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An Optimization of the Flexural Strength of Fibre Cement Mixtures Using Scheffe's Simplex Lattice

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ABSTRACT

Thirty two Flexural strength tests were conducted on Fibre Cement (FC) prism samples composed of cement, sand, fibre (coir) and water combined in specified proportions after curing for 28 days. A scientific approach is introduced to optimize flexural strengths of the FC composites by generating a mathematical model that can predict the mix proportions given specified strengths; in doing this, Scheffe's simplex lattice was adopted. The mix proportions were generated from matrix transformations after prescribing typical mix proportions at each of the four points of the tetrahedron lattice (i.e. Scheffe's factor space). The results of the tests were used to develop a mathematical model of the form; $y = f(x_1, x_2, x_3, x_4)$*; where* $f(x_1, x_2, x_3, x_4, \ )$ *is a real – valued polynomial function of volumetric proportions of water* x_1*, cement* x_2*, sand* x_3 *and coir* x_4 *respectively. Statistical adequacy tests carried out on the model using student t and Chi – square confirmed the adequacy of the model. A computer programme was developed based on the model to generate mix proportions for any specified strength and to predict optimum strengths. The optimum flexural strength (measured as modulus of rupture) is 7.102079N/mm*^{2} *after 28 days of curing with six mix combinations. The FC elements producer can use the model to generate combinations of mix proportions from which a final and economic FC product could be produced.*

1.0 INTRODUCTION

Fibre Cement (FC) elements are used as roofing sheets, tiles, partitions and ceiling boards[1]. They possess low thermal conductivity, noise dampening properties and appreciable mechanical strengths[2]. The production process required low capital outlay and the materials are relatively cheap and available. The FC consists of cement, fine aggregate (sand), fibre (coir) and water. A traditional practice in the construction industry is to specify the target strength of a concrete/mortar product and the producer or contractor is left to achieve the specified strength usually by means of trial proportional mixing based on intelligent guesses; this is the design mix approach[3]. However, a better approach to mix proportioning that eliminates repeated and laborious trial mixing is formulated. This scientific approach seeks to optimize strength while minimizing production cost. The approach is based on the use of a mathematical model that can predict the mix proportions of FC elements given specified strengths without compromising flexibility and economy.

The model is formulated from an Experimental Mixture Design theory developed by Scheffe[4] using simplex lattice. The purpose of the theory is the empirical prediction of the response (a real-valued function) to any mixture of the components, when the response depends only on the proportion of the components and not on the total amount. This response function serves as a mathematical model that is expressed in regression polynomial form whose predictions are then tested for adequacy against experimental control results obtained in the laboratory.

In 1999, Khuri et al[5] used a Scheffe quadratic model to predict the optimum fertilizer constituents (N, P, K, Ca) proportions needed to restore the health of young citrus trees that had been infected by blight. Other applications of this theory have been recorded: Akpokwe[6] worked to optimize a 1:2:4 concrete mix and obtained an optimum compressive strength of $36.088N/mm^2$ after 28 days of curing with optimum mix ratio of 1:1.92:4.14 and a w/c ratio of 0.55, Obam[7] obtained an optimum strength of $22.35N/mm^2$ with a computer print out of 51 possible mix combinations in a palm kernel shell aggregate concrete work, Nwajagu[8] optimized clay tile properties from different sources and discovered that "Nsu" clay produced clay tiles having optimum properties when the constituent proportions are 47.8%clay, 33.7%Quartz and 18.5%Calcium Carbonate and work on cement-rice husk - clay composite optimization done by Nwakonobi[9] equally produced similar results. The efficacy of Scheffe's simplex lattice as a tool for mixture design optimization is no more in doubt.

In this paper, a mathematical model was formulated for a FC mixture with coir as fibre. The results showed that when a desired strength of $7.0N/mm^2$ was prompted, it generated a total of 27 mix combinations and the optimum flexural strength is $7.102079N/mm^2$ at 28 days with six mix combinations. A producer of FC is expected to choose a most economic mix from the combinations.

