CREEP RESPONSE ANALYSIS OF QUASI - ISOTROPIC BAMBOO FIBRE REINFORCED COMPOSITE STRUCTURES

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1.0 ABSTRACT

This study used experimental method and analytical tools of engineering as mixed method to model Bamboo fibre reinforced polyester resin matrix composites under creep failure mode. Multiple linear regression and the general Power law equations were employed to model the bamboo fibre composites creep limit responses, while the creep strains and moduli were modeled graphically from where interpolations can be made. The creep limit estimated with analytical tools correlated with the experimental data .The creep modulus was found to vary nonlinearly with temperature and time applied, while the creep limit was found to be time and temperature dependent. The established mechanical properties of creep limit, creep modulus, and strain, for Bamboo fibre composites at temperatures 30^0C ,50^0C and 100^0C were evaluated and coordinated as (8.6775 Mpa, 3.7728 GPa, 0.0023), (8.6667 MPa, 8.6667 GPa, 0.001) and (8.6645 MPa, 14.439 GPa, 0.0006) respectively. The after-effects of increasing temperatures on creep properties of Bamboo fibre composites include reduction of strength, stiffness and toughness of composites as well as increased initial modulus of composites following creep modulus reduction.

2.0 NOMENCLATURE

Symbol	Definition	Symbol	Definition
A	Material constant	σ	Creep stress
A	Constant for stress	m	Material Constant independent of stress
A_i	Modifying constant for stress	E_i	Residual
N	Material constant	S_r	Sum of Squares of residuals
t_c	Creep time	Pm(x)	Dependent parameter (Variable)
k	Material constant	x1i	Independent polynomial variable
β	Material constant depending	x2i	Second Independent variable

y	Material constant depending on stress and temperature (material constants)
ε	Transient Creep strain
ε_0	Initial creep strain
ε^1	Minimum creep rate
T	Temperature
nB 30	strain at temperature 30°C
nB 50	strain at temperature 50°C
nB 100	strain at temperature 100°C
σ 30	stress at temperature 30°C
σ 50	stress at temperature 50°C
a_1	Linear polynomial coefficient
a_2	Quadratic polynomial coefficient
n	Number of points or coordinates during experimental data
σ 100	stress at temperature 100°C
B30	modulus at temperature 30°C
B50	modulus at temperature 50°C
B100	modulus at temperature 100°C

3.0 INTRODUCTION

On account of the fast development of machine building, increasingly vital importance is being attached to strength analysis of machine parts working for long periods at high temperatures. Such parts include discs and blades of steam and gas turbines, pipes and other elements of steam generators, various parts of internal combustion engines, jet engines, chemical plants, etc.The behavior of the materials of such parts is affected by the absolute temperature as well as the duration for which the parts work at the high temperatures. Viscoelastic materials such as polymers/plastics show a time dependent response to applied stress. Because of involvement of plastics in most recent designs such as in multi-layer moldings, Design of snap Fits, Design of Ribbed sections and in design of light weight structures, in every day use, the creep limit of plastics need to be established. One of the major objectives of this paper is to establish whether some of the models established for metal creeps can be applied for plastics creep. Study also aims at obtaining the creep limit for bamboo fibre reinforced polyester matrix composites. Black and Adams [1] reported the influence of high temperature on tensile strength; yield strength and elongation of materials. There is a general loss of strength at high temperatures.

The maximum temperature in steam and gas turbine castings is limited to about 1000°F, although experimental value is in the range of 1400°F to 1600°F.

In addition to loss of strength at high temperature, steel and other metals exhibit the phenomenon of creep, which is the gradual elongation of the entire member at high temperatures over a long period of time. High temperature and high stresses increase the creep rate so that at high temperatures the part will not elongate beyond a certain amount over a designed life of the machine or equipment. Shigley and Mischke [2] reported that a part may fail with a load that induced stresses in it that lie between the yield strength and the tensile strength of the material even if the load is steady and constant rather than alternating and repeating as in fatigue failure. [4, 5] and Classical reports show that creep can occur at low temperature in aluminum and polymeric.

