SIMPLIFIED ESTIMATION OF TURBULENT HEAD LOSSES IN WATER DISTRIBUTION PIPES

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ABSTRACT

Turbulent flows in pipes are difficult to treat analytically and thus numerous semi-empirical and fully empirical friction loss relationships have been developed for them. The most accurate of these is the Colebrook-White formula. Unfortunately, computation of the head loss with it requires iteration leading engineers to adopt simpler, albeit less accurate formulas which do not require iteration. This paper develops a relationship for computing head losses in pipes, which does not require iteration but still has an adequate accuracy over the entire range of flows in the water supply industry. A comparison with existing formulas confirms its significant superiority over the existing non-iterative procedures.

INTRODUCTION

The problem of determining head losses in pipes is quite common in hydraulic engineering and needs to be carefully handled. An overestimation of head losses will tend to lead to an overestimation of required conduit sizes, which implies a high initial cost of conduit installation. Too small a conduit size from underestimation will lead to functional inadequacies.

Analytical treatment of laminar flows has led to the straightforward and well-tested friction loss relationship for laminar flow, the Hagen-Poiseuille equation, which shows a linear relationship between head loss and pipe discharge. The problem of computing head losses in laminar flow is thus simple. Attention here will be focused on turbulent flows.

Turbulent flows are more difficult to treat analytically and thus, numerous semi-empirical and fully empirical friction loss relationships have been developed for them. Aiyesimoju [Ref. 1] has compared the accuracy of the most commonly used ones in water supply practice and confirmed that the Colebrook-White formula yields the best results. The problem is that computation of head losses with the formula involves iteration. Numerous charts and graphical aids such as Moody's Chart [Ref. 2] and Asthana's Diagram [Ref. 3] have been developed to simplify using this formula. However, these graphical aids are often not available and when they are, the errors introduced in scaling the graphs are often significant.

Recent work in this area has been about assessment or proper use of existing methods. For example, Cristensen et al. [Ref. 4] was concerned only with the proper use of the Hazen-Williams formula whilst Khatibi et al. [Ref. 5] focussed mainly on the estimation of friction factors for flow in nearly flat tidal channels. Aiyesimoju [Ref. 6] was concerned with relating Hazen-Williams coefficients for pipes with the equivalent roughness.

The aim of this paper is to develop an **explicit** relationship that will enable computing turbulent head losses in pipes over the range of flow of practical interest in water supply practice, without iteration. The developed relationship as well as three previously developed relationships that are generally applicable in practical situations, and that do not involve any iteration, will be compared with the most accurate Colebrook-White formula. These three other methods are

1) Ranga Raju and Garde equation

2) Manning's formula

3) Hazen-Williams formula

The comparison will be in the context of several examples of flow situations designed to reflect extremes of practical flow situations. The specific range of flow these are as follows

1. Pipe equivalent roughness height ε between 1 μm (less than the value for drawn tubing) and 1 cm (corresponding to riveted steel).

2. Pipe diameter D between 2cm and 2m.

3. Pipe relative roughness ε/D up to 0.033 (limit of Nikuradse's original data, upon which Colebrook-White formula is based [see Ref. 2]).

4. Flow Reynold's Number R above 4000 (lower limit for turbulent pipe flow).

5. Maximum velocity V_{max} of water which is the fluid of interest, will be limited to 10m/s here (usually less than 3m/s in practice [Ref. 2]) which implies maximum Reynolds's Number R_{max} in a

pipe of diameter D, is $R_{max} = V_{max}*D/\nu = 10D/\nu$.

6. Minimum kinematic viscosity ν_{min} of water will be taken as 3E-7 m^2/s (a little below that for water at 100°C) and maximum ν_{max} will be taken as 2E-6 m^2/s (a little more than that for water at 0°C).

2. FRICTION LOSS RELATIONSHIPS

2.1 Colebrook-White Formula

The best known friction loss relationship for turbulent flows is the D'arcy-Weisbach formula

$$S_f = f\frac{V^2}{2gD} \tag{1}$$

where S_f is the friction slope (head loss h over pipe length L), V is flow velocity, D is pipe diameter, g is acceleration due to gravity and f is the friction factor. There are numerous relationships for the friction factor but the best is the Colebrook-White formula,

$$\frac{1}{\sqrt{f}} = -2\log_{10}(\frac{\varepsilon}{3.7D} + \frac{2.51}{R\sqrt{f}}) \tag{2}$$

