Neural Network Approach to Economic Dispatch: Case Study: Nigerian Thermal Stations

By

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ABSTRACT

This paper presents an application of a new Hopfield model with simulated annealing in solving the economic dispatch problem of power systems. Factors such as power mismatch, total fuel cost and the transmission line losses are used in defining the energy function to solve the economic dispatch problem using the Hopfield model. Each term of the energy function is multiplied by a weighting factor which can either be logically selected or directly estimated according to the specified power mismatch. A linear input-output model that takes into account the transmission losses was used here instead of the usual sigmoidal neuron model. It is observed that economic scheduling is an optimization problem. In Hopfield network approach the objective function of the economic dispatch problem is transformed into a Hopfield energy function, which is then minimized through iterations. Computational results based on the proposed method are compared with well-known classical solution approach and found satisfactory. The result reveals that the new algorithm is relatively simple and fast in respect of computational requirement when compared with conventional.

Keywords: *Cost of thermal generation, optimal generation, modeling of Hopfield Neural Network, Lambda-iteration method.*

1.0 INTRODUCTION

Utilization of energy has played a vital role in the development of civilization, and it is an index for Industrial and all round development of any country.

Over the last few decades, human population and energy consumption has been on the increase. To this extent the size of electric power systems are increasing rapidly, to meet the energy requirement. Consequently, a number of power plants are connected in parallel to supply the system load. With this development, it becomes necessary to operate the system most economically, by scheduling generators with the aim to guarantee at all times the optimum combination of generators connected to the system to supply the load demand. The problem of economic scheduling is divided into two steps, namely, unit commitment and on-line economic dispatch. The unit commitment is the selection of unit that will supply the anticipated load of the system over a required period of time at minimum cost as well as provide a specified margin of the operating reserve (Spinning Reserve). The function of on-line economic dispatch is to distribute the load among the generating units actually paralleled with the system in such a manner as to minimize the total cost of supplying the minute requirement of the system.

It is observed that economic scheduling is an optimization problem. Traditionally, to solve the problem, a lagrangian-augmented function is first formulated [1], and the optimal conditions are obtained by partial derivation of this function, calculation of the penalty factor as well as incremental loss is imperative in the solution algorithm. In the past, the incremental loss was determined by -coefficient method [2], which states that the transmission losses can be expressed in quadratic forms of the generation powers, and solved by employing the -iteration method [5]. It has also been solved using Dynamic programming, Lagrange classical method, Branch and Bound method. With recent development in optimization algorithm, the problem is amenable also to Genetic algorithm, Fuzzy logic and Neural Network [6], [7]. In Hopfield neural network approach the objective function of the economic dispatch problem is transformed into a Hopfield energy function, which is then minimized through iterations. The energy function is composed of power mismatch, fuel cost and transmission line losses. The optimal generations are the ones, which result in the minimum energy function value.

Hopfield network follows a gradient decent rule. Once it reaches a local minimum, it is stuck there. To find the global minimum of a function using only local information, some randomness must be added to the gradient decent rule to increase the chance of hitting the global minimum; hence, Simulated Annealing was employed. Simulated annealing, [9] is a method that introduces randomness to allow the system to jump out of a local minimum. The method draws an analogy between optimization of a cost function in a large number of variables with minimization of an energy function in statistical mechanics.

The primary aim of this project is to proffer an algorithm for economic scheduling of N.E.P.A thermal systems, using Hopfield neural network.
The project is limited to economic dispatch of N.E.P.A. thermal stations in Nigeria. It also excludes all private and independent electric power supply.

2.1 COST OF THERMAL GENERATION

The input-output curve of a generating unit specifies the input energy rate $F_i(P_{Gi})$ or cost of the fuel used per hour $C_i(P_{Gi})$ as a function of generator power output P_{Gi}. It is usually assumed that the fuel cost is the major cost factor of operating a thermal plant.

$P_{\min}(MW)$ is the minimum loading limit below which it is uneconomical to operate and $P_{\max}(MW)$ is the output maximum limit. The output-Input curve can be obtained from the heat-rate curve as

$$F_i(P_{Gi}) = P_{Gi} \times H_i(P_{Gi}) \quad \left(MKJ/_h\right)$$

Where $H_i(P_{Gi})$ is the heat-rate in MKJ/h. The graph of $F_i(P_{Gi})$ is the input-output curve Figure 2-1. If the cost of the fuel is $k\left(Naira/_{MKJ}\right)$. Then the input fuel-cost $C_i(P_{Gi})$ is

$$C_i(P_{Gi}) = k \times F_i(P_{Gi}) = k \times P_{Gi} \times H_i(P_{Gi}) \quad \left(Naira/_h\right) \qquad 2.1.2$$

The heat rate curve Fig.2.1.2 may be approximated by

$$H_i(P_{Gi}) = \left(a_i^{/}/P_{Gi}\right)_i + b_i^{/} + c_i^{/} P_{Gi} \quad \left(MKJ/_{MWh}\right) \qquad 2.1.3$$

With all coefficient positive from equation 2.1 and equation 2.3, gives a quadratic expression for input energy rate $F_i(P_{Gi})$ with positive coefficient of the form

$$F_i(P_{Gi}) = a_i^{/} + b_i^{/} P_{Gi} + c_i^{/} P_{Gi}^2 \qquad \left(MKJ/_h\right) \qquad 2.1.4$$

Also $C_i(P_{Gi})$ is quadratic i.e

$$C_i(P_{Gi}) = ka_i^{/} + kb_i^{/} P_{Gi} + kc_i^{/} P_{Gi}^2$$
$$C_i(P_{Gi}) = a_i + b_i P_{Gi} + c_i P_{Gi}^2 \qquad \left(Naira/_h\right) \qquad 2.1.5$$