2.0 THEORY

Scheffe's theory postulates that the factor space for experiments with mixtures is a regular (q-1) dimensional simplex and for the mixture the relationship holds.

$$\sum_{i=1}^{q} x_i = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Where $x_i \geq 0$ is the component concentration in volume fraction and q is the components number. In exploring the whole factor space of a mix design with a uniformly spaced distribution of points over the factor space, we have what is called a {q,m} **Simplex Lattice**. The proportions used for each factor have the m+1 equally spaced values from

0 to 1, $x_i = 0, 1/m, 2/m, \ldots\ldots, 1$. ....(2)

For a {4,2} lattice, there exists three levels as follows: $x_4 = 0, \frac{1}{2}, 2/2$

All possible mixtures with these proportions for each factor are used. Also the number of points in the simplex lattice design is derived from the statistical equation shown in (3) below:

$$\left[\frac{m+q-1}{m}\right] = \frac{q(q+1)\ldots(q+m-1)}{1.2\ldots.m} \qquad \text{or}$$

$$_{q+m+1}C_m = \frac{(q+m-1)!}{m!(q+m-1-m)!} \quad \ldots\ldots\ldots(3)$$

Where m is the number of equal spaces between two vertices of a simplex lattice, C is probability combination and q is defined above.

Figure 1 shows a {4,2} quadratic simplex lattice with $\frac{4(4+1)}{1x2} = 10\text{points}$

The property (response) of the mixture is assumed to be a real-value function on a simplex, to which an appropriate form of Polynomial regression model is introduced. The polynomial function of degree, n, in the q variables $x_1, x_2, \ldots, x_q$ must subject to equation (1) and to be called a {q,n} Polynomial of the form:

$$\hat{y} = b_0 + \Sigma b_i x_i + \Sigma b_{ij} x_i x_j + \Sigma b_{ijk} x_i x_j x_k + \Sigma b_{i1i2\ldots in} x_{i1} x_{i2} \ldots x_{in} \ldots\ldots\ldots\ldots\ldots(4)$$

$1 \le i \le q \qquad 1 \le i \le j \le q \qquad 1 \le i \le j \le k \le q \qquad 1 \le i \le i2 \le \ldots \le in \le q$

Where the coefficients b are constants and $_{q+n}C_n$ is the number of coefficients and $\hat{y}$ is the response.

Design Matrix for The {4,2} Matrix

Using equation (2) and with 10 points on the lattice, the design matrix is given in Table 3.

Six control test points are also chosen. The values for the four constituent points are within the constraints of Equation 1. Table 4 shows the design matrix for control test points

The Polynomial Regression Model

The general polynomial for a {4,2} model is given as follows [10]

$$\hat{y} = \sum_{1\le i\le q} \beta_i x_i + \sum_{1\le i\le j\le q} \beta_{ij} x_i x_j \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Hence the (4,2) model has a reduced Polynomial of the form:

$$\hat{y} = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{14} x_1 x_4 + \beta_{23} x_2 x_3 + \beta_{24} x_2 x_4 + \beta_{34} x_3 x_4 \ldots\ldots\ldots(6)$$

and the general equations for the coefficients are as follows:

$$\beta_1 = n_1 \quad \beta_2 = n_2 \quad \beta_3 = n_3 \quad \beta_4 = n_4 \quad \beta_{12} = 4n_{12} - 2n_1 - 2n_2 \quad \beta_{13} = 4n_{13} - 2n_1 - 2n_3$$
$$\beta_{14} = 4n_{14} - 2n_1 - 2n_4 \quad \beta_{23} = 4n_{23} - 2n_2 - 2n_3 \quad \beta_{24} = 4n_{24} - 2n_2 - 2n_4 \quad \beta_{34} = 4n_{34} - 2n_3 - 2n_4 \ \ldots\ldots\ (7)$$

Matrix Transformation of Components

In view of equation 1, it is not possible to use normal mix ratios like 1:2:4. This is why a transformation of the actual components is imperative. The transformed fractions, say, $x_i^{(i)}$,

$x_2^{(i)}$, $x_3^{(i)}$ and $x_4^{(i)}$ for the i^{th} experimental points, are called "Pseudo - components". Taking prescribed mix proportions as follows: A_1 (0.48:1:1.0:025), A_2 (0.50:1:1.5:0.030), A_3 (0.65:1:2.0:0.04) and A_4 (0.6:1:1.8:0.035) representing water, cement, fine aggregate and fibre fractions respectively.