where ε is the pipe roughness height (a measure of the typical height of protrusions from the pipe material surface), D is pipe diameter and R is pipe Reynold's number. The Colebrook-White formula is based on the results of experiments on sand-roughened pipes (see Ref. 2), with appropriate modification of the results to reflect observations of commercial pipes. The well known Moody's Chart [Ref. 2] is simply a graphical presentation of this. If Q is pipe discharge, ν is the fluid kinematic viscosity and g is assumed as 9.8 m/s^2, then

$$V = \frac{4Q}{\pi D^2} \tag{3}$$

and $$R = \frac{VD}{\nu} = \frac{4Q}{\pi\nu D} \tag{4}$$

From Eqn. 1,

$$f = \frac{2gDS_f}{V^2}$$

and using Eqn. 3 above gives

$$f = \frac{12.1D^5S_f}{Q^2} \tag{5}$$

Substituting Eqns. 4 and 5 in Eqn. 2 yields

$$Q = -6.957D^{5/2}S_f\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{0.567\nu}{\sqrt{D^3S_f}}\right) \tag{6}$$

Obviously, determining head loss h with the above formula is cumbersome and thus numerous simpler, less accurate formulas, in some cases with limited range of applicability, have been developed as follows.

2.2 Ranga Raju and Garde's method

Ranga Raju and Garde [Ref. 7] proposed equations for head loss, which can be rewritten as follows:

$$\frac{\varepsilon^3 g S_f}{\nu^2} = 0.1043\left[\frac{Q}{\nu\varepsilon}\right]^{1.852}\left[\frac{\varepsilon}{D}\right]^{2.633}$$ for

$(Q/\nu\varepsilon)(\varepsilon/D)^{2.633} \leq 0.282$ and

$$\frac{\varepsilon^3 g S_f}{\nu^2} = 0.1241\left[\frac{Q}{\nu\varepsilon}\right]^{1.992}\left[\frac{\varepsilon}{D}\right]^{2.633}$$ for

$(Q/\nu\varepsilon)(\varepsilon/D)^{2.633} > 0.282$

2.3 Manning's formula

$$V = \frac{1}{n}R^{2/3}S_f^{\ 1/2}$$

where n is Manning's roughness coefficient [Ref.2]. For pipes flowing full, the above yields

$$S_f = \frac{10.29Q^2n^2}{D^{5.333}}$$

2.4 Hazen-Williams formula

This is widely used in water supply industry.

$$V = 0.85C_{HW}R^{0.63}S_f^{\ 0.54}$$

where C_{HW} is the Hazen-Williams coefficient [Ref. 2]. For pipes flowing full, the above yields

$$S_f = \left[\frac{3.587Q}{D^{2.63}C_{HW}}\right]^{1.852}$$

2.5 Proposed method

Obviously, use of Equation 6 to determine head loss would involve iteration. The reason for this is basically because Equation 2, upon which Eqn. 6 is based, is implicit in f. Experience in use of Equation 2 suggests that the f on the left hand side of the equation is not very sensitive to the f on the right hand side. This suggests that reasonably accurate friction factor can be obtained by assuming a constant value for the f on the right hand side (say f_o). The best value of f_o over the entire range of flow of practical interest will be determined in this study. This implies

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{2.51}{R\sqrt{f_o}}\right) \qquad (7)$$

Substituting Eqns 4 and 5 in Eqn. 7 and rearranging, yields

$$S_f = \left(\frac{-0.1437Q}{D^{5/2}\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{1.972\nu D}{Q\sqrt{f_o}}\right)}\right)^2 \qquad (8)$$

3. TEST PROBLEMS

The various friction loss relationships above are so different and involve so many variables that a basis for comparison is difficult to obtain. Aiyesimoju [Ref. 1] has however designed eight test problems which cover up to the extreme range of practical flow situation. These are applicable here and will be adopted. They can be characterized as follows.

1) low κ, low R, large D
2) low κ, large R, large D
3) large κ, low R, large D
4) large κ, large R, large D
5) low κ, low R, small D
6) low κ, large R, small D
7) large κ, low R, small D
8) large κ, large R, small D

The problem in each case is to determine the head loss for the different extreme flow conditions [Ref. 1] listed in Table 1. The performance of a method in a test problem will be measured by the factor (relative to Colebrook-White method) within which it is able to estimate the head loss for that test problem.

