIONS

The spline function used for the analyses reported in this paper is the cubic B-spline function of equal section length defined by the following equations:

For displacement function formulation, a number of cubic B-splines are usually required. These functions can be suitably modified to satisfy the boundary conditions often encountered in most bridge superstructures. Details of the modification schemes have been provided by Cheung and Fan [1983] together with the procedure for the formulation of the stiffness matrix expressions. In another formulation by Cheung et al[1986], spline functions with unequal sections were used for the analysis of skew and irregularly shaped plates. Uko[1984] presented more details on the application of spline finite strip method to the analysis of box-girder bridge decks.

$$\phi_i = \frac{1}{6h^3}\begin{cases} 0 & y < y_{i-2} \\ (y - y_{i-2})^3 & y_{i-2} < y < y_{i-1} \\ h^3 + 3h^2(y - y_{i-1}) + 3h(y - y_{i-1})^2 - 3(y - y_{i-1})^3 & y_{i-1} < y < y_i \\ h^3 + 3h^2(y_{i+1} - y) + 3h(y_{i+1} - y)^2 - 3(y_{i+1} - y)^3 & y_i < y < y_{i+1} \\ (y_{i+2} - y)^3 & y_{i+1} < y < y_{i+2} \\ 0 & y_{i+2} < y \end{cases}$$

Recent developments in the finite strip method give good promise of handling non-prismatic box-girder bridge decks. Swannel et a [1993] have developed a more versatile spline finite strip formulation that could find application in the analysis of these bridge decks. Dawe and Wang [1997] applied the spline finite strip procedure recently to buckling and vibration analysis of composite prismatic structures while Wang and Dawe [1999] have developed an application suitable for analysis of buckling of shell structures.

STIFFNESS MATRIX EVALUATION

The formulation of the stiffness matrix expressions follows finite element procedure with the application of the principle of minimum total potential energy. The stiffness matrix expressions for plate bending analysis using fifth order displacement interpolation functions with curvature compatibility at the nodal lines have been given by Uko[5]. Since the B-spline function used is cubic, it is to be expected that the highest power to be integrated in the stiffness matrix expressions will be six. This can be integrated exactly by using a Gauss integration scheme with four points between section knots. The use of two or three integration points between section knots amounts to reduced integration and should result in reduced computational resource requirements. Researchers working on finite element method have used reduced integration approach in a number of instances.

The evaluation of the stiffness matrix expressions is done by incorporating the rigidities of the strip, calculated from the estimated thickness of the strip at the Gauss integration points as the numerical integration procedure reaches the Gauss point. The stiffness matrix elements for plate bending are as shown below:

$$S_{11} = 12/b^3.D_x.\phi_{wi}.\phi_{wi} - 6/5b.D_1.[\phi_{wi}.\phi''_{wi} + \phi''_{wi}.\phi_{wi}] + 13b/35.D_y.\phi''_{wi}.\phi''_{wi} + 24/5b.D_{xy}.\phi_{wi}.\phi'_{wi}$$

$$S_{12} = 6/b^2.D_x.\phi_{wi}.\phi_{\theta i} - D_{1/10}.[\phi_{wi}.\phi_{\theta i} + 11\phi''_{wi}.\phi_{\theta i}] + 11b^2/210.D_y.\phi_{wi}.\phi_{\theta i} + 2/5.D_{xy}.\phi'_{wi}.\phi_{\theta i}$$

$$S_{13} = -12/b^3.D_x.\phi_{wi}.\phi_{wj} + 6/5b.d_1.[\phi_{wi}.\phi''_{wj} + \phi''_{wi}.\phi_{wj}] + 9b/70.D_y.\phi''_{wi}.\phi''_{wj} - 24/5b.D_{xy}.\phi'_{wi}.\phi_{wj}$$

$$S_{14} = 6/b^2.D_x.\phi_{\theta j} - D_1/10.[\phi_{wi}.\phi''_{\theta j} + \phi''_{wi}.\phi_{\theta j}] - 13b^2/420.D_y.\phi''_{wi}.\phi''_{\theta j} + 2/5.D_{xy}/\phi'_{wi}.\phi_{\theta j}$$