From Table 3, the required transformation can be depicted thus[6]:

$$Z\begin{bmatrix} 0.48 & 0.50 & 0.65 & 0.60 \\ 1 & 1 & 1 & 1 \\ 1 & 1.5 & 2.0 & 1.8 \\ 0.025 & 0.030 & 0.040 & 0.035 \end{bmatrix} \xrightarrow{\text{Transformation}} X\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

If X denotes the Pseudo -matrix of the i^{th} experimental points, then observing from Table 3 (points 1-4), $BZ = X = I$, where B is the transformed matrix and I is the unit Matrix.

For any pseudo – component $\begin{bmatrix} X_1^{(i)} \\ X_2^{(i)} \\ X_3^{(i)} \\ X_4^{(i)} \end{bmatrix}$

The actual component is given by

$$\begin{bmatrix} z_1^{(i)} \\ z_2^{(i)} \\ z_3^{(i)} \\ z_4^{(i)} \end{bmatrix} = \begin{bmatrix} 0.48 & 0.5 & 0.65 & 0.6 \\ 1 & 1 & 1 & 1 \\ 1 & 1.5 & 2.0 & 1.8 \\ 0.025 & 0.030 & 0.040 & 0.035 \end{bmatrix} \begin{matrix} x_1^{(i)} \\ x_2^{(i)} \\ x_3^{(i)} \\ x_4^{(i)} \end{matrix} \qquad \ldots\ldots\ldots\ldots(8)$$

Matrix equation (8) is employed to determine all other actual components for the {4,2} lattice in Tables 3 and 4 using 'Matlab' Software (see appendix 2). Tables 5 and 6 show the values of the actual components for the experimental and control tests points respectively.

3.0 MATERIALS AND METHODS

3.1 Preparation of Samples

Cement

Ordinary Portland Cement (OPC) was used. It exhibited the qualities of good cement by visual means, touch and hydration.

Fine Aggregate (Sand)

Sharp river sand was used. Sieving was done with mechanical shaker in accordance with BS 1377 [12].

Fibre

Coconut fibre (coir) was extracted from the coconut pod, cleaned and chopped to lengths of 15-40mm. Efforts were made to ensure that they separated into single fibres and in compliance with BS3681[13]

Water

Clean tap water was used.

3.2 Mixing

Mix proportions were specified by volume. This is important for light- weight mortar/concrete because the bulk relative densities are not in the same order due to different sizes of grains[14]. The mix ratios are based on the actual components in Tables 5 and 6 resulting from the matrix transformations. In the mixing process, dry samples were first mixed before water was added and further mixing done.

3.3 Casting and Compaction

32No. wet mortar was cast into prism moulds of sizes 160x40x40mm. The Compaction was done by tamping in accordance with BS 1881[15]. The specimens were demoulded after 24 hours.

3.4 Curing

Curing was done for 28 days by plastic sheeting before testing.

3.5 Testing

This was carried out on the prismatic samples in accordance with BS 1881 and ASTM C348-80 on a "Maurice Perrier" Flexural Testing Machine with a maximum capacity of 150KN and loading speed of 5 bars/second. The third point loading arrangement was adopted and fractures occurred within middle third. The Modulus of Rupture $= \frac{PL}{bd^2}$. Where P = maximum total failure load, L = Span, b =Width , d = Depth.