Though classical studies abound for metal creep, not enough work has been done in modeling viscoelastic behaviour of plastics especially composites made of local fibre. This study used experimental method and analytical tools of engineering in conjunction with parametric methods. Multiple linear regression and the general Power law model were employed to model the bamboo fibre composites creep limit response, while the creep strains and moduli were modeled graphically.

4.0 THEORETICAL REVIEW AND ANALYSIS

The properties of materials change considerably at high temperatures, therefore the known properties of strength and ductility at normal (room) temperature cannot serve for the analysis of elements of the same materialss working at high temperatures. The variation of strength and plastic properties with the increase in temperature is of vital importance in the design of element of machines and structures. Craw ford [4] reported the following Sternet creep-strain equations for plastics:

$$\varepsilon = a + bt^c \text{ (parabolic)} \quad (1)$$

$$\varepsilon = a + bt^{1/3} + ct \text{ (power)} \quad (2)$$

$$\varepsilon = at/1+bt \text{ (Hyperbolic)} \quad (3)$$

$$\varepsilon = 1/1+at^b \text{ (Hyperbolic)} \quad (4)$$

Creep curves are plotted as log (strain) against log (time). This is convenient because quite often this results in straight line plots suggesting that the creep behaviour can be described by an equation of the type

$$\varepsilon(t) = At^n \quad (5)$$

Wherea = Material constant

A = Constant which depends on stress level. To be strictly accurate, the right hand side of Eq (5) should be of form($A_o + At^n$) so that

$$\varepsilon(t) = A_o + At^n \quad (6)$$

to allow for the instantaneous strain at zero time. But for long creep times, sufficient accuracy can be obtained by ignoring A_o

Andrade in 1910 was the first to establish a relationship between creep strain and time to describe primary and secondary creep, Benham and War nock [5] in the form

$$\varepsilon = kt^{1/3} + \beta t \quad (7),$$

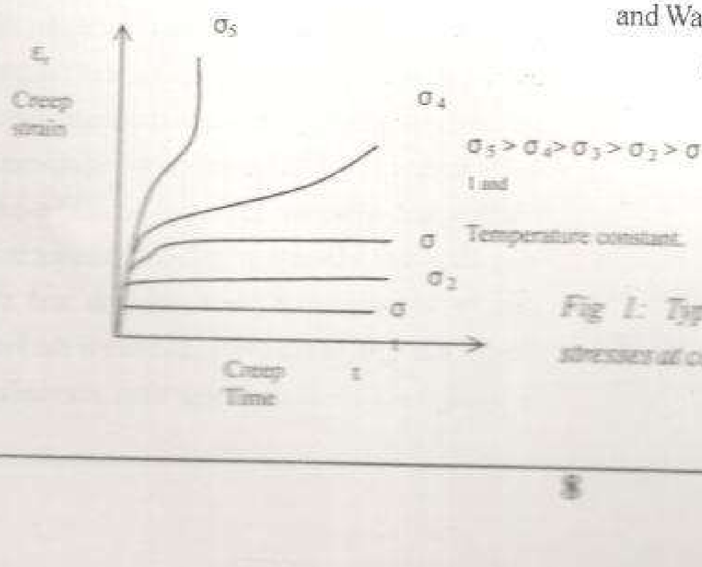


Fig 1: Typical creep curves for various stresses at constant temperature

β= material constants which are functions of stress and temperature

Graham and Walles extended the analysis to cover the tertiary stage by proposing a relationship of the form

$$\varepsilon = kt^{1/3} + \beta t + yt^3 \quad (8)$$

Where y = material constants also dependent on stress and temperature.

A family of curves usually obtained by considering stress and temperature as parameters for creep analysis is presented in Fig 1 showing increasing deformation rate at increasing stress. The straight line portion of creep curve decreases as temperature or stress increases.

4.1 Estimation Of Creep Time

When the tertiary term of equation (8) is neglected and the primary is replaced by constant ε_0 being the intercept of the extrapolated secondary stage on the strain axis of fig 1,

$$\varepsilon_t = \varepsilon_0 + \varepsilon^1 \quad (9)$$

$\varepsilon^1 = d\,\varepsilon/dt$, the minimum creep rate. It has been found experimentally that the minimum creep rate and stress are related as

$$\varepsilon^1 = A^m \quad (10)$$

Hence a log-log plot of ε^1 against σ yields a straight line (power law).