The slope of the fuel-cost curve dc_i/dP_{Gi} is called incremental fuel cost (IC). If from equation 2.1.3 c_i is negligible, then the heat rate may be approximated by

$$H_i(P_{Gi}) = \left(a_i^{/}/P_{Gi}\right) + b_i \qquad \left(MKJ/_{MWh}\right) \qquad 2.1.6$$

The shape of the heat-rate curve given by equation 2.1.6 is that of rectangular hyperbola as shown in Figure 2-3. For large value of power P_{Gi}, the input energy rate $H_i(P_{Gi}) \cong b$, a constant

From equations 2.1, 2.1 and 2.6, approximation for the input energy rate $F_i(P_{Gi})$ and fuel cost $C_i(P_{Gi})$ are gotten in form of

$$F_i(P_{Gi}) = a_i^{/} + kb_i^{/} P_{Gi}$$
$$C_i(P_{Gi}) = a_i + b_i P_{Gi} \qquad \left(Naira/_h\right) \qquad 2.1.7$$

2.2 OPTIMAL GENERATION OF THERMAL GENERATION

If it is known a priori which generators are to run to meet a particular committed and active power load P_D given, the real power generation P_{Gi} for each generator has to be allocated so as to minimize the total cost.

$$C = \sum_{i=1}^{k} C_i(P_{Gi}) \qquad \left(Naira/_h\right) \qquad 221$$

subject to the inequality constraint

$$P_{Gi}(\min) \leq P_{Gi} \leq P_{Gi}(\max) \qquad 222$$
$$i = 1,2,\cdots k$$

Where $P_{Gi}(\min)$ and $P_{Gi}(\max)$ are lower and upper real power generation limits of the i[th] generator

$$\sum_{i=1}^{k} P_{Gi}(\max) \geq P_D$$

In addition, consideration of spinning reserve requires that

$$\sum_{i=1}^{k} P_{Gi}(\max) > P_D \qquad 2.2.4$$

$$\overline{C} = C - \lambda\left(\sum_{i=1}^{k} P_{Gi} - P_D\right) \qquad 2.2.6$$

where λ is the lagragian multiplier. Minimization is achieved by the condition

$$\frac{\partial \overline{c}}{\partial P_{Gi}} = 0$$

$$\Rightarrow \frac{dc_i}{dP_{Gi}} = \lambda \qquad ;i = 1,2,\cdots k \qquad 2.2.7$$

$\frac{dc_i}{dP_{Gi}}$ is the incremental cost of the i[th] generator

It is assumed that the cost C is independent of Q_{Gi} i.e the reactive power, $C_i(P_{Gi})$ is non-linear and $P_L = 0$, where P_L is the power loss then

$$\sum_{i=1}^{k} P_{Gi}(\max) = P_D$$

2.2.3

The cost of operating unit i, i.e. C_i is independent of the power generated by unit j if ($j \neq i$). Optimizing using lagrangian multiplier gives

$$\frac{dc_1}{dP_{G1}} = \frac{dc_2}{dP_{G2}} = \cdots = \frac{dc_k}{dP_{Gk}} = \lambda \qquad 2.2.8$$

From the equations above, the optimal loading of generators correspond to the equal incremental cost point of all the generators. Equation 2.2.8 is called coordination equation. When these k equations are solved with load-demand equation 2.2.5; a solution for the Lagrange multiplier λ and the optimal generation of k generators results.

Algorithm for determining λ in equation 2.2.8 is as follows

1. Choose a trial value of λ i.e IC =$(IC)^0$
2. Solve for P_{Gi} (i =1, 2 ... k) from equation 2.1.7
3. If $\left|\sum_{i=1} P_{Gi} - P_D\right| < \varepsilon$, the optimal solution is reached otherwise
4. (i) Increment IC by ΔIC if $\left|\sum_{i=1} P_{Gi} - P_D\right| < 0$ or
   (ii) Decrement IC by ΔIC if $\left|\sum_{i=1} P_{Gi} - P_D\right| > 0$ and repeat from step 2. ε is the error tolerance.

The effect of the generator limits given by the inequality constraint of equation 2.2.2 is such that, as IC is increased or decreased in the iterative process, if a particular generator loading P_{Gi} reaches the limit $P_{Gi}(max)$ or $P_{Gi}(min)$, its loading from then on is held fixed at this value and the balance load is shared between the remaining generator on equal incremental cost basis. The fact that this is optimal follows from Kuhn-Tucker theorem.

2.3 OPTIMAL GENERATION WITH LOSS CONSIDERATION

A modern electric utility serves over a vast area of relatively low load density and the transmission loss may be appreciable, hence, it is essential to account for losses while developing an economic dispatch policy. When losses are present, simple equal incremental cost criterion no longer hold. Considering a system in which all the generators at each bus are identical, equal incremental cost criterion would dictate that the total load should be shared equally by all the generators, but considering line losses, it will be cheaper to draw more power from the generators which are closer to the loads. When the distances of generating plants are different, the cost of transmission losses will affect the economic distribution.

The overall cost of generation is

$$C = \sum_{i=1}^{m} C_i(P_{Gi}) \qquad 231$$

With the constraint

$$\sum_{i=1}^{m} P_{Gi} - \sum_{i=1}^{n} P_{Di} - P_L = 0 \qquad 232$$

Where: m is the total number of generating plants

n is the total number of load buses

P_{Gi} is the generation of the i[th] plant

$P_D = \sum_{i=1}^{n} P_{Di}$ Sum of load demand at all buses (symmetric load is assumed)

P_L is the total system transmission loss

The lagrangian of cost is

$$\overline{C} = \sum_{1=1}^{m} C_i(P_{Gi}) - \lambda\left(\sum_{i=1}^{m} P_{Gi} - P_D - P_L\right) \quad \left(Naira/_h\right) \qquad 2.3.3$$