4.0 RESULTS AND DISCUSSIONS

Table 8 shows the response (average Modulus of Rupture) and the variance of the actual FC mixtures at each of the 16 points (10+6 controls). They are replicated to give 32 results.

The mean and variance are calculated from the expressions given in equations (9) and (10) respectively, where M is the number of replication and y_r is the response.

$$\tilde{y} = \left(\sum_{r=1}^{M_i} y_r\right)/M \quad \text{..............(9)}$$

$$S_i^2 = \frac{1}{M_{i-1}}\left[\sum_{r=1}^{M_i} y_r^2 - \frac{1}{M_i}\left(\sum_{r=1}^{M_i} y_r\right)^2\right] \quad \text{......(10)}$$

Degree of Freedom:

$$v_e = \sum_{i=}^{N} v_i = \sum_{i=1}^{16} (m_{i-1}) = 16$$

Replication Variance,

$$S_y^2 = \frac{1}{v_e} \sum_{i=1} S_i^2 = \frac{5.595}{16} = 0.3497$$

Replication Error, $S_y = \sqrt{0.3497} = 0.5913$

The Mathematical Model

The coefficients of the quadratic polynomial are determined from equations (7) and Table 8. The results are given below as follows:

$\beta_1 = 7.05$, $\beta_2 = 6.35$, $\beta_3 = 3.5$, $\beta_4 = 6.3$, $\beta_{12} = 1.2$, $\beta_{13} = 0.7$, $\beta_{14} = -0.5$, $\beta_{23} = 4.3$, $\beta_{24} = -0.7$, $\beta_{34} = -1.6$

Thus, from Equation (6), the mathematical model for the flexural strength of cement mortar stabilized with coir after 28 days of curing is:

$$\hat{y} = 7.05x_1 + 6.35x_2 + 3.50x_3 + 6.30x_4 + 1.2x_1x_2 + 0.7x_1x_3 - 0.5x_1x_4 + 4.3x_2x_3 - 0.7x_2x_4 - 1.6x_3x_4 \quad \text{... (11)}$$

Adequacy Tests for the Mathematical Model

The Student t and Chi-Square tests are employed respectively.

For this the statistical hypotheses are:

1. Null hypothesis, Ho: There is no significant difference between the experimental and theoretical results
2. Alternative hypothesis, H1: There is a significant difference between the two results.

To test for adequacy at each control point, the following statistic is built [16]

$$t = \frac{\Delta y}{\sqrt{(S_y^2 + S_y^2)}} = \frac{\Delta y \sqrt{n}}{S_y\sqrt{(1+\xi)}}$$

Where the **t** statistic has the student distribution, l is the number of control points, S_y is the replication variance, $\Delta y = y_{exp} - y_{theory}$, n = number of parallel observations at every point and ξ is error for predicted values of the response. The calculated t values for the 6 control points are; 1.742, 1.09, 0.328, 0.265, 0.949, 0.292. From the student t distribution table, $t_{table} = 2.58$, which is greater than any of the calculated t values.

The test statistic for goodness of fit is also given by [16, 17]

$$\chi^2 = \sum [(O - E)^2 / E]$$

Where, χ^2 is the Chi-Square Statistic, O is the observed (experimental) values from control test points and E is the expected (theoretical) values from the mathematical models.

The Chi-Square (χ^2) calculated values for the 6 control points are; 0.131, 0.050, 0.011, 0.638, 0.004, 0.834.

From the chi-square distribution table, χ_{table} = 9.49, which is greater than the calculated χ values. Hence for the two statistical tests, the Null Hypothesis Ho is accepted.

Computer Programme Results

A computer programme developed for the model was able to give possible mix combinations to match an inputted strength within a tolerance of ± 0.001N/mm^2 (see appendix 1). It also prints out the optimum strength predictable by the model and when inputted also generates combinations of mixes from which a producer could select. Appendix 3 shows two results' print-outs.