The constants k and n can be determined by linear transformation using power law method and time to reach a specified value of total creep strain in the secondary stage evaluated with

$$t = (\varepsilon_t\ \varepsilon_0)/\ k\sigma^m \quad (11)$$

Where t = tim,k

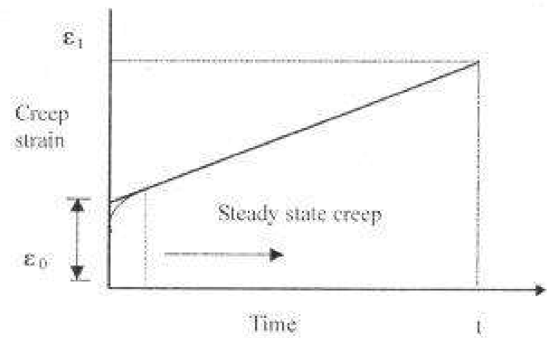


Fig 2: Simplified creep curve

4.2 Analytical/computational Method

The time stress developed during creeping of material depends greatly on temperature and time. A correlation between true stress, temperature and time will describe the time dependent response of polymeric material. Creep occurs at stress level within the elastic limit of material, so a tensile test to establish elastic limit of material is necessary. A creep test is conducted and true stress vs time data is obtained. A plot of stress time data is necessary to establish the material response. A nonlinear relationship is usually exhibited by creep stress-time graphics so a power law model and nonlinear regression and Gauss-Newton iterative models can be fit to creep data.

By subjecting the test material to three different temperatures a system of logarithmic transformed linear equations are obtained. Master curve is established for the prediction of material constants that are temperature dependent or line of best fit method can be used to predict the material constants and predictions made with the model established.

4.3 Multiple Linear Regression Model

For modeling of nonlinear responses, Canale[8] and Zill and Cullen[9] expressed as follows :

$$P_m(x) = a_0 + a_1 x_{1i} + a_2 x_2 \qquad (12)$$

Following minimization of sum of squares of residuals as applied in polynomial regression.

$$E_i = p_m(x) - y_i = \text{Residual} \qquad (13)$$

$$S_r = \Sigma (a_0 + a_1 x_{1i} + a_2 x_{2i}) - y_i \qquad (14)$$

By differentiating with respect to polynomial coefficients, a_0, a_1 and a_2

$$\frac{\partial s_r}{\partial a_0} = 2 \Sigma (a_0 + a_1 x_1 i + a_2 x_{2i}) - y_i \qquad (15)$$

$$\frac{\partial s_r}{\partial a_1} = 2 \Sigma x_{1i} (a_0 + a_1 x_1 i + a_2 x_{2i}) - y_i \qquad (16) \qquad \partial a_1$$

$$\frac{\partial s_r}{\partial a_2} = 2 \Sigma x_{2i} (a_0 + a_1 x_1{}^i + a_2 x_{2i}) - y_i \qquad (17)$$

By setting Eq 15 Eq 17 to zero, the following system of linear equations is obtained.

$$n a_0 + a_1 \Sigma x_{1i} + a_2 \Sigma x_{2i} = \Sigma y_i \qquad (18)$$

$$a_0 x_{1i} + a_1 \Sigma x_{1i}{}^2 + a_2 \Sigma x_{1i} x_{2i} = \Sigma x_{1i} y_i \qquad (19)$$

$$a_0 ?x_{2i} + a_1 \Sigma x_{2i} + a_2 {}_{\Sigma} x1ix_{2i} = \Sigma x_{2i} y_i \qquad (20)$$

Putting Eq (18 20) in matrix form

$$\begin{array}{ccc} n & \Sigma x1i & \Sigma x_2 i \\ \end{array} \quad a_0 \quad \Sigma yi$$

$$\begin{pmatrix} \Sigma x1i & \Sigma x1i^2 & \Sigma x_1 ix_2 i \\ \Sigma x2i\Sigma & x1ix2i & \Sigma x_2 i^2 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \begin{pmatrix} \Sigma x1t_i yi \\ \Sigma x1t_i yi \end{pmatrix} \qquad (21)$$

Where x_1 = temperature, x_2 = time, t

This system can be solved by any direct methods of solving system of linear equations such as Crammer's Rule or Gauss - Jordan elimination and LU decomposition as found in [8,9]

The parameters, a_0 and a_1 a_2 are material constants that depends greatly on temperature and structure of material.

