Prediction of Flexural Strength of Rice Husk Ash Concrete

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ABSTRACT

Pozzolanas are materials containing reactive silica and/or aluminum. When the material is mixed with lime in powdered form and in the presence of water, it will set and harden like cement though with higher initial setting time than Ordinary Portland Cement (OPC). Rice Husk Ash (RHA) is a form of natural pozzolana. It is found locally in abundance in Nigeria and in other parts of the world. Udeala (2003), produced a mix containing RHA with 45 per cent slaked lime. This work uses Scheffe's (1958) simplex theory to optimize the flexural strength of concrete made from RHA pozzolan based on (4,2) simplex lattice. The strengths predicted by the model are in good agreement with their corresponding experimentally obtained values. The statistical student's t-test was used to test the adequacy of the model equation. The results are in affirmative. With the model, any desired strength of hardened concrete, given any mix proportions, is easily evaluated. Conversely, the various mix proportions matching any stipulated strength are also easily obtained. The computer program is written in BASIC language.

Keywords

Flexural Strength, Optimization, Rice-Husk-Ash, Pozzolan, Simplex Lattice, and Regression.

INTRODUCTION

Concrete, a product of water, cement and aggregate, when sufficiently hardened, is used in various forms to resist load. The cost of one of its ingredients, Ordinary Portland Cement (OPC) is rising rapidly. Cheap and replaceable or complementary substitutes are being developed (Neville, 1996).

Rice Husk Ash (RHA) is an agro-waste material, found in abundance in Nigeria and in many other parts of the world. RHA is one of the natural Pozzolanas. Pozzolanas are materials containing reactive silica and/or aluminum. When the material is mixed with lime in powdered form and in the presence of water, it will set and harden like cement (ASTM 618-94(a)). The Indian ITDG

(2005), has it that Greeks and the Romans were the first civilization known to have used Pozzolanas in lime mortars. Udeala (2003), produced RHA with 45 per cent slaked lime mix. The IS 4098 (1967), stipulates specific characteristics of different grades of lime-pozzolan mixture. The strength and other properties are affected by lime-pozzolan ratio.

Although concrete is not normally designed to resist direct tension, the knowledge of tensile strength is of value in estimating the load under which cracking will develop (Neville, 1996). The absence of cracking is important in maintaining the continuity of a concrete structure and in the prevention of corrosion of reinforcements. Flexural strength is calculated using the following equation:

$$\sigma_T = \frac{2p}{\pi LD} \qquad (1)$$

where

P is third-point maximum total load on the beam

L is span of the beam

D is the depth of the beam

Scheffe's Optimization Theory

Strength of concrete depends on the adequate proportioning of its ingredients. H. Scheffe (1958), developed an optimization theory that is used to optimize the strength of concrete.

The Simplex

According to Jackson, N. (1983) "simplex is the structural representation (shape) of the line or planes joining the assumed positions of the constituent materials (atoms) of the mixture".

H. Scheffe (1958), considered experiments with mixtures of which the property studied depends on the proportions of the components but not their quantities in the mixture. An obvious example of such a study is the relationship between the compressive strength of concrete and the proportion of w/c (water- cement), cement, sand and coarse aggregate.

If a mixture has a total q components and X_i, be the proportion of the components (ingredients) of the ith component in the mixture such that

$$X_i \geq 0 \quad (i = 1\text{-}4) \qquad (2)$$

Then, if we assume the mixture to be a unit quantity then the sum of all the proportions of the components must be unity. That means that

$X_1+X_2+X_3+X_4=1$

$$\sum_{i=1}^{4} x_i = 1 \qquad (3)$$

where in this case,

X_1 = proportion of water/cement (w/c) ratio

X_2 = proportion of cement

X_3 = proportion of sand

X_4 = proportion of crushed stone.

Thus, the factor space is a regular (q-1) dimensional simplex

Simplex Lattice

For a 4- component mixture, we have a tetrahedron simplex-lattice (fig. 1). Taking a whole factor space in the design. We will have (q.m) simplex lattice whose properties are defined as follows.