Discussions

The computer programme is found to be an efficient tool... Each desired strength that is inputted generates a combination of mixes. For example a flexural strength of 7.0N/mm^2 at 28 days of age generated a total of 27 combinations (see appendix 3). The trend shows that higher strength values have more limited choice of mix combinations. There are other trends from the results that confirm the scientific basis of the model, these include the fact that higher flexural strengths generated less water contents. This is in conformity with the known fact that the lower the w/c ratio, the higher the resultant strength. Higher strengths also generated less fine aggregates and fibre ingredients respectively. These entirely buttress the established fact that higher cement ingredient in mix combinations will enhance strength. The optimum flexural strength of 7.102079 N/mm^2 at 28 days of age generated six combinations.

It is observed that the optimum flexural strength of 7.102079N/mm^2 at 28 days of age ›m the model almost equaled the maximum strength value determined from the laboratory experiment (which is 7.05 N/mm^2) with a difference of only + 0.502079. This greatly accords the model a high credibility. It is not possible to compare the optimum value with the values obtained from cited previous works as they were compressive strength values which as expected are higher than the flexural strength value. However the generation of mix combinations followed the same pattern. Also, the optimum flexural strength obtained is higher than the optimum range values of 4.2 – 4.5 N/mm^2 obtainable for plain concrete of the same age[18]. The higher strength is likely due to the coir fibre component which is expected to reduce the brittleness of the FC component.

The model has some limitations. This is because in formulating it, certain parameters that affect FC strength were not considered; these include the workability of the wet mix, temperature and other weather considerations. Also the models cannot generate just any amount of strength beyond their optimum values as presented in the computer print-outs.

5.0 CONCLUSIONS

Scheffe's simplex design has been applied to optimize FC mixes. Also the strength of concrete varies with the proportion of the ingredient and not the total amount.

The mathematical model has been established in the form $\hat{y} = f(x_1, x_2, x_3, x_4,..)$ where x_1, x_2, x_3, and x_4 are the pseudo- components ratios of the various ingredients of the FC, i.e. water, cement, fine aggregate and fibre proportions. It has been established that the flexural strength of the model produced a maximum mean strength of 7.102079N/mm^2 after 28 days of curing at water/cement ratio of 0.484.

Tests for adequacy on the model which include student t- test and chi- square χ^2 - test showed acceptance of the results of the models. The advantage of this model over conventional mix proportioning is that a producer who wants a particular composite strength, may input the value and the computer programme will yield all possible discrete component combinations from which a choice can be made thereby avoiding the labourous and time wasting "trial and error" or "rule of the thumb" approach of conventional mix proportioning. If prices of ingredients are known and cost is a primary consideration, then for each of the possible predictions can be calculated a cost. This cost function can serve as decision criteria provided other considerations like workability are considered acceptable.

It is important to make the following recommendations; the model can be used for optimization of strength of FC made from other fibres other than coir, an index of workability specifying acceptable workability in relation to mix proportions may be introduced into the model, temperature and other weather conditions may also be introduced to enhance the models, and accuracy of the model may improve by taking more orders of polynomials of the simplex and further work on the modeling and optimization of Impact and Tensile strengths, Water Absorption Capacity, Permeability, Thermal Conductivity and Sound Proofness of Fibre Cement are suggested.

REFERENCES

1. Kateregga, J.K; Production process, Applications and Acceptance of Fibre concrete Roofing Products, Journal of the Network of African Countries on Local Building Materials and Technologies.Vol. (4), UN Centre for Human Settlements (Habitat), Nairobi, 1991, pp. 3-5.

2. Ndububa, E.E. An Appraisal of Vegetable Fibre Reinforcement in Construction, Project Report (M.Engr) - University of Nigeria, Nsukka, 1995. pp. 102-130.

3. Hover, C. K, Concrete Mixture Proportioning with Water-Reducing Admixtures to Enhance Durability: A Quantitative Model. Journal of Cement and Concrete Composites. Vol. 20, 1998, pp.113-119.