4.4 The general power law-multiple regression model.

For the modeling of nonlinear responses, [8] expressed:

$$y = a_0 x_{1i}^{a1} x_{2i}^{a2} x_{3i}^{a3} \ldots\ldots\ldots x_{mi}^{am} \tag{22}$$

By linear transformation of (22)

$$\log y = \log a_0 + a_1 \log x_1 i + a_2 \log x_2 i \ldots\ldots a_m \log x_m i \tag{23}$$

By comparing (12) and (23)

$$\log a_0 = a_0, \log x_1 i = x_1 i, \log x_2 i = x_2 i \tag{24}$$

so that the constants a_0 and a_1 a_2 can be solved with matrix

$$\begin{pmatrix} n & \Sigma \log x_1 i & \Sigma \log x_2 i \\ \Sigma \log x_1 i & \Sigma(\log x_1 i)^2 & \Sigma \log x_1 i \log x_2 i \\ \Sigma \log x_2 i & \Sigma \log x_1 i \log x_2 i & \Sigma(\log x_2 i)^2 \end{pmatrix} \begin{bmatrix} \log a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{pmatrix} \Sigma \text{Log} yi \\ \Sigma \log x_1 i \text{Log} yi \\ \Sigma \log x_2 i \text{Log} yi \end{pmatrix} \tag{25}$$

Where x_1 - temperature, x_2 - time, t

This system can be solved by any direct methods of solving system of linear equations such as Crammer's Rule or Gauss - Jordan elimination and LU decomposition as found in [8,9]

The parameters, a_0 and a_1 a_2 are material constants that depends greatly on temperature and structure of material.

5.0 METHODOLOGY

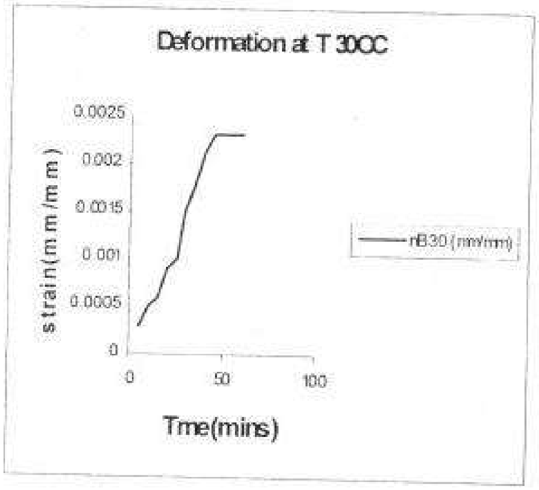
The mixed-methodology of this work consists of three parts, Hand-Lay up method to form Raffia fibre composites Testing or Experimentation to obtain creep failure data and Analytical or computational method to model and analyse creep data.

5.1 Hand-lay Up Method

Raffia fibre composites are formed. The full details of the processes involved are found in [4, 7]. Replicated samples of composites are formed for creep tests following standard procedures.

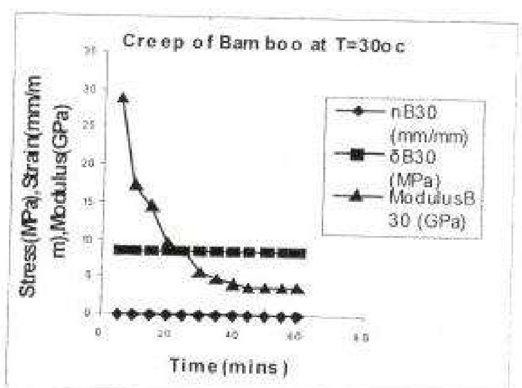
5.2 Experimentation And Method

Creep testing equipment was used on replicated samples of Raffia composite considering temperatures 30°c, 50°c and 100°c and deformation response data recorded as presented in table 1. The full details of the equipment used are found in Anambra State University mechanical engineering laboratory (Equipment is fabricated to suit testing of composite rectangular samples).The equipment is a modification of TECHEQUIPMENT creep equipment for room conditions