(a) The factor space has uniformly space distribution of points

(b) The proportions used for the factor has m+1 equally spaced values from 0 to 1, that is $X_1 = 0, \frac{1}{m}, \frac{2}{m}, ...1$, and all possible mixtures with these proportions for the factor. H. Scheffe (1958), also showed that the number of points in (q,m) lattice is given by

$$\frac{q(q+1)...(q+m-1)}{m!} \qquad (4)$$

where m is a digit number and

q is number of independent variables

This implies that for a (4.2) lattice, the number of points =

$$\frac{4(4+1)}{1x2} = 10 \quad \text{(See fig.1)}$$

DESIGN POINTS

The criterion of eqn.3 makes it impossible to use the normal mix ratios such as 1:2:4 at a given water/cement ratio. Therefore, a transformation of the actual components, i.e. the ingredient proportion is necessary. The transformed proportions x_i (i=1-4) for each experimental point are called "pseudo-components" for actual components Z, the pseudo-components X is given by $X=AZ$ (5)

where A is the inverse of Z matrix. Similarly the inverse transformation from pseudo-components to actual components is expressed as

$Z=BX$ (6)

where B is the inverse of A, matrix. the mix ratios (Z values) shown in table 1.0 for points 1-4, which also form a 4x4 z matrix, were selected for the four design points (i.e.x_1,x_2,x_3,and x_4 of fig.1). The pseudo-components, arbitrarily selected to conform to eqn.3, and the transformed actual components, using eqn.6, for design points 5-10 and the control points 11-20 are shown in table 1.0. This brings the total design points to 20. The components x_i (i=1- 4) represents water/cement ratio, cement, sand and crushed stone.

The Simplex Canonical Polynomials

RESPONSE

The properties studied in the assumed polynomial are real-valued functions on the simplex. They are termed **responses**. In our study the target strength is the response. H. Scheffe (1958), assumed that a polynomial function of degree m, in the q (q=4) variables X_1, X_2...X_q subject to eqn.3 will be called a "(q, m) polynomial" and that it will be of the form

$$Y=b_o+\sum_{i=1}^{q} b_i x_i + \sum \sum_{1\le i\le q, 1\le j\le q} b_{ij} + \sum_{1\le i\le q, 1\le j\le q, 1\le k\le q} b_{ijk} x_i x_j x_k + \sum b\, i_1 i_2 \ldots i_n x_{i1} x_{i2} \ldots x_{in} \qquad (7)$$

b= is a constant coefficient.

He also showed that the number of coefficients in eqn. 7 is given by C^{n}_{q+n-1}. But eqn 7 must be subject to the condition of eqn.3, hence b_i values are not unique.

If from eqn .3. We let

$X_q = 1 - \Sigma x_i$

Substituting the values of x_q into eqn.7, the number of coefficients b_i will reduce to C^{n}_{q}+n-1 or

$$\frac{q(q+1)\ldots(q+n-1)}{n!} \qquad (8)$$

where n is a digit number. This implies that the number of the coefficients equal to a (q,n) lattice. The implication here, therefore, is that the values of a (q,n) polynomial can be assigned arbitrarily on a (q,n) lattice and values on the simplex (eqn 3) are then uniquely determined.

Let $\alpha'_i = b_o+b_i+b_{ii}$ and $\alpha_{ij} = b_{ij}-b_{ii}-b_{jj}$ (9)

The second degree polynomial of eqn.7 can therefore, be represented as

$$Y=\alpha_1x_1+\alpha_2x_2+\alpha_3x_3+\alpha_4x_4+\alpha_{12}x_1x_2+\alpha_{13}x_1x_3+\alpha_{14}x_1x_4+\alpha_{23}x_2x_3+\alpha_{24}x_2x_4+\alpha_{34}x_3x_4 \qquad (10)$$

It was shown (Akhnazarova, 1959) that the following relations hold:

$\alpha_i = y_i$ and $\alpha_{ij} = 4y_{ij}-2y_i-2y_j$ (α=response) (11)

MATERIALS AND METHODS

The main material for this research is the Rice Husk Ash (RHA)-slaked lime mix.

The mix ratios used for the simplex design points were as a result of preliminary research findings about the concrete made from the pozzolan.