4. Scheffe, H. Experiments with Mixtures. Journal of the Royal Statistical Society, Ser. B., 20 (1958), pp. 344-360.

5. Khuri, A. I Harrison, J. M Cornell, J.A. Using Quantile Plots of the Prediction Variance for Comparing Designs for a Constrained Mixture Region: An Application Involving a Fertilizer Expt. Journal of Appl. Statistics,Vol.48, Part4,1999, pp.521-532

6. Akpuokwe, C.V., A computer Model for Optimization in Concrete Mix Proportioning, Project Report (B.Engr) - University of Nigeria, Nsukka, 1996.

7. Obam. S.O., A Mathematical Model for Optimization of Strength of Palm Kernel Shell Aggregate Concrete, Project Report (M.Engr) – Univ.of Nigeria, Nsukka, 1998.

8. Nwajagu, C.O, Use of Simplex Method in Determination of Optimal Rational Composition of Tiles Producible from Nigerian Clays. Nigerian Journal of Technology. Vol. 16, No.1, 1995, pp 36-45

9. Nwakonobi, T.U., A computer Optimization Model for mix Proportioning of Clay - Rice Husk - Cement Mixture for partition Walls of Animal Building, Project Report (M.Engr) University of Nigeria, Nsukka, 1997.

10. Akhnazarova, S. And Kafarov, V., Experiment Optimization in Chemistry and Chemical Engineering, Translated from Russian by Matskovsky, Vladimir, M. And Repyers, A., Moscow: MIR Publishers. 1982. Pp. 240-248.

11. Ndububa, E. E., Mathematical Modeling and Optimization of Strength of Fibre Cement Mixtures Using Scheffe's Simplex Lattice, Thesis (PhD) - University of Nigeria, Nsukka, 2004.

12. British Standards Institution BS 1377, BSI, London. 1975.

13. British Standards Institution BS 3681: Methods for the Sampling and Testing of Light-weight Aggregate for Concrete, Part 2, BSI, London. 1973.

14. Ndububa, E. E. and Ndububa, O. I., Performance Characteristics of Sawdust Concrete, Proceedings of the 5th National

Engineering Conference of The Kaduna Polytechnic, Kaduna. Vol. 5, No. 1, 1998, pp.47-52.

15. British Standards Institution BS 1881: Methods of Testing Concrete for Strength, Parts3 and 4, BSI, London. 1970. p.25.

16 Frank, H and Althoen, S. C., Statistics Concepts and Applications, Cambridge: Cambridge University Press. Low Priced Edition, 1995, p. 625.