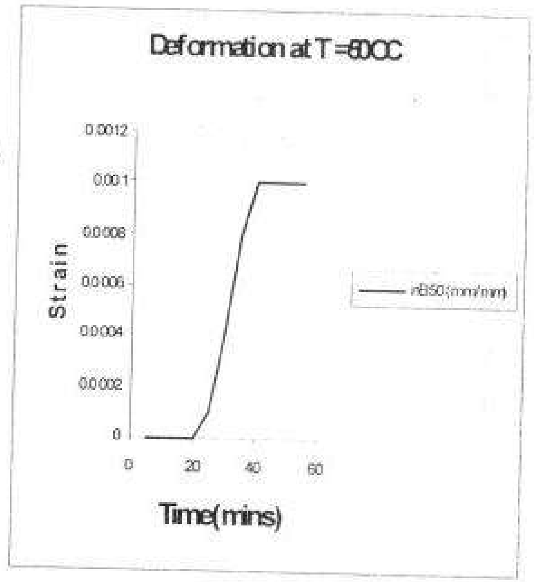
5..3 Computational Modeling Of Creep Data

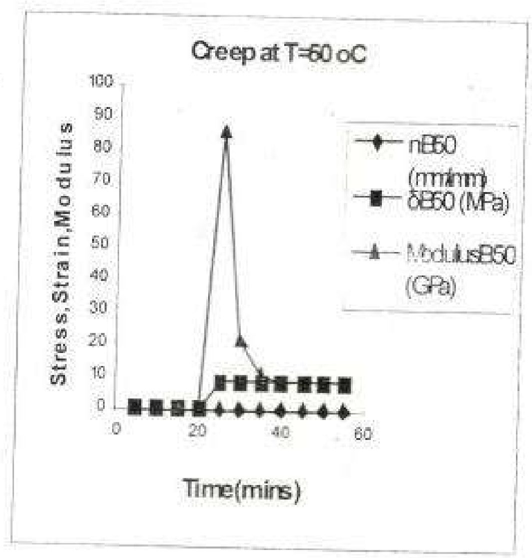
Creep limit, modulus and strain are time dependent responses and are functions of two variables, temperature and time. This work considered three temperature levels, 30°C, 50°C and 100°C. Table 3.1 shows that at 30°C, the creep limit is 8.6675 MPa at time 45 mins and at 50°C, creep limit is 8.6667 MPa at time 40 mins and that at 100°C creep limit is 8.6645MPa at time 20mins.These information are correlated with multiple linear regression and general power law model.



Excel Graphics of Results at various Temperatures

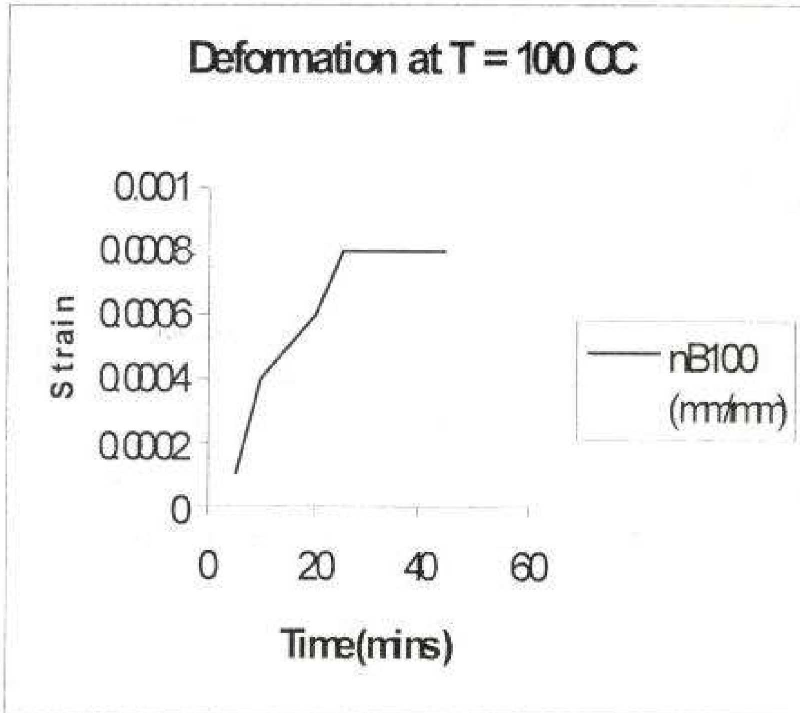
Experimental results of table 1 are fed into excel package to obtain graphics responses of creep at various temperatures as shown below.