Preparation of samples

a. the RHA was used as supplied.

b. aggregates:

i. sand

The sand was collected from River Benue, Makurdi-Nigeria. It was prepared to BS 1017: parts 1 and 2 and BS 882: 1992. The grading was carried out to BS 812:103:1985. The sand belongs to zone C (Neville, 1996).

ii. coarse aggregate (crushed stone)

The crushed granite chippings were collected from Kwande, Benue State-Nigeria. The nominal size of aggregate used was 20mm.

Flexural Strength

Concrete cylinders of size 150 diameter by 300mm height (length) were cast from predetermined proportions of water, cement, sand and crushed granite chippings. The cylinders were demoulded after 3 days (72 hours) and immediately transferred to the curing tank at room temperature for 56 days. The cylinders were then tested for flexure in accordance with BS 1881-117:1983.

RESULTS AND ANALYSIS

Flexural strength test results, based on Scheffe's (4,2) simplex lattices.

The results of the flexural strength test results, based on Scheffe's (4,2) simplex lattices are shown in table 2.

The Regression Equation

Based on eqn. 11 and table 2, the coefficients of the (4,2) polynomial are determined as follows:

$\alpha_1 = 0.23$, $\alpha_2 = 0.23$, $\alpha_3 = 0.22$, $\alpha_4 = 0.23$

$\alpha_{12} = 4*0.21-2*0.23-2*0.23S = -0.08$

Similarly,

$\alpha_{13} = -0.14$, $\alpha_{14} = 0.00$, $\alpha_{23} = 0.02$, $\alpha_{24} = 0.00$, $\alpha_{34} = 0.02$

Thus, from equation 10, we have

$$\hat{Y} = 0.23x_1 + 0.23x_2 + 0.22x_3 + 0.23x_4 - 0.08x_1x_2 - 0.14x_1x_3 + 0x_1x_4 + 0.02x_2x_3 + 0x_2x_4 + 0.02x_3x_4 \quad (12)$$

Equation 12 is the mathematical model for the optimization of the flexural strength of the Rice Husk Ash pozzolan concrete, based on Scheffe's (4,2) polynomial.

TEST OF THE ADEQUACY OF THE MODEL

Equation 12, the model equation, was subjected to statistical student's t- test for adequacy against the controlled experimental results. It was proved adequate. A typical result of an executed program is shown in appendix A.

CONCLUSION

The research showed that the Rice Husk Ash (RHA) produced an average flexural strength of 0.23 N/mm^2.(see table 2).

The model equation was tested for adequacy using the student's t-test (see appendix C).The strengths predicted by the model are in good agreement with the corresponding experimentally obtained results. With the model, any desired strength of hardened concrete, given any mix proportions, is easily evaluated. Conversely, the various mix proportions matching any stipulated strength are also easily obtained using simple BASIC computer program (See appendix B).

Legend : $\bar{y} = \frac{\sum_{r=1}^{m} y_r}{m_r}$; $S_i^2 = \frac{1}{m_{i-1}}\left[\sum_{r=1}^{mi} y_r^2 - \frac{1}{m_i}\left(\sum_{r=1}^{m_i} y_r\right)^2\right]$

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Appendix A

An executed program for the flexural strength (Scheffe's)

Desired strength? 0.231

Counter	x1	x2	x3	x4	y	z1	z2	z3	z4
1	0.000	0.810	0.190	0.000	0.231	0.858	1.000	2.095	3.905
2	0.000	0.820	0.140	0.040	0.231	0.858	1.000	2.078	3.890
3	0.000	0.820	0.150	0.030	0.231	0.858	1.000	2.081	3.895
4	0.000	0.820	0.160	0.020	0.231	0.858	1.000	2.084	3.900
5	0.000	0.820	0.170	0.010	0.231	0.856	1.000	2.090	3.190
6	0.000	0.860	0.140	0.000	0.231	0.859	1.000	2.075	3.930