$$S_{22} = 4/b.D_x.\phi_{\theta i}.\phi_{\theta i} - 2b/15.D_1.[\phi_{\theta i}.\phi''_{\theta i} + \phi''_{\theta i}.\phi_{\theta i}] + b^3/105.D_y.\phi''_{\theta i}.\phi''_{\theta i} + 8b/15.D_{xy}.\phi'_{\theta i}.\phi_{\theta i}$$

$$S_{23} = -6/b^2.D_x.\phi_{\theta i}.\phi_{wj} + D_{1/10}.[\phi_{\theta i}.\phi''_{wj} + \phi''_{\theta i}.\phi_{wj}] + 13b^3/420.D_y.\phi''_{\theta i}.\phi''_{wj} - 2/5.D_{xy}.\phi'_{\theta i}.\phi'_{wj}$$

$$S_{24} = 2/b.D_x.\phi_{\theta i}.\phi_{\theta j} + b/30.D_1.[\phi_{\theta i}.\phi''_{\theta j} + \phi''_{\theta i}.\phi_{\theta j}] - b^3/140.D_y.\phi''_{\theta i}.\phi''_{\theta j} - 2b/15.D_{xy}.\phi'_{\theta i}.\phi'_{\theta j}$$

$$S_{33} = 12/b^3.D_x.\phi_{wj}.\phi_{wj} - 6/5b.D_1.[\phi_{wj}.\phi''_{wj} + \phi''_{wj}.\phi_{wj}] + 13b^3/35.D_y.\phi''_{wj}.\phi''_{wj} + 24/5b.D_{xy}.\phi'_{wj}.\phi'_{wj}$$

$S_{34} = -6/b^2.D_x.\phi_{wj}.\phi_{\theta j} + D_1/10.[\phi_{wj}\,\phi''_{\theta j} + \phi''_{wj}.\phi_{\theta j}] + 11b^2/210.D_y.\phi''_{wj}.\phi''_{\theta j} - 2/5.D_{xy}.\phi'_{wj}.\phi'_{\theta j}$

$S_{44} = 4/b.D_x.\phi_{\theta j}.\phi_{\theta j} - 2b/15.D_1.[\phi_{\theta j}.\phi''_{\theta j} + \phi''_{\theta j}.\phi_{\theta j}] + b^3/105.D_y.\phi''_{\theta j}.\phi''_{\theta j} - 8b/15.D_{xy}.\phi'_{\theta j}.\phi'_{\theta j}$

where b = width of strip

ϕ_{ac} = represents the selected cubic B-spline functions for displacement a at nodal line c

ϕ'_{ac} is the first derivative of ϕ_{ac}

ϕ''_{ac} = is the second derivative of ϕ_{ac}.

D_x, D_y, D_{xy} and D_1 are the bending rigidities defined earlier

$\phi_{ac} \cdot \phi_{de}$ is the integral of the product of ϕ_{ac} and ϕ_{de}

SIMULATION OF NON-UNIFORM STRIPS

The simulation of non-uniform structures by the finite element method is usually done by sub-dividing the structure into elements with constant thicknesses. Thus the simulation gets more accurate as the number of elements gets larger. This, however, means substantial increase in computer resources and greater difficulty at output interpretation level. Another finite element simulation uses three-dimensional brick elements with as many as twenty-four nodes in some cases. This element is computationally very expensive and is rarely used for large problems.

The spline finite strip simulation presented in this paper makes use of the estimated thickness of the strip at the Gauss integration points. For input purposes, the thickness of the strip at the ends and midpoints of spline sections are provided. The thicknesses at the Gauss integration points are interpolated linearly from the input values.

Figure 1 shows the location of the Gauss integration points within a cubic B-spline function. While the ordinates of the three and four Gauss point schemes are fairly well distributed between the section knots, those of the two-point scheme are very close to the section knots.

APPLICATION TO THE ANALYSIS OF SLABS

The proposed integral schemes were employed in the analysis of the propped cantilever shown in Fig. 2. Figure 2[a] shows the variable thickness propped cantilever of the same span and width as the constant depth slab. The thickness of the propped cantilever varied from 600mm at the fixed end to 200mm at the propped end. Figure 2[b] shows the finite element simulation for the deck using isoparametric quadrilateral elements while Figure 3c shows the simulation scheme adopted for the variable thickness of the deck.