4. DISCUSSION AND CONCLUSION

The results for all the test cases are summarized in Table 1. These show clearly that Ranga Raju's method predicted head loss within a factor of accuracy ranging from 1.05 to 8.34, For Hazen-William's formula, this was from 1.01 to 2.08, for Manning's formula, from 1.01 to 4.04, and for the proposed method from 1.00 to 1.09. A kinematic viscosity coefficient of 3E-7 is shown in the table but increasing this value up to the maximum of 2E-6 had no noticeable effect on the results.

The closer the factor of accuracy is to unity the better the prediction, which indicates clearly that the proposed method is superior to the other methods. In the proposed method, f_o corresponds to a value of 0.031 which gave the least value of the worst factor of accuracy (of 1.09) over all the eight test problems. This results in a final recommendation for the proposed method as follows:

$$\frac{h}{L} = \frac{1}{48.4D^5}\left(\frac{Q}{\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{11.2\nu D}{Q}\right)}\right)^2 \qquad (9)$$

This formula yields head loss with less than 10% error in the extreme cases but it should be noted that only the extreme cases have been presented. It in fact achieves an error of less than 2% error in most practical cases. This performance is quite acceptable even in the extreme cases.

5. REFERENCES

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6. NOTATION

C_{HW}	=	Hazen-Williams coefficient
D	=	pipe diameter
f	=	friction factor
g	=	acceleration due to gravity
h	=	head loss
n	=	Manning's roughness coefficient
Q	=	pipe discharge
R	=	hydraulic radius
R	=	Reynold's Number
S_f	=	friction slope
V	=	flow velocity
ε	=	pipe equivalent roughness height
κ	=	pipe relative roughness
ν	=	fluid kinematic viscosity

TABLE 1: Test Cases and the computed head losses and Factors of Accuracy for different methods

Test Case No.	Rough. Height (m)	Pipe Dia. (m)	Relative Rough.	Reynold's Number	Kinematic Viscosity	Discharge (m^3/s)	Colebrook-White		Ranga Raju		Manning's			Hazen-William's			Proposed	
							Friction factor	Friction Slope	Friction Slope	Factor of Accuracy	Mann. Rough Coeff.	Friction Slope	Factor of Accuracy	Haz. Will. Coeff.	Friction Slope	Factor of Accuracy	Friction Slope	Factor of Accuracy
	ϵ	D	$K = \epsilon/D$	R	υ	Q	f	S_f	S_f		n	S_f		C_{HW}	S_f		S_f	
1	1.E-06	2.00	5.00E-07	3.98E+03	3.E-07	1.9E-03	4.0E-02	3.63E-10	2.5E-10	1.45	0.01	9.0E-11	4.04	143	3.3E-10	1.10	3.8E-10	1.05
2	1.E-06	2.00	5.00E-07	1.00E+06	3.E-07	4.7E-01	1.2E-02	6.69E-06	7.0E-06	1.05	0.01	5.7E-06	1.18	143	9.2E-06	1.38	6.1E-06	1.09
3	1.E-02	2.00	5.00E-03	3.98E+03	3.E-07	1.9E-03	4.5E-02	4.07E-10	2.4E-09	5.93	0.019	3.2E-10	1.25	86	8.5E-10	2.08	4.3E-10	1.05
4	1.E-02	2.00	5.00E-03	1.00E+06	3.E-07	4.7E-01	3.0E-02	1.75E-05	1.5E-04	8.34	0.019	2.0E-05	1.17	86	2.4E-05	1.35	1.7E-05	1.00
5	1.E-06	0.02	5.00E-05	3.98E+03	3.E-07	1.9E-05	4.0E-02	3.64E-04	2.8E-04	1.30	0.01	4.2E-04	1.14	143	3.6E-04	1.01	3.8E-04	1.05
6	1.E-06	0.02	5.00E-05	1.00E+06	3.E-07	4.7E-03	1.3E-02	7.26E+00	7.8E+00	1.08	0.01	2.6E+01	3.62	143	1.0E+01	1.38	6.9E+00	1.05
7	1.E-02	0.30	3.33E-02	3.98E+03	3.E-07	2.8E-04	6.6E-02	1.79E-07	5.7E-07	3.21	0.019	1.8E-07	1.01	86	2.6E-07	1.46	1.9E-07	1.04
8	1.E-02	0.30	3.33E-02	1.00E+06	3.E-07	7.1E-02	6.0E-02	1.02E-02	3.5E-02	3.41	0.019	1.1E-02	1.12	86	7.3E-03	1.40	1.0E-02	1.00