17 Owen, F and Jones, R., Statistics, London: Financial Times/Pitman Publishing. 4th Edition, 1995, pp.430-435.

18 Neville, A. M. Properties of Concrete, Delhi: Pearson Education Pte Ltd. 4th Edition, 2003, p. 603.

APPENDIX 1

```
10 REM A QBASIC PROGRAMME THAT OPTIMIZES AND PREDICTS FIBRE CEMENT MIX PROPORTIONS
20 REM VARIABLE USED
30 REM Z1, Z2, Z3, Z4, X1 X2, X3, X4, Ymax, Yout, Yin
40 REM BEGIN MAIN PROGRAMME
50 LET COUNT = 0
60 CLS
70 GOSUB 100
80 END
90 REM END OF MAIN PROGRAMME
100 REM PROCEDURE BEGIN
110 LET YmAX = 0
120 PRINT
130 PRINT
131 PRINT "A MATHEMATICAL MODEL FOR THE COMPUTATION OF FIBRE CEMENT Mix"
PRINT "PROPORTIONS CORRESPONDING TO A DESIRED FLEXURAL STRENGHT"
170 PRINT
185 INPUT "ENTER DESIRED STRENGTH =", Yin
190 GOSUB 400
200 FOR X1 = 0 TO 1 STEP 01
210 FOR X 2 = 0 TO 1 – X1 – X2 STEP 01
230 FOR X 4 = 0 TO 1 – X1 – X2 – X3
240 LET Yout = 7.05 *X1 + 6.35* X2 + 3.5* X3 + 6.3* X4 + 1.2* X1*X2 +0.7*X1*X3
              - 0.5 X1*X4 + 4.3 * X2 * X3 – .7 X2*X4 – 1.6* X3*X4
250 GOSUB 500
260 IF (ABS(Yin – Yout) <=.001) THEN 270 ELSE 290
270 LET COUNT = COUNT + 1
280 GOSUB 600
290 NEXT X 4
291 NEXT X 3
292 NEXT X 2
291 NEXT X 1
295 PRINT
300 IF (COUNT>0) THEN GOTO 310 ELSE GOTO 340
310 PRINT "THE OPTIMUM VALUE OF STRENGTH PREDICTIBLE BY THIS MODEL IS "; YmAX; "N/sq. mm"
320 SLEEP 2
330 GOTO 360
340 PRINT "SORRY, DESIRED STRENGTH OUT OF RANGE OF MODEL"
350 SLEEP 2
360 RETURN
400 REM PROCEDURE PRINT HEADING
410 PRINT
415 LPRINT "PLEASE WAIT... CALCULATION IN PROGRESS"
418 PRINT
420 LPRINT "COUNT X1 X2 X3 X4 Y Z1 Z2 Z3 Z4"
430 PRINT
440 RETURN
500 REM PROCEDURE CHECK MAX
510 IF YmAX<Yout THEN YmAX = Yout ELSE YmAX = YmAX
520 RETURN
600 REM PROCEDURE OUT RESULTS
610 LET Z1 = 48* X1 +.5 * X2 +.65* X3 +.6* X4
620 LET Z2 = X1 + X2 + X3 + X4
630 LET Z3 = X1 + 1.5* X2 +21* X3 + 1.8 * X4
640 LET Z4 = 025* X1 +.03* X2 +.04 * X3 +.035* X4
650 PRINT TAB (1); COUNT; USING "###.###"; X1; X2; X3; X4; Yout; Z1; Z2; Z3; Z4
660 RETURN
```

APPENDIX 2

Computer print out of Matrix transformations and calculations

```
Z =[.48 .54 .65 .6;1 1 1 1;1 1.5 2 1.8; .025 .03 .04 .035]

Z =

    0.4800  0.5000  0.6500  0.6000
    1.0000  1.0000  1.0000  1.0000
    1.0000  1.5000  2.0000  1.8000
    0.0250  0.0300  0.0400  0.0350

>> B = inv (Z)

B =

    5.5556    -0.2778    -2.7778     55.5556
  -22.2222     5.1111     1.1111    177.7778
  -11.1111    -2.4444    -4.4444    488.8889
   27.7778    -1.3889     6.1111   -722.2222

>> X = [.5 .5 .5 0 0 0 .75 .75 0 0 .25 .25 .25 .25 .25 .5 .5 0 0 .5 .25 0 0 0 0
0 .25 .25 .25 .25;.5 0 0 .5 .5 0 .25 0 0 .75 0 .75 0 0 .25 0 .25 0 .25 .5 .25 .
25 .75 .25 .25 0 .25 .5 0 .5 .25;0 .5 0 .5 0 .5 0 .25 0 .25 .75 0 .75 0 .5 .25 .
25 .25 .25 .25 0 0 0 .75 0 .25 .5 .25 .5 0 .25;0 0 .5 0 .5 .5 0 0 .25 0 .25 0 0
.75 0 .5 0 .25 .5 .25 .25 .5 .25 0 .75 .75 .25 0 .25 .25 .25]

X =
Columns 1 through 7

    0.5000    0.5000    0.5000         0         0         0     0.750
    0.5000         0         0    0.5000    0.5000         0    0.2500
         0    0.5000         0    0.5000         0    0.5000         0
         0         0    0.5000         0    0.5000    0.5000         0
>> Z*X
ans =
Columns 1 through 7

    0.4900    0.5650    0.5400    0.5750    0.5500    0.6250    0.4850
    1.0000    1.0000    1.0000    1.0000    1.0000    1.0000    1.0000
    1.2500    1.5000    1.4000    1.7500    1.6500    1.9000    1.1250
    0.0275    0.0325    0.0300    0.0350    0.0325    0.0375    0.0262
```

APPENDIX 3

A Mathematical Quadratic Model for the Computation of Fibre Cement Mix Proportions Corresponding to a Desired Flexural Strength (i.e. Modulus of Rupture).