From equation 2.3.2, for a given real load P_{Di} at all buses, the system loss P_L is a function of active power generation at each plant. If the power factor of load at each bus is assumed to remain constant, the system loss P_L is

$$P_L = P_L(P_{G1}, P_{G2}, \cdots P_{Gm}) \qquad 2.3.4$$

The optimum real power dispatch is

$$\frac{\partial \overline{c}}{\partial P_{Gi}} = \frac{dc_i}{dP_{Gi}} - \lambda + \lambda\frac{\partial P_L}{\partial P_{Gi}} = 0 \qquad 2.3.5$$

$i = 1,2,\cdots m$

$$\Rightarrow \lambda = \frac{dc_i/dP_{Gi}}{\left(1 - \partial P_L/\partial P_{Gi}\right)} \qquad 2.3.6$$

$i = 1,2,\cdots m$

defining

$$L_i = \frac{1}{\left(1 - \partial P_L / \partial P_{Gi}\right)}$$

$$\lambda = IC \times L_i \qquad 2.3.7$$

L_i is called penalty factor of the i[th] plant. λ has the unit of Naira per Megawatt-hour. Equation 2.3.7 implies that minimum fuel cost is obtained, when the incremental fuel cost of each plant multiplied by its penalty factor is the same for all the plants. The (m+1) variables (P_{G1}, P_{G2}, ... P_{Gm}, λ) can be obtained from m optimal dispatch equation 2.3.7 together with power balance equation 2.3.2. The $\partial P_L / \partial P_{Gi}$ is referred to as the incremental transmission loss (ITL) associated with the i[th] generating plant.

$$IC_i = \lambda\left(1 - ITL_i\right) \qquad 2.3.8$$
$$i = 1, 2, \cdots m$$

$$P_L = \sum_{i=1}^{m} \sum_{j=1}^{m} P_i B_{ij} P_j \qquad 2.3.9$$

P_i, P_j are real power injection at i[th] and j[th] buses, B_{ij} is loss coefficient which are constant under assumed operating conditions, m is the number of generator buses.

From equation 2.1.5, the incremental cost

This equation is called exact coordination equation. To solve the optimum load-scheduling problem, it is necessary to compute ITL for each plant; therefore, the functional dependence of transmission loss on real power of generating plants must be determined. Optimum generator allocation with line losses is obtained by operating all generators such that the product, $IC_i \times L_i = \lambda$

for every generator. All generators which are not at or within their limits are operated such that $IC_i \times L_i = \lambda$ for each of them. The generators, which are at their limits are or beyond are operated at their limits.

The transmission loss in a system is expressed in terms of active power generation using B-coefficient method.

$$\frac{dc_i}{dP_{Gi}} = b_i + 2c_i P_{Gi} \quad \left(\text{Naira}/_{\text{MWh}}\right) \qquad 2.3.10$$

substituting $\frac{dc_i}{dP_{Gi}}$ and $\frac{dP_L}{dP_{Gi}}$ from equation 2.3.9 in the equation 2.3.5

$$b_i + 2c_i P_{Gi} + \lambda \sum_{j=1}^{m} 2B_{ij} P_j = \lambda$$

substituting $P_{Gi} = P_i + P_{Di}$ and collecting like terms of P_i

$$P_i = \frac{1 - \frac{b_i}{\lambda} - \sum_{\substack{j=1 \\ j \neq i}}^{m} 2B_{ij} P_j}{2\left(\frac{c_i}{\lambda} + B_{ii}\right)} \qquad 2.3.12$$

$i = 1, 2 \cdots m$

For any particular values of λ, equation 2.3.12 can be solved iteratively by assuming initial value of P_i equation 2.3.12 along with power balance equation 2.3.2 for a particular load demand P_D are solved iteratively as follows:

1. Initially choose $\lambda = \lambda_0$
2. Assume $P_i = 0$; i = 1, 2, …m
3. Solve equation 2.3.12 iteratively for P_i
4. Calculate P_L using equation 2.3.9
5. Check if Power balance equation 2.3.2 is satisfied $\left|\sum_{i=1}^{m} P_i - P_L\right| < \varepsilon$ if yes stop, otherwise step 6
6. Increase λ by $\Delta\lambda$ if $\left(\sum_{i=1}^{m} P_1 - P_L\right) < \varepsilon$ or decrease λ by $\Delta\lambda$ if $\left(\sum_{i=1}^{m} P_1 - P_L\right) > \varepsilon$

Repeat from step 3. ε is error tolerance.

2.4 HOPFIELD NEURAL NETWORK

The basic model consists of a set of processing elements that computes the weighted sum of the input and threshold the output. Associated with a model is an energy function which is partly akin to the Lyapunov function occurring in stability investigation of non-linear dynamic systems. The outputs of the model converge to a stable local energy minimum, which is referred to as equilibrium point.

$$C_i \frac{du_i}{dt} = \sum_j w_{ij} y_j - \frac{u_i}{R_i} + I_i$$

where for unit i, C_i is the membrane capacitance, R_i is the transmembrane resistance, I_i is the external input current, y_i is the output potential of a unit, u_i is the instantaneous transmembrane potential, and w_{ij} represent the connection weight conductance from each of the other units. If the activation function is defined as $y_i = g(u_i)$ of a neuron be a nonlinear and monotone increasing function. It is essential that

$$E = -\frac{1}{2}\sum_i \sum_j w_{ij} y_i y_j - \sum_i I_i y_i + \sum_i \frac{1}{R_i} \int_0 g_i^{-1}(y)dy$$

the function g_i^{-1} *is related to* g_i *in* $y_i = g(u_i)$

through the constraint equation

$$g_i^{-1}[g_i(.)] = g_i\left[g_i^{-1}(.)\right] = 1$$

The function g_i is assumed to be monotonically increasing differentiable function. This guarantees

$$\frac{dE}{dt} = -\sum_i \frac{dy_i}{dt}\left(\sum_j w_{ij} y_j - \frac{u_i}{R_i} + I_i\right)$$

2.4.1 MODELLING OF HOPFIELD NETWORK

The differential equation describing a network with graded response units are given as

2.4.1

Hopfield network converge when using it to solve an optimization problem.