PAFEC77 software was utilised in the finite element analysis of the decks. Only one load case adjudged to be very critical was considered for the two selected decks. This consisted of a single concentrated load of 1000KN placed at an edge of the deck[i.e. 4metres from the centreline] at the midspan.

RESULTS AND DISCUSSION

The results of the analysis are shown on Tables 1 to 3. Due to the closeness of the values of the parameters considered, it became apparent that a tabular vis-à-vis a graphical presentation would be most appropriate, bearing mind also the subject matter of the investigation. Deflections, longitudinal bending stresses and transverse bending stresses are presented for selected o utput s ections. F or the constant deck slab, the transverse distribution of the parameters are given at a distance of 2.25m from the left support [i.e. at the midspan]. For the propped cantilever, the longitudinal distribution along the loaded edge of the slab and also along the longitudinal centreline are presented on Tables 3.

It can be observed that there is very close agreement between the finite element results and that produced by using the three finite strip analysis using 2-Gauss points, 3-Gauss points and 4-Gauss points for the stiffness matrix integrals. Results for deflection are generally in very good agreement at all locations. For longitudinal and transverse bending stresses, the finite element analysis returned higher values at locations close to the concentrated loads. This behaviour of the finite element analysis is not peculiar to this analysis as Uko[1984] and other researchers have reported similar tendencies even when comparisons were made with experimental results on models and prototypes. Away from the concentrated loads, there is good agreement between the finite element and the three finite strip simulations. The results also indicate very little difference between the values returned by the finite strip simulations using 3-Gauss point and 4-Gauss point stiffness matrix integrals.

The use of reduced integration in the spline finite strip simulations also results in savings in computation resources in terms of central processing unit [CPU] time requirement. Table

4 shows the CPU time requirement for the three spline finite strip requirements.

CONCLUSIONS AND RECOMMENDATIONS

The 2-Gauss point and 3-Gauss point spline finite simulations have proved very satisfactory in the analysis of the two decks considered. The results compare extremely well with the 4-Gauss point integral scheme which provides exact integration for the stiffness matrix elements. The 3-Gauss point integral scheme is just as good as the 4-Gauss point scheme and they compare very favourably with the finite element analysis. Effort should be made to apply the reduced integration to box-girder type structures to investigate the performance of the reduced integral schemes in the analysis of in-plane stresses.

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TABLE 1 – PROFILES FOR CONSTANT DECK AT Y=2.25m

[a] DEFLECTION [in mm] PROFILE

OUTPUT SECTION	FINITE ELEMENT	4-SECTION FINITE STRIP			8-SECTION FINITE STRIP		
		2 POINT	3 POINT	4 POINT	2 POINT	3 POINT	4 POINT
0.00	-7.171	-7.128	-7.139	-7.139	-7.175	-7.174	-7.174
1.00	-6.359	-6.346	-6.351	-6.351	-6.359	-6.359	-6.359
2.00	-5.605	-5.606	-5.607	-5.607	-5.604	-5.605	-5.605
3.00	-4.927	-4.932	-4.931	-4.931	-4.926	-4.926	-4.926
4.00	-4.337	-4.343	-4.341	-4.341	-4.336	-4.336	-4.336
5.00	-3.840	-3.844	-3.842	-3.842	-3.839	-3.839	-3.839
6.00	-3.430	-3.433	-3.431	-3.431	-3.429	-3.429	-3.428
7.00	-3.095	-3.096	-3.095	-3.095	-3.093	-3.093	-3.093
8.00	-2.814	-2.815	-2.815	-2.815	-2.813	-2.813	-2.813

[b] LONGITUDINAL BENDING STRESS [in N/mm^2] PROFILE

OUTPUT SECTION	FINITE ELEMENT	4-SECTION FINITE STRIP			8-SECTION FINITE STRIP		
		2 POINT	3 POINT	4 POINT	2 POINT	3 POINT	4 POINT
0.00	-2.680	-2.291	-2.376	-2.376	-2.892	-2.879	-2.899
1.00	-2.823	-2.701	-2.734	-2.734	-2.797	-2.797	-2.797
2.00	-2.728	-2.778	-2.778	-2.778	-2.719	-2.723	-2.723
3.00	-2.582	-2.655	-2.642	-2.642	-2.574	-2.575	-2.575
4.00	-2.392	-2.448	-2.434	-2.434	-2.377	-2.378	-2.378
5.00	-2.230	-2.228	-2.217	-2.217	-2.172	-2.172	-2.172
6.00	-2.022	-2.029	-2.021	-2.021	-1.985	-1.984	-1.984
7.00	-1.851	-1.865	-1.859	-1.859	-1.828	-1.828	-1.828
8.00	-1.657	-1.734	-1.730	-1.730	-1.702	-1.703	-1.703