The maximum value of strength predictable by this model is 0.232 N/sq.mm

Appendix B

Program for Scheffe's model

```
10 REM A GW BASIC V2.02 program that computes the proportions of concrete mixes to a desired
strength.
20 REM Scheffe's model
30 COUNT = 0
40 GOSUB 100
50 END
100 REM procedure begins
110 YMAX = 0
120 PRINT
130 PRINT "A Model for Computation of Concrete Mix Proportions to Desired Strength"
140 PRINT
```

```
150 INPUT "Desired Strength"; YIN

160 GOSUB 600

170 FOR X1 = 0 TO 1 STEP .01

180 FOR X2 = 0 TO 1-X1 STEP .01

190 FOR X3 = 0 TO 1-X1-X2 STEP .01

200 X4 = 1-X1-X2-X3

300 REM Assign Coefficients

310 A1 =

320 A2 =

330 A3 =

340 A4 =

350 A12 =

360 A13 =

370 A14 =

380 A23 =

390 A24 =

400 A34 =

410 YOUT =

A1*X1+A2*X2+A3*X3+A4*X4+A12*X1*X2+A13*X1*X3+A14*X1*X4+A23*X2*X3+A24*X2*X4+A

34*X3*X4

420 GOSUB 700

430 IF (ABS (YIN-YOUT)<=0.001) THEN 440 ELSE 460

440 COUNT = COUNT+1

450 GOSUB 800

460 NEXT X3
```

```
470 NEXT X2

480 NEXT X1

490 PRINT

500 IF (COUNT>0) THEN 510 ELSE 530

510 PRINT "The max. Value of strength predictable by this model is"; ymax; "N/sq.mm."

520 GOTO 540

530 PRINT "Sorry! Desired strength out of range of the model"

540 RETURN

600 REM print heading

610 PRINT

620 PRINT "COUNT  X1   X2   X3   X4   Y   Z1  Z2  Z3   Z4"

630 RETURN

700 REM Checkmax

710 IF YMAX<YOUT THEN YMAX = YOUT ELSE YMAX = YMAX

720 TETURN

800 REM Out results

810 Z1 = 0.88*X1+0.86*X2+0.85*X3+0,84*X4

820 Z2 = X1+X2+X3+X4

830 Z3 = 2.8*X1+2.0*X2+2.5*X3+2.2*X4

840 Z4 = 4.5*X1+4.0*X2+3.5*X3+3.0*X4

850 PRINT TAB (1); COUNT; USING "####.##"; X1;X2;X3;X4;YOUT;Z1;Z2;Z3;Z4

860 RETURNS
```

Appendix C

Table 1 (Appendix C) t-Statistic for the controlled points, flexural strength, based on Scheffe's (4,2) regression polynomial

Response symbol	ϵ	$\bar{y}$	$\hat{Y}$	t
C_1	0.6093	0.18	0.20	3.15
C_2	0.4842	0.20	0.22	3.20
C_3	0.7343	0.21	0.23	3.04
C_4	0.5937	0.19	0.21	3.17
C_5	0.2893	0.22	0.22	0.00
C_6	0.8593	0.23	0.23	0.00
C_7	0.5937	0.22	0.23	1.58
C_8	0.4833	0.20	0.21	1.64
C_9	0.6405	0.22	0.23	1.56
C_{10}	0.4697	0.22	0.23	1.65

Legend:

$\varepsilon = \Sigma a^2_i + \Sigma a^2_{ij}$: $a_i = x_i(2x_i-1)$: $a_{ij} = 4x_ix_j$ and x is a variable and i varies from 1 to 4

$t = \dfrac{\Delta y\sqrt{n}}{S_y(1+\varepsilon)}$ where $\Delta y = y_o - y_t$ (o = observed, t = theory)

n = number of replicate observations at every point

t Value From Table

Significant level, $\alpha = 0.05$ and $t_{\alpha/1}(V_e) = t_{0.05/10} = t_{0.005}(9) = 3.69$ (From t-test chart)

This is greater than any of the t values calculated in table 1 of appendix C, hence, we accept that the regression equation is adequate.

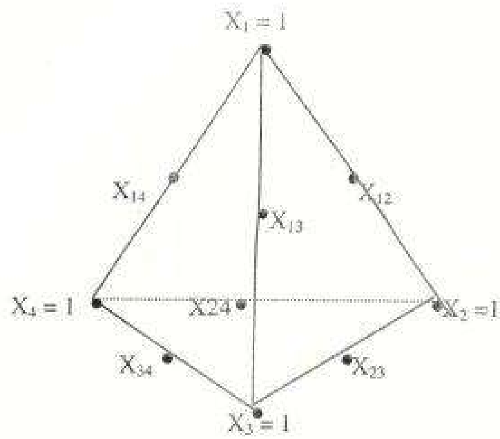

Fig. 1a: (4,2) Simplex Lattice

Legend

C_i, Y_i, Y_{ij} (i=1- 4 and j=2-4), the 'response', represents the expected compressive strength of concrete.