Let the energy function of model in equation 2.4.1 be defined as E. E is a lyapunov-like function of the system represented by equation 2.4.1

2.4.2

the existence of the inverse function, which is also monotonically increasing. Therefore g_i^{-1} has a positive slope everywhere.

For a symmetric connection

2.4.3

Comparing 2.4.1 with 2.4.3

$$\frac{dE}{dt} = -\sum_i C_i \frac{dy_i}{dt}\frac{du_i}{dt} = -\sum_i C_i \left(g_i^{-1}(y_i)\right)' \left(\frac{dy_i}{dt}\right)^2 \qquad 2.4.4$$

Since $g_i^{-1}(y_i)$ is a monotonically increasing function, its derivative $g_i^{-1}(y_i)' > 0$. Also C_i is positive. Therefore

$\frac{dE}{dt} \le 0 \text{ and } \frac{dE}{dt} = 0 \rightarrow \frac{dy_i}{dt} = 0 \text{ for all } i$ Using the boundedness properties of E, it can be concluded that the time evolution of the network seeks out the minima of E in the state space and stops at such points.

If we assume that the transmembrane resistance R_i is extremely large, to first approximation $R_i \rightarrow \infty$ and normalizing equation 2.4.1

$$\frac{du_i}{dt} = \sum_j w_{ij} y_j + I_i \qquad 2.4.5$$

3.0 MATHEMATICAL FORMULATION

3.1 ENERGY FUNCTION FORMULATION

The aim of economic dispatch in electricity generation is as follows

(i) *Minimization of the fuel cost* $\sum_i (a_1 + b_i P_i + c_i P_i^2)$

(ii) *Minimization of power loss* P_L

(iii) *Minimization of power mismatch* $P_m = P_D + P_L - \sum P_i$

An energy function is formulated incorporating the three objectives

$$E = \left(\frac{A}{2}\right)\left[P_D + P_L - \sum P_i\right]^2 + \left(\frac{B}{2}\right)\sum_i (a_1 + b_i P_i + c_i P_i^2) + \left(\frac{C}{2}\right)P_L$$

where A, B, and C are positive weight factors, their values introduces their relative importance. 3.1.1a

The power loss P_L may be expressed as

$$P_L = P_{Lo} + \Delta P_L \cong P_{Lo} + \sum_i I_{Lio}(P_i - P_{io}) \qquad 3.1.2$$

Changing the generation output of unit i from P_{io} to P_i the transmission losses change from P_{Lo} to P_L.

ΔP_L is the linearized power loss change and I_{Lio} is the incremental loss of unit i at power generation P_{io}.

3.2 **SOLUTION TECHNIQUE**

The energy function in relation with optimal power generation, is derived by substituting equation 3.1.2 into equation 3.1.1

$$E = \left(\frac{A}{2}\right)\left[P_D + P_L - \sum_i P_i\right]^2 + \left(\frac{B}{2}\right)\sum_i (a_1 + b_i P_i + c_i P_i^2) + \left(\frac{C}{2}\right)\left(P_{Lo} + \sum_i I_{Lio}(P_i - P_{io})\right)$$

$$= \left(\frac{A}{2}\right)(P_D + P_L)^2 - A(P_D + P_L)\left(\sum_i P_i\right) + \left(\frac{A}{2}\right)\sum_i\sum_j P_i P_j + \left(\frac{B}{2}\right)\left(\sum_i a_i\right) + \left(\frac{B}{2}\right)\left(\sum_i b_i P_i\right) +$$
$$\left(\frac{B}{2}\right)\left(\sum_i c_i P_i^2\right) + \left(\frac{C}{2}\right)P_{Lo} + \left(\frac{C}{2}\right)\left(\sum_i I_{Lio} P_i\right) - \left(\frac{C}{2}\right)\left(\sum_i I_{Lio} P_{io}\right)$$

$\Rightarrow$

assuming $\Delta P_L << P_{Lo}$

$$E = \left(\frac{A}{2}\right)(P_D + P_L)^2 - \sum_i\left[A(P_D + P_{Lo}) - \frac{Bb_i}{2} - \frac{CI_{Lio}}{2}\right]P_i + \left(\frac{B}{2}\right)\left(\sum_i c_i P_i^2\right)$$
$$\left(\frac{A}{2}\right)\sum_i\sum_j P_i P_j + \left(\frac{B}{2}\right)\left(\sum_i a_i\right) + \left(\frac{C}{2}\right)P_{Lo} - \left(\frac{C}{2}\right)\left(\sum_i I_{Lio} P_{io}\right) \qquad 3.2.1$$

$$let \left(\frac{A}{2}\right)(P_D + P_L)^2 + \left(\frac{B}{2}\right)\left(\sum_i a_i\right) + \left(\frac{C}{2}\right)\left(P_{Lo} - \sum_i I_{Lio} P_{io}\right) = k$$

where k is a constant for a given set of generator and power demand

$$E = k + \left(\frac{B}{2}\right)\left(\sum_i c_i P_i^2\right) + \left(\frac{A}{2}\right)\sum_i\sum_j P_i P_j - \sum_i\left[A(P_D + P_{Lo}) - \frac{Bb_i}{2} - \frac{CI_{Lio}}{2}\right]P_i \qquad 3.2.2$$

Comparing equation 3.2.2 with equation 2.4.2

$$\left.\begin{aligned} w_{ii} &= -A - Bc_i \\ w_{ij} &= -A \\ I_i &= A(P_D + P_{Lo}) - \frac{Bb_i}{2} - \frac{CI_{Lio}}{2} \end{aligned}\right\} \qquad 3.2.3$$