[c] TRANSVERSE BENDING STRESS[in N/mm^2] PROFILE

OUTPUT SECTION	FINITE ELEMENT	4-SECTION FINITE STRIP			8-SECTION FINITE STRIP		
		2 POINT	3 POINT	4 POINT	2 POINT	3 POINT	4 POINT
0.00	-0.515	-0.012	-0.041	-0.040	0.017	0.018	0.018
1.00	0.043	-0.030	-0.028	-0.027	0.135	0.129	0.129
2.00	0.256	0.180	0.202	0.203	0.276	0.274	0.274
3.00	0.392	0.368	0.386	0.386	0.415	0.415	0.415
4.00	0.439	0.460	0.470	0.470	0.482	0.483	0.483
5.00	0.352	0.458	0.462	0.462	0.468	0.468	0.468
6.00	0.333	0.377	0.377	0.377	0.380	0.381	0.381
7.00	0.221	0.224	0.223	0.223	0.225	0.226	0.226
8.00	-0.354	-0.000	-0.001	-0.001	0.001	0.002	0.002

TABLE 2 – PROFILES FOR PROPPED DECK ALONG LOADED EDGE

[a] DEFLECTION [in mm] PROFILE

OUTPUT	FINITE	4-SECTION FINITE STRIP			8-SECTION FINITE STRIP		
SECTION	ELEMENT	2 POINT	3 POINT	4 POINT	2 POINT	3 POINT	4 POINT
0.750	-0.615	-0.525	-0.526	-0.526	-0.596	-0.594	-0.594
2.250	-4.731	-4.567	-4.566	-4.566	-4.777	-4.770	-4.770
3.750	-11.815	-12.158	-12.128	-12.128	-11.841	-11.844	-11.844
5.250	-20.483	-20.669	-20.603	-20.602	-20.629	-20.620	-20.619
6.000	-24.234	-23.793	-23.724	-23.723	-24.189	-24.171	-24.171
6.750	-25.828	-25.467	-25.414	-25.413	-25.818	-25.811	-25.811
8.250	-24.595	-24.385	-24.384	-24.385	-24.401	-24.403	-24.403
9.750	-18.362	-17.840	-17.854	-17.855	-18.011	-18.003	-18.003
11.250	-7.100	-6.786	-6.782	-6.781	-6.803	-6.805	-6.805

[b] LONGITUDINAL BENDING STRESS [in N/mm^2] PROFILE

OUTPUT	FINITE	4-SECTION FINITE STRIP			8-SECTION FINITE STRIP		
SECTION	ELEMENT	2 POINT	3 POINT	4 POINT	2 POINT	3 POINT	4 POINT
0.750	14.497	15.909	15.898	15.900	16.531	16.494	16.495
2.250	9.428	13.033	12.909	12.907	10.239	10.301	10.301
3.750	4.515	3.485	3.446	3.444	6.395	6.317	6.317
5.250	-6.239	-10.642	-10.469	-10.466	-10.235	-10.187	-10.187
6.000	-36.454	-16.578	-16.333	-16.328	-23.851	-23.528	-23.525
6.750	-13.500	-14.959	-14.858	-14.855	-17.082	-17.067	-17.067
8.250	-9.658	-12.195	-12.321	-12.322	-11.005	-11.041	-11.041
9.750	-8.118	-9.007	-9.036	-9.039	-9.487	-9.479	-9.479
11.250	-4.422	-5.231	-5.076	-5.074	-4.733	-4.669	-4.670