ponses at T= 30°C

Fig. 4: Strain and Stress responses at T= 50°C



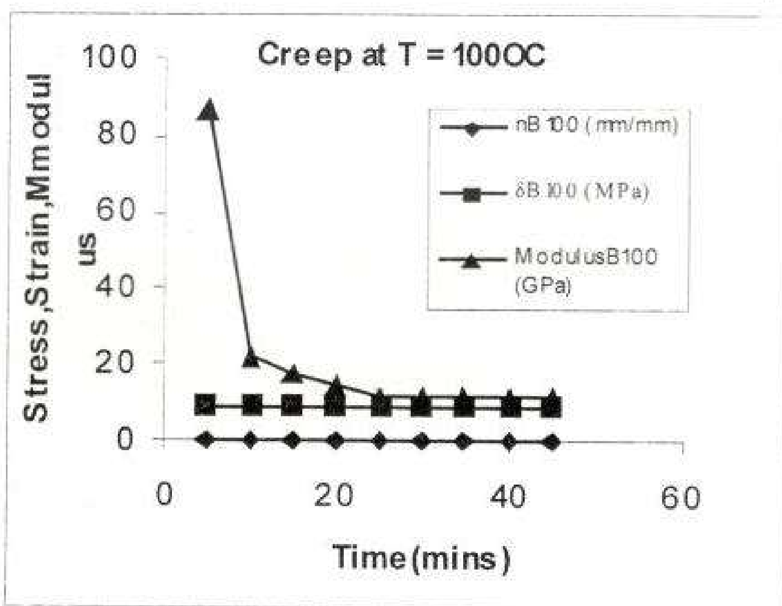


Fig. 5 a ,b: Strain and Stress responses at T= 100°C

Table 2: Limit stress responses at Elevated temperatures

$X1^{O}C$	X2(mins)	y (MPa)	M(MPa)	N(mm/mm)
30	45	8.6775	3.7728	0.0023
50	40	8.6667	8.6667	0.001
100	20	8.6645	14.439	0.0006