Substituting equation 3.2.3 into equation 2.4.5

$$\frac{du_i}{dt} = \sum_j w_{ij} y_j + I_i$$

$$\frac{du_i}{dt} = \sum_i - Ay_i + A(P_D + P_{Lo}) - \frac{Bb_i}{2} - \frac{CI_{Lio}}{2} + w_{ii} y_i$$

$$= -(A + Bc_i)y_i + \sum_j - Ay_j + A(P_D + P_{Lo}) - \frac{Bb_i}{2} - \frac{CI_{Lio}}{2}$$

$$= A\left[P_D + P_{Lo} - \sum_j y_j\right] - Bc_i P_i - \frac{Bb_i}{2} - \frac{CI_{Lio}}{2}$$

$$\frac{du_i}{dt} = A\left[P_D + P_{Lo} - \sum_j y_j\right] - \left(\tfrac{B}{2}\right)\left[b_i + 2c_i y_i\right] - \tfrac{CI_{Lio}}{2} \qquad 3.2.4a$$

Letting the output of neuron j represent the power generation of unit j

$$\frac{du_i}{dt} = A\left[P_D + P_{Lo} - \sum_j P_j\right] - \left(\tfrac{B}{2}\right)\left[b_i + 2c_i P_i\right] - \tfrac{CI_{Lio}}{2} \qquad 3.2.4b$$

Using linear input-output model instead of the usual sigmoidal neuron model.

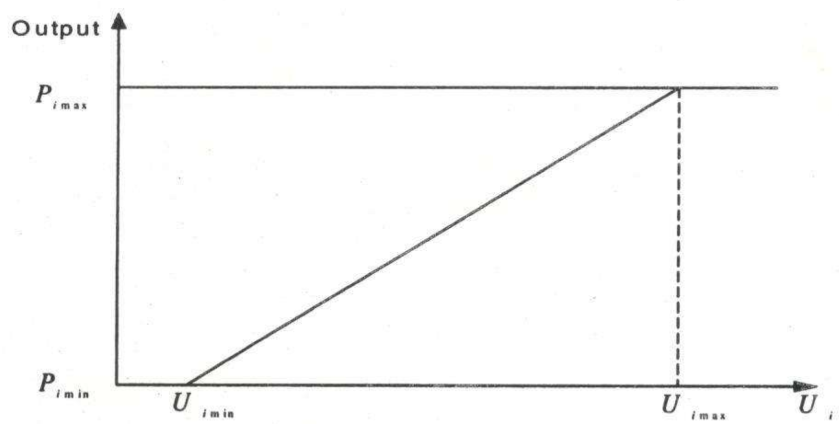

Figure 3-1: Linear input - output neuron model

$$P_i = h_i(u_i) = \left[\left(u_i - u_{i\min}\right) \Big/ \left(u_{i\max} - u_{i\min}\right) \right] \left(P_{i\max} - P_{i\min}\right) + P_{i\min}$$

$$\alpha_i u_i + \beta_i \quad \forall \quad u_{i\min} \le u_i \le u_{i\max} \qquad 3.2.5$$

where both α_i *and* β_i are constant

$$\alpha = \left(\left(u_i - u_{i\min}\right) \Big/ \left(u_{i\max} - u_{i\min}\right) \right)$$

$$\beta = -\alpha u_{i\min} + P_{i\min}$$

$$P_i = h_i(u_i) = P_{\max} \quad \forall\, u_i > u_{i\max}$$

$$and\ P_i = h_i(u_i) = P_{i\min} \quad \forall\, u_i < u_{i\max}$$

Differentiating equation 3.2.5 with respect to *t*

$$\frac{dP_i}{dt} = \alpha_i \frac{du_i}{dt} \qquad 3.2.6a$$

$$P_D + P_{Lo} - \sum_j P_j = P_m \qquad 3.2.6b$$

Substituting equation 3.2.4b into equation 3.2.6

$$\frac{dP_i}{dt} = \alpha_i \left[A\left(P_D + P_{Lo} - \sum_j P_j \right) - \left(B/2 \right)\left[b_i + 2c_i P_i \right] - CI_{Lio}/2 \right] \qquad 3.2.7a$$

$$but\ P_D + P_{Lo} - \sum_j P_j = P_m$$

Hence

$$\frac{dP_i}{dt} = \alpha_i A P_m - \alpha_i b_i B/2 - \alpha_i\, CI_{Lio}/2 - \alpha_i c_i B P_i \qquad 3.2.7b$$

$$Let\ M_i = \alpha_i A P_m - \alpha_i b_i B/2 - \alpha_i\, CI_{Lio}/2$$

$$and\ N_i = \alpha_i c_i B$$

$$\frac{dP_i}{dt} = M_i + N_i P_i \qquad 3.2.7c$$

taking the laplace transform of equation 3.2.7c

$$sP_i(s) - p(0) = \frac{M_i}{s} + N_i P_i(s)$$

$$P_i(s) = \frac{p(0)}{s - N_i} + \frac{M_i}{s(s - N_i)} \qquad 3.2.8$$

taking the inverse laplace transform of 3.2.8

$$P_i(t) = P(0)e^{N_i t} - \frac{M_i}{N_i} + \frac{M_i}{N_i}e^{N_i t}$$

$$P_i(t) = \left[P(0) + \frac{M_i}{N_i}\right]e^{N_i t} - \frac{M_i}{N_i} \qquad 3.2.9$$

substituting for M_i *and* N_i

$$P_i(t) = \left[P(0) + \frac{AP_m - b_i B/2 - CI_{Lio}/2}{Bc_i}\right]e^{\alpha_i c_i Bt} + \frac{AP_m - b_i B/2 - CI_{Lio}/2}{Bc_i} \qquad 3.2.10$$

Since the output of the neuron as time tends to infinity settles down to give minimum function energy i.e. an optimal value, from equation 3.2.10 as $t \rightarrow \infty$

$$P_i(\infty) = \frac{AP_m - b_i B/2 - CI_{Lio}/2}{Bc_i} \qquad 3.2.11$$

$P_i(\infty)$ represent the optimal generation of *unit i.* Substituting equation 3.2.11 into equation 3.2.6b

$$P_m = P_D + P_{Lo} - \sum_{i=1}^{N}\left[\frac{AP_m - b_i B/2 - CI_{Lio}/2}{Bc_i}\right] \qquad 3.2.12$$

$$P_m = \frac{\left(P_D + P_{Lo} + \sum_{i=1}^{N}\left[\left(\frac{Bb_i}{2} + \frac{CI_{Lio}}{2}\right)\Big/(Bc_i)\right]\right)}{1 + (A/B)\sum_{i=1}^{N}\frac{1}{c_i}} \qquad 3.2.13$$

$P_i(0)$ and $P_i(\infty)$ are the values of $P_i(t)$ as $t = 0$ and $t = \infty$ respectively. $P_i(\infty)$ represent the optimal generation of unit i, which is the solution to the problem.