[c] TRANSVERSE BENDING STRESS [in N/mm^2] PROFILE

OUTPUT	FINITE	4-SECTION FINITE STRIP			8-SECTION FINITE STRIP		
SECTION	ELEMENT	2 POINT	3 POINT	4 POINT	2 POINT	3 POINT	4 POINT
0.750	3.668	2.313	2.273	2.276	1.896	1.811	1.815
2.250	2.226	1.936	1.757	1.766	1.392	1.396	1.395
3.750	0.995	1.858	1.805	1.807	2.141	2.148	2.149
5.250	-1.161	1.310	1.475	1.466	1.722	1.719	1.719
6.000	-4.251	0.494	0.634	0.627	0.295	0.388	0.381
6.750	-2.770	0.112	0.083	0.084	-0.277	-0.296	-0.296
8.250	-2.138	-2.120	-2.363	-2.350	-2.711	-2.633	-2.633
9.750	-1.793	-4.718	-4.452	-4.460	-3.175	-3.392	-3.391
11.250	-0.988	-3.618	-3.116	-3.136	-5.567	-4.950	-4.957

TABLE 3 – PROFILES FOR PROPPED DECK ALONG CENTRELINE

[a] DEFLECTION [in mm] PROFILE

OUTPUT SECTION	FINITE ELEMENT	4-SECTION FINITE STRIP			8-SECTION FINITE STRIP		
		2 POINT	3 POINT	4 POINT	2 POINT	3 POINT	4 POINT
0.750	-0.203	-0.193	-0.192	-0.192	-0.180	-0.180	-0.180
2.250	-1.688	-1.615	-1.610	-1.610	-1.593	-1.593	-1.593
3.750	-4.269	-4.135	-4.129	-4.129	-4.121	-4.121	-4.121
5.250	-7.149	-7.013	-7.007	-7.007	-7.005	-7.005	-7.005
6.000	-8.362	-8.230	-8.222	-8.222	-8.244	-8.244	-8.244
6.750	-9.220	-9.114	-9.101	-9.101	-9.150	-9.150	-9.149
8.250	-9.454	-9.472	-9.450	-9.450	-9.497	-9.495	-9.495
9.750	-7.301	-7.344	-7.331	-7.331	-7.409	-7.407	-7.407
11.250	-2.832	-2.842	-2.845	-2.845	-2.875	-2.875	-2.875

[b] LONGITUDINAL BENDING STRESS [in N/mm^2] PROFILE

OUTPUT SECTION	FINITE ELEMENT	4-SECTION FINITE STRIP			8-SECTION FINITE STRIP		
		2 POINT	3 POINT	4 POINT	2 POINT	3 POINT	4 POINT
0.750	5.869	5.648	5.632	5.632	5.609	5.610	5.610
2.250	4.225	4.141	4.152	4.152	4.217	4.214	4.214
3.750	1.420	1.564	1.569	1.569	1.541	1.544	1.544
5.250	-1.852	-1.757	-1.781	-1.781	-1.630	-1.632	-1.632
6.000	-3.339	-3.171	-3.206	-3.207	-3.132	-3.143	-3.143
6.750	-4.258	-3.933	-3.940	-3.940	-4.088	-4.090	-4.090
8.250	-4.944	-5.161	-5.118	-5.118	-5.019	-5.016	-5.016
9.750	-4.198	-4.264	-4.247	-4.247	-4.424	-4.422	-4.422
11.250	-2.039	-2.089	-2.142	-2.142	-1.965	-1.969	-1.969

[c] TRANSVERSE BENDING STRESS[in N/mm^2] PROFILE

OUTPUT SECTION	FINITE ELEMENT	4-SECTION FINITE STRIP			8-SECTION FINITE STRIP		
		2 POINT	3 POINT	4 POINT	2 POINT	3 POINT	4 POINT
0.750	1.062	0.968	0.967	0.967	0.980	0.980	0.980
2.250 -	1.625	1.573	1.580	1.580	1.621	1.620	1.620
3.750	2.327	2.501	2.502	2.502	2.459	2.458	2.458
5.250	2.944	3.124	3.111	3.111	3.126	3.126	3.126
6.000	3.026	3.142	3.126	3.126	3.184	3.183	3.183
6.750	2.888	2.967	2.960	2.960	3.013	3.011	3.011
8.250	2.054	2.036	2.047	2.047	2.086	2.086	2.086
9.750	1.008	1.028	1.042	1.042	1.002	1.003	1.003
11.250	0.209	0.193	0.192	0.192	0.217	0.216	0.216