Table 1: Pseudo (x_i) and actual (z_i) components (points 1-20)

N	X_1	X_2	X_3	X_4	Response	Z_1	Z_2	Z_3	Z_4
1	1	0	0	0	Y_1	0.88	0.86	0.85	0.84
2	0	1	0	0	Y_2	1	1	1	1
3	0	0	1	0	Y_3	2.8	2.0	2.5	2.2
4	0	0	0	1	Y_4	4.5	4	3.5	3.0
5	½	½	0	0	Y_{12}	0.87	1	2.4	4.25
6	½	0	½	0	Y_{13}	0.865	1	2.65	4.0
7	½	0	0	½	Y_{14}	0.86	1	2.5	3.75
8	0	½	½	0	Y_{23}	0.855	1	2.25	3.75
9	0	½	0	½	Y_{24}	0.85	1	2.1	3.5
10	0	0	½	½	Y_{34}	0.845	1	2.35	3.25
Control Points									
11	½	¼	¼	0	C_1	0.85	1	2.525	4.125
12	¼	¼	¼	¼	C_2	0.858	1	2.375	3.75
13	0	¼	0	¾	C_3	0.845	1	2.15	3.25
14	½	0	¼	¼	C_4	0.863	1	2.575	3.875
15	½	¼	0	¼	C_5	0.865	1	2.45	4.0
16	0	¾	¼	0	C_6	0.858	1	2.125	3.625
17	0	½	¼	¼	C_7	0.853	1	2.175	3.625
18	¼	⅛	½	⅛	C_8	0.858	1	2.475	3.75
19	¼	¼	0	½	C_9	0.855	1	2.3	3.625
20	⅛	⅛	¼	½	C_{10}	0.85	1	2.325	3.438

Table 2: Flexural Strength (y_r) Test Results and Variance (s_i) Based on Scheffe's (4,2) Simplex Lattices

Expt. No.	Replication	Response Y_r N/mm²	Response Symbol	ΣY_r	$\bar{y}$	$(\Sigma yr_r)^2$	S_i^2
1	1A 1B	0.23 0.22	Y_1	0.45	0.23	0.2025	0.0001
2	2A 2B	0.23 0.23	Y_2	0.46	0.23	0.2116	0.0000
3	3A 3B	0.24 0.20	Y_3	0.44	0.22	0.1936	0.0008
4	4A 4B	0.23 0.23	Y_4	0.46	0.23	0.2116	0.000
5	5A 5B	0.20 0.21	Y_{12}	0.41	0.21	0.1681	0.0005
6	6A 6B	0.17 0.20	Y_{13}	0.37	0.19	0.1369	0.0005
7	7A 7B	0.23 0.23	Y_{14}	0.46	0.23	0.2116	0.0000
8	8A 8B	0,23 0.23	Y_{23}	0.46	0.23	0.2116	0.0000
9	9A 9B	0.22 0.23	Y_{24}	0.45	0.23	0.2025	0.0005
10	10A 10B	0.23 0.22	Y_{34}	0.45	0.23	0.2025	0.0005
			Control				
11	11A 11B	0.18 0.18	C_1	0.36	0.18	0.2116	0.0000
12	12A 12B	0.22 0.18	C_2	0.40	0.20	0.1444	0.0002
13	13A 13B	0.21 0.21	C_3	0.42	0.21	0.1764	0.0000
14	14A 14B	0.18 0.20	C_4	0.38	0.19	0.1444	0.0002
15	15A 15B	0.22 0.21	C_5	0.43	0.22	0.1089	0.0005
16	16A 16B	0.23 0.23	C_6	0.46	0.23	0.2116	0.0000
17	17A 17S	0.22 0.22	C_7	0.44	0.22	0.1936	0.0000
18	18A 18B	0.18 0.22	C_8	0.40	0.20	0.2916	0.0002
19	19A 19B	0.22 0.22	C_9	0.44	0.22	0.4356	0.0000
20	20A 20B	0.23 0.21	C_{10}	0.44	0.22	0.1936	0.0005
						Σ	**0.0045**