From equation 2.4.5

$$\frac{dF_i}{dP_i} = b_i + 2c_i P_i = \left(\frac{2}{B}\right)\left(AP_m - \frac{CI_{Li0}}{2}\right) \qquad 3.2.14$$

The equal-incremental cost criterion requires that

$$PF_1\frac{dF_1}{dP_1} = PF_2\frac{dF_2}{dP_2} = \cdots = PF_n\frac{dF_n}{dP_n} \qquad 3.2.15$$

$$PF_i = \frac{1}{1 - I_{Lio}}$$

PF_i is the penalty factor for unit *i*.

Solving equation 3.2.14 and equation 3.2.15 for *C*

$$C = 2AP_m \qquad 3.2.16$$

4.0 PROCEDURE, PRESENTATION OF RESULT AND DISCUSSION OF RESULT

4.1 PROCEDURE

The computational procedures include a series of adjustments of the weighting factor associated with the transmission line losses. After each adjustment of the weighting factor, unit power and incremental losses are reevaluated. The newly obtained power generations and incremental losses are applied, to compute power mismatch and re-adjusted until optimal generations are obtained. The optimal power generations are the ones, which result in the minimum energy function value.

The data used is given in Table 2, Table 3 and Table 4. Table 2 shows the loss coefficients of transmission lines, Table 3 shows the operational cost parameter and Table 4 shows the power limit of all thermal generation units used.

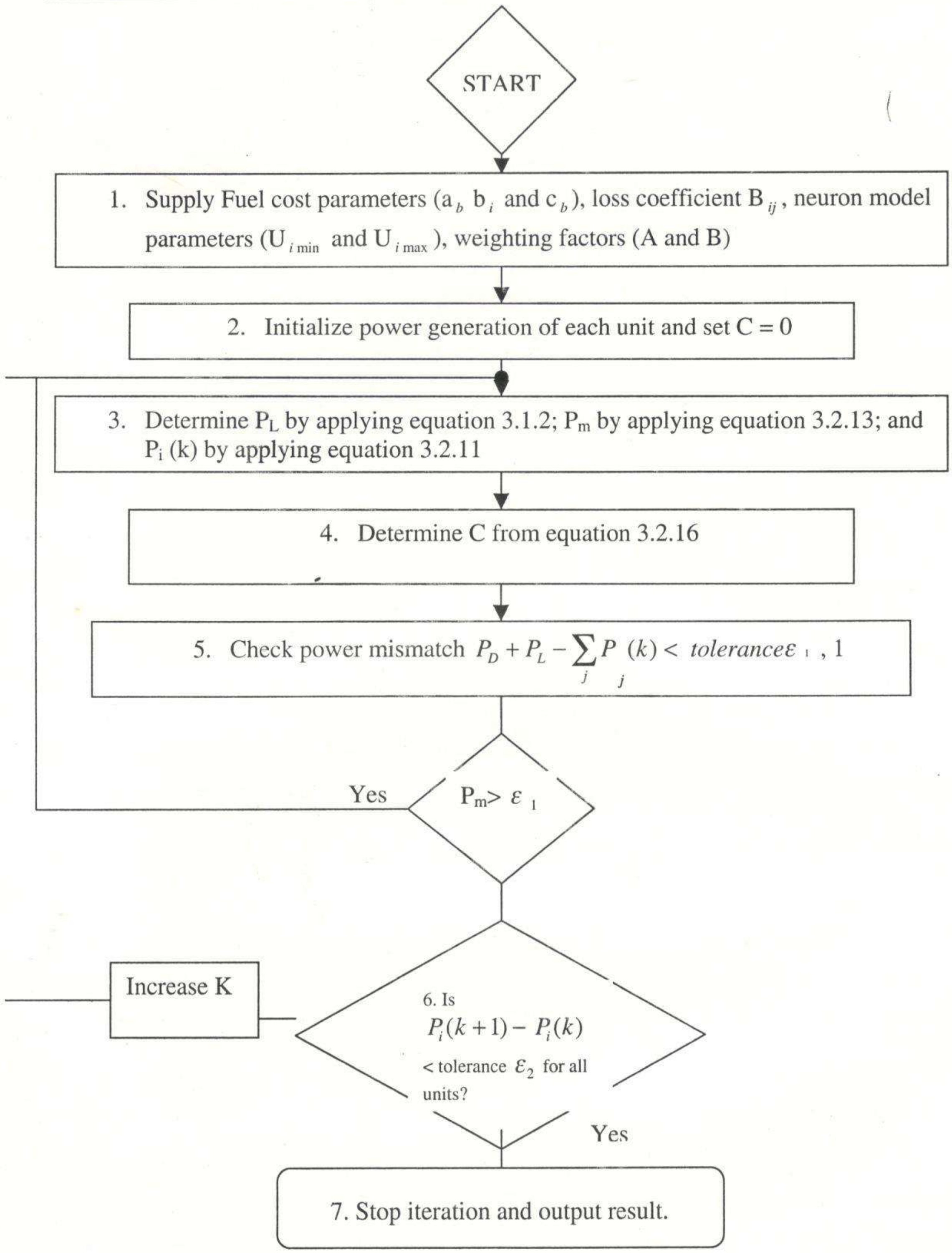

Fig. 3-2: Flow Chart for Computation of Power Mismatch

Table 1: Loss coefficient B_{ij} in 10^{-3} p.u.