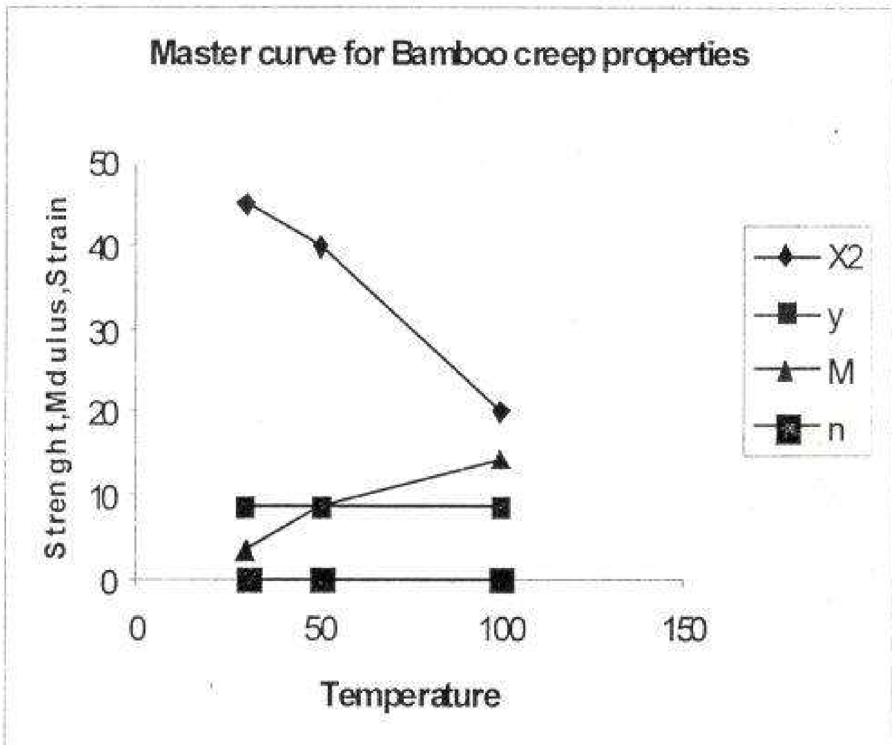


Fig. 6 :Master curve for Bamboo creep properties

? **Multiple linear Regression Modeling**

The data of table 2 is presented as table 3 following Eq. (21)

	x1	x2	y	x1^2	x2^2	x1x2	x1y	x2y
	30	45	8.6775	900	2025	1350	260.325	390.4875
	50	40	8.6667	2500	1600	2000	433.335	346.668
	100	20	8.6645	10000	400	2000	866.45	173.29
SUM	180	105	26.0087	13400	4025	5350	1560.11	910.4455

When appropriate values of table 3 were used in Eq

(21) a system of 3x3 matrix equationis obtained as

When appropriate values of table 3 were used in Eq (21) a system of 3x3 matrix equationis obtained as

$$\begin{pmatrix} 3 & 180 & 105 \\ 180 & 13400 & 5350 \\ 105 & 5350 & 4025 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 26.0087 \\ 1560.1100 \\ 910.4455 \end{pmatrix}$$

The variable parameters a_0, a_1, a_2 were obtained by direct method of solving system of linear equations of Crout method (LU - decomposition) as

$a_0 = 8.8673$, $a_1 = -0.00136$, $a_2 = -0.003306$

These parameters were substituted in Equation (1) to obtain multiple linear regression creep model as $y = 8.8673 - 0.00136x_1 - 0.003306x_2$ (26)

- Power law Modeling

When appropriate values of table 4 were used in Eq (25) a system of 3x3 matrix equation was obtained as.

$$\begin{pmatrix} 3.0000 & 5.1761 & 5.5563 \\ 5.1761 & 9.0684 & 7.7659 \\ 4.5563 & 7.7659 & 6.9925 \end{pmatrix} \begin{pmatrix} \log a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 2.8140 \\ 4.8550 \\ 4.2739 \end{pmatrix}$$ Similarly, this equation is s

Similarly this equation is solved by LU decomposition obtaining

$\text{Log } a_0 = 0.9552967$,

$a_0 = \text{Antilog } 0.9552967 = 10^{0.9552967} = 9.0219$

$a_1 = -0.112965, a_2 = -0.1753288$

When these constant coefficients were substituted in Eq (12) the following power law model is obtained

$$Y = 9.0219 x_1^{0.112965} x_2^{0.17533} \quad (27)$$

Discussion of results

The graphics of fig 3, 4, 5 shows that creep of Bamboo fibre composite is time and temperature dependent and creep stress and creep toughness decreases with increasing time and temperature.

Table 1and 2 gave the creep limit, y, creep modulus, m and strain, n for Bamboo fibre composite at temperatures 30°C, 50°C and 100°C as (8.6775 MPa, 3.7728 GPa, 0.0023), (8.6667 MPa, 8.6667 GPa, 0.001) and (8.6645 MPa, 14.439 GPa, 0.0006) respectively. Creep modulus was found to increase with temperature but at reduced time because modulus is estimated as stress divided by strain which decrease with time as increased deformation is favored by high temperature and elapsed time.

Power law creep equations and multiple linear regression equations were established for Bamboo fibre composites for temperature range of 30°C - 100°C. The predictions of these equations correlated well with experimental results.