TABLE 4 : CPU REQUIREMENTS [in seconds] FOR SPLINE FINITE STRIP ANALYSIS

NO. OF SPLINE SECTIONS	2-GAUSS POINTS	3-GAUSS POINTS	4-GAUSS POINTS
4	0.66	0.70	0.73
8	1.30	1.38	1.43

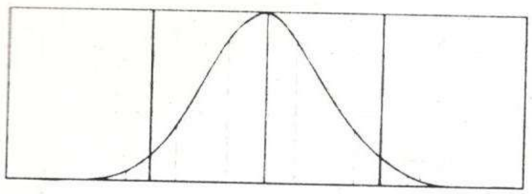

Fig. 1[a] Spline Function with 2 - Gauss Integration Points per Section

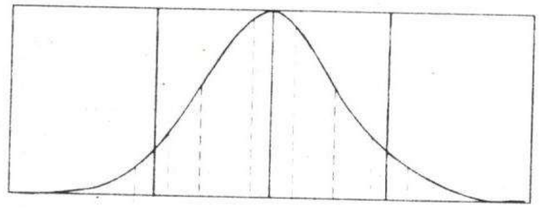

Fig. 1[b] Spline Function with 3-Gauss Integration Points per Section

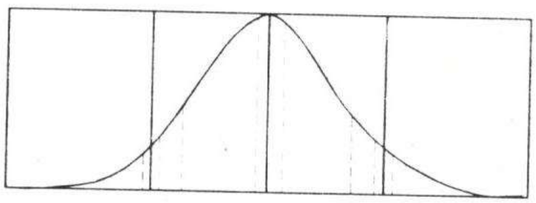

Fig. 1[c] Spline Function with 4-Gauss Integration Points per Section

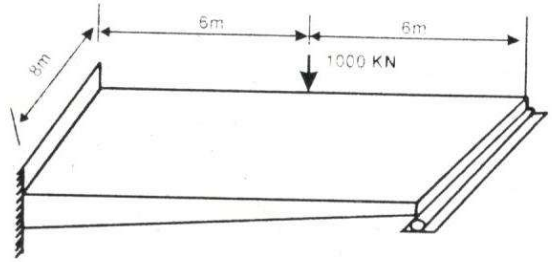

Fig. 2[a] Deck considered in the Example

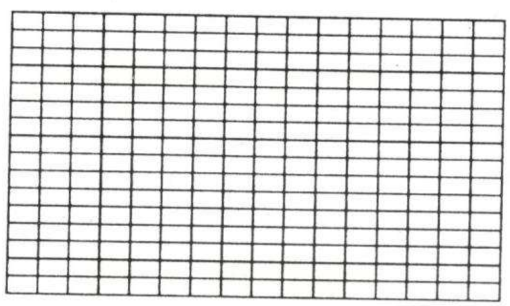

Fig. 2[b] Finite Element Mesh Used in the Analysis

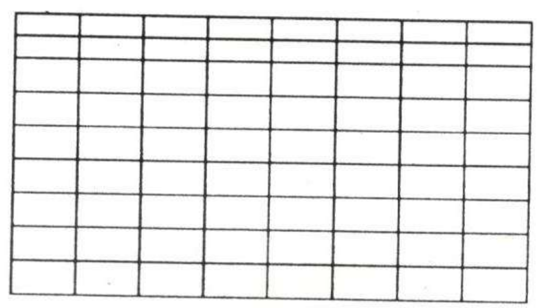

Fig. 3[a] Finite Strip Simulation - 9 Strips and 8 Sections

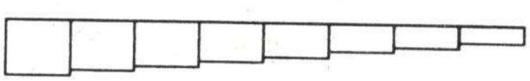

Fig. 2[c] Finite Element Thickness Simulation

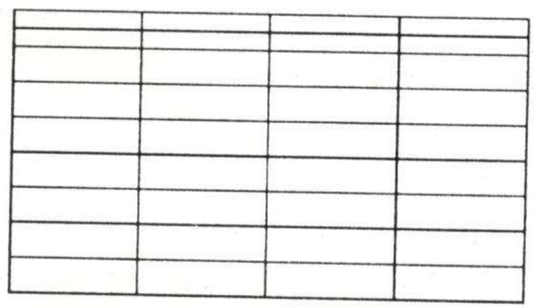

Fig. 3[b] Finite Strip Simulation - 9 Strips and 4 Sections