8.7	0.43	-4.61	0.36	0.32	-0.66	0.96	-1.6	0.8	-0.1	3.6	0.64	0.79	2.1	1.7	0.8	-3.2	0.7	0.48	-0.7
0.43	8.3	-0.97	0.22	0.75	-0.28	5.04	1.7	0.54	7.2	-0.28	0.98	-0.46	1.3	0.8	-0.2	0.52	-1.7	0.8	0.2
-4.6	-0.97	9	-2	0.63	3	1.7	-4.3	3.1	-2	0.7	-0.77	0.93	4.6	-0.3	4.2	0.38	0.7	-2	3.6
0.36	0.22	-2	5.3	0.47	2.62	-2	2.1	0.67	1.8	-0.45	0.92	2.4	7.6	-0.2	0.7	-0.1	0.86	1.6	0.87
0.32	0.75	0.63	0.47	8.6	-0.8	0.37	0.72	-0.9	0.69	1.8	4.3	-2.8	-0.7	2.3	3.6	0.8	0.2	-3	0.5
-0.7	-0.28	3	2.62	-0.8	11.8	-4.9	0.3	3	-3	0.4	0.78	6.4	2.6	-0.2	2.1	-0.4	2.3	1.6	-2.1
0.96	5.04	1.7	-2	0.37	-4.9	8.24	-0.9	5.9	-0.6	8.5	-0.83	7.2	4.8	-0.9	-0.1	1.3	0.76	1.9	1.3
-1.6	1.7	-4.3	2.1	0.72	0.3	-0.9	1.2	-0.96	0.56	1.6	0.8	-0.4	0.23	0.75	-0.6	0.8	-0.3	5.3	0.8
0.8	0.54	3.1	0.67	-0.9	3	5.9	-0.96	0.93	-0.3	6.5	2.3	2.6	0.58	-0.1	0.23	-0.3	1.5	0.74	0.7
-0.1	7.2	-2	1.8	0.69	-3	-0.6	0.56	-0.3	0.99	-0.66	3.9	2.3	-0.3	2.8	-0.8	0.38	1.9	0.47	-0.26
3.6	-0.28	0.7	-0.5	1.8	0.4	8.5	1.6	6.5	-0.66	10.7	5.3	-0.6	0.7	1.9	-2.6	0.93	-0.6	3.8	-1.5
0.64	0.98	-0.77	0.92	4.3	0.78	-0.8	0.8	2.3	3.9	5.3	8	0.9	2.1	-0.7	5.7	5.4	1.5	0.7	0.1
0.79	-0.46	0.93	2.4	-2.8	6.4	7.2	-0.4	2.6	2.3	-0.6	0.9	11	0.87	-1	3.6	0.46	-0.9	0.6	1.5
2.1	1.3	4.6	7.6	-0.7	2.6	4.8	0.23	0.58	-0.3	0.7	2.1	0.87	3.8	0.5	-0.7	1.9	2.3	-0.97	0.9
1.7	0.8	-0.3	-0.2	2.3	-0.2	-0.9	0.75	-0.1	2.8	1.9	-0.7	-1	0.5	11	1.9	-0.8	2.6	2.3	-0.1
0.8	-0.2	4.2	0.7	3.6	2.1	-0.1	-0.56	0.23	-0.8	-2.6	5.7	3.6	-0.7	1.9	10.8	2.5	-1.8	0.9	-2.6
-3.2	0.52	0.38	-0.1	0.8	-0.4	1.3	0.8	-0.3	0.38	0.93	5.4	0.46	1.9	-0.8	2.5	8.7	4.2	-0.3	0.68
0.7	-1.7	0.7	0.86	0.2	2.3	0.76	-0.3	1.5	1.9	-0.6	1.5	-0.9	2.3	2.6	-1.8	4.2	2.2	0.16	-0.3
0.48	0.8	-2	1.6	-3	1.6	1.9	5.3	0.74	0.47	3.8	0.7	0.6	-1	2.3	0.9	-0.3	0.16	7.6	0.69
-0.7	0.2	3.6	0.87	0.5	-2.1	1.3	0.8	0.7	-0.26	-1.5	0.1	1.5	0.9	-0.1	-2.6	0.68	-0.3	0.69	7

Table 2 : Cost coefficient of thermal units

Unit	a	B	c
1	1000	18.19	0.00068
2	970	19.26	0.00071
3	600	19.8	0.0065
4	700	19.1	0.005
5	420	18.1	0.00738
6	360	19.26	0.00612
7	490	17.14	0.0079
8	660	18.92	0.00813
9	765	18.27	0.00522
10	770	18.92	0.00573
11	800	16.69	0.0048
12	970	16.76	0.0031
13	900	17.36	0.0085
14	700	18.7	0.00511
15	450	18.7	0.00398
16	370	14.26	0.0712
17	480	19.14	0.0089
18	680	18.92	0.00713
19	700	18.47	0.00622
20	850	19.79	0.00773

Table 3: Economic Power limit of thermal units

Unit	Pmin	Pmax
1	150	600
2	50	200
3	50	200
4	50	200
5	50	160
6	20	100
7	25	125
8	50	150
9	50	200
10	30	150
11	100	300
12	150	500
13	40	160
14	20	130
15	25	185
16	20	80
17	30	85
18	30	120
19	40	120
20	30	100