Master curve was established as fig 6 for prediction of creep properties of Bamboo fibre composites of polyester resin composites working within temperature range of 30°C- 100 °C

The low creep properties of Bamboo fibre composites may be attributed to the low aspect ratio due to fibre interfacial debonding, existence of micro cracks and composite manufacturing method of hand lay up [10, 11, 12, 13, and 14] as well as the sensitivity of available creep equipment used. Ihueze [7] reported that composites have random martial properties that may result from laying methods or due to porosities in composites laminates, non- uniformity in fibre size, all these

these contribute to low strengths of local fibre reinforced composites. The strength of the matrix material does not determine the strength of the composites because in composites the fibre or the reinforcement carries the load. This is supported by the fact that though koshal [15] and Hamrock etal [16] gave the tensile strength of polyester as 40 90 MPa, the creep strength of Quasi-Isotropic Bamboo fibre composites is not up to 10 Mpa.

6.0 CONCLUSION

Bamboo fibre composites properties can be used for economic design and construction of light weight structures such as automobile bodies serving under prevailing environmental conditions of temperature that is not steady, preferably at a range 30°C - 100°C. However the properties can be optimized to widen the design application of Bamboo fibre composites by a treatment that will improve the aspect ratio of Bamboo fibre. Use of uniform size fibres will also improve the mechanical properties as well as the creep properties.

The after-effects of increasing temperatures on creep properties of Bamboo fibre composites include reduction of strength, stiffness and toughness of composites as well as reduced modulus of composites.

7. REFERENCES

1 Black,P.H and Adams,O.E ,Machine Design, McGraw- Hill,Inc.Tokyo, 1981

2 Joseph E. Shigley and Charles R. Mischke, Mechanical Desi, 1989

3 Ihueze, C.C., Hygrothermal Behavior of Plant Fibre Reinforced Composites, Nigerian Journal of Technology(NIJOTECH), Faculty of Engineering University of Nigeria , 2007 (Awaiting publication)

4 Crawford, R.J. ,Plastics Engineering, 3red,BUTTERWORTH. HEINMANN, Oxford, 1998

5 Benham, P.P. and Warnock, F.V, Mechanics of Solids and Structures, pitman books, Toronto.1981.

6 Muben,Abdul. , Machine Design , 4ed ,Khanna Publishers New Delhi India, 2003

7 Ihueze, C.C., Optimum Buckling Response Model of GRP Composites, Ph.D Thesis, University of Nigeria, 2005.

8 Canale, R.P and Chapra, S.C., Numerical Methods for Engineers, McGraw-Hill Publishers, 3rd edition, Boston, N.Y.. 1998.

9 Zill, Denis G. and Cullen. Michael R., Advanced Engineering Mathematics.

Jones and Bartlett Publishers, Sudbury, Massachusetts, Boston, 1996.

10 Mikell P. Grover ,Fundamentals of model manufacturing Engineering, 3rd and John willy and sons, Inc. USA. 2007

11 Gillat, O.P, and Broutman, L.J, ,Moisture in Epoxy Resin composites, journal Adhesion 12:153 1981.

12 Osswald, T.A., Fundamental principles of polymer composites: processing and design. In proc. 5th international conference on wood fibre-plastic composite; may 26-27, 1999, Madison., 1999 page 475.

13 Kun z, S.M, chia, k, Mc Murtry, C.H, and Boecker. ,Composite Applications Society of Manufacture Engineers, Dearborn, Michigan, 1989. P.38.4

14. Zhu, Y.T, Valdez, T.A, Beyerlein, I.J, stout, M.G, zhotu, S.J, Liu, c, Bult, D.P, and Dowe, T.C (in press)., Mechanical properties of Bone-shapped-short Fibre Reinforced Composites Materials Science and Technology Division, MSG 755, Los Alamos National Laboratory Los Alamos, NM 87545 Acta Materialia (in press), LAUR N98-5326

15 Hamrock, B. J, Jacobson, B.O and Schmid. S R. , Fundamentals of Machine Elements. WCB/Mc Graw-Hill, Boston , 1999

16 koshal, D. , Manufacturing Engineers Reference Book.utter Worth Heinemann, Oxford, 1993.