Table 4: Optimum power generated by thermal units using the proposed algorithm

Unit	Unit Power(MW) Optimal Technique	Unit Power (MW) Classical Approach
1	521.0085044	512.7805
2	195.9020562	169.1033
3	125.786112	126.8898
4	102.7142951	102.8657
5	114.2976138	113.6836
6	70.48867348	73.571
7	116.0966622	115.2878
8	116.0195387	116.3994
9	105.0821036	100.4062
10	65.59932526	106.0267
11	130.3187606	150.2394
12	307.2581799	292.7648
13	121.9730105	119.1154
14	31.00012211	30.834
15	121.1383808	115.8057
16	35.93005392	36.2545
17	64.53894732	66.859
18	90.6362459	100.8033
19	103.1797461	54.305
20	53.01276141	91.967

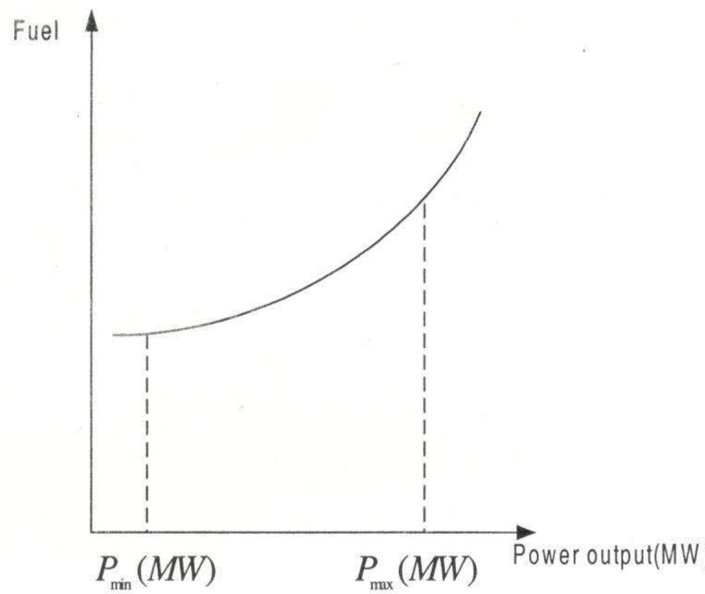

Figure 2-1: Input-Output curve of a typical generating unit (Fuel - Power)

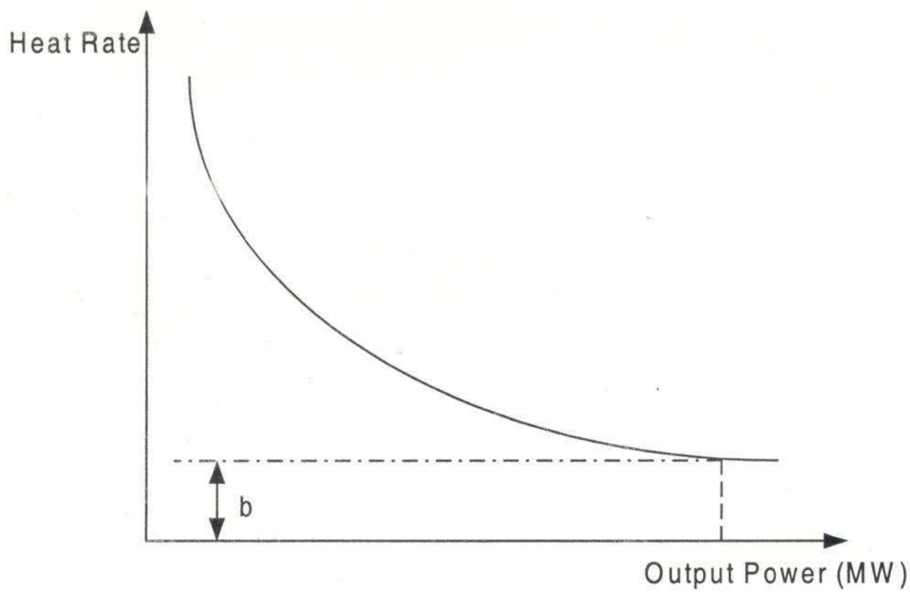

Figure 2-2: Heat rate - output power of a generating unit

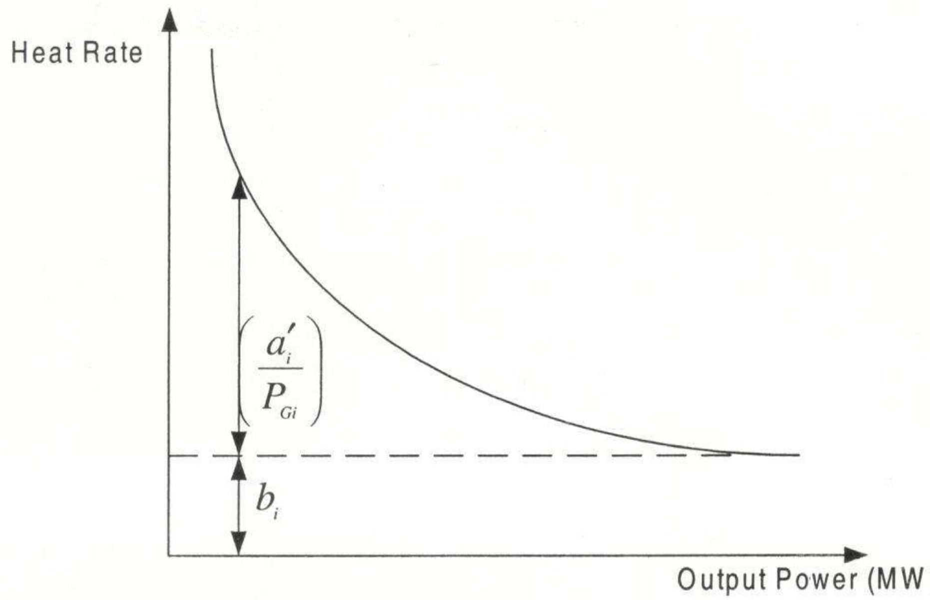

Figure 2-3: Heat rate - output power curve