Enter Desired strength = 7.0

Count	x1	x2	x3	x4	y	z1	z2	z3	z4
1	0.500	0.500	0.000	0.000	7.000	0.490	1.000	1.250	0.028
2	0.520	0.470	0.000	0.010	7.001	0.491	1.000	1.243	0.027
3	0.540	0.440	0.000	0.020	7.001	0.491	1.000	1.236	0.027
4	0.540	0.440	0.010	0.010	7.001	0.492	1.000	1.238	0.027
5	0560	0.410	0.000	0.030	6.999	0.492	1.000	1.229	0.027
6	0.570	0.400	0.030	0.000	7.001	0.493	1.000	1.230	0.027
7	0.590	0.370	0.000	0.040	7.001	0.492	1.000	1.217	0.027
8	0.610	0.350	0.030	0.010	6.999	0.493	1.000	1.213	0.027
9	0.620	0.330	0.000	0.050	7.000	0.493	1.000	1.205	0.027
10	0.630	0.330	0.040	0.000	7.001	0.493	1.000	1.205	0.027
11	0.660	0.280	0.000	0.060	6.999	0.493	1.000	1.188	0.027
12	0.660	0.290	0.020	0.030	7.000	0.493	1.000	1.189	0.027
13	0.700	0.240	0.010	0.050	6.999	0.493	1.000	1.170	0.027
14	0.710	0.230	0.010	0.050	7.000	0.492	1.000	1.165	0.027
15	0.710	0.240	0.030	0.020	6.999	0.493	1.000	1.166	0.027
16	0.720	0.230	0.030	0.020	7.000	0.492	1.000	1.161	0.027
17	0.730	0.220	0.030	0.020	7.000	0.492	1.000	1.156	0.027
18	0.740	0.210	0.030	0.020	6.999	0.492	1.000	1.151	0.027
19	0.790	0.150	0.010	0.050	7.000	0.491	1.000	1.125	0.026
20	0.800	0.140	0.010	0.050	7.000	0.491	1.000	1.120	0.026
21	0.800	0.160	0.040	0.000	7.000	0.490	1.000	1.120	0.026
22	0.820	0.130	0.020	0.030	7.000	0.490	1.000	1.109	0.026
23	0.840	0.120	0.030	0.010	7.001	0.489	1.000	1.098	0.026
24	0.870	0.070	0.000	0.060	7.000	0.489	1.000	1.083	0.026
25	0.920	0.030	0.000	0.050	7.001	0.487	1.000	1.055	0.026
26	0.930	0.040	0.020	0.010	6.999	0.485	1.000	1.048	0.026
27	0.960	0.000	0.000	0.040	7.001	0.485	1.000	1.032	0.025

The optimum value of Strength predictable by this model is 7.102079 N/sq. mm.

A Mathematical Quadratic Model for the Computation of Fibre Cement Mix Proportions Corresponding to a Desired Flexural Strength (i.e. Modulus of Rupture).

Enter Desired Strength = 7.102079

Count	x1	x2	x3	x4	y	z1	z2	z3	z4
1	0.770	0.230	0.000	0.000	7.102	0.485	1.000	1.115	0.026
2	0.780	0.220	0.000	0.000	7.102	0.484	1.000	1.110	0.026
3	0.790	0.210	0.000	0.000	7.102	0.484	1.000	1.105	0.026
4	0.800	0.200	0.000	0.000	7.102	0.484	1.000	1.100	0.026
5	0.810	0.190	0.000	0.000	7.102	0.484	1.000	1.095	0.026
6	0.820	0.180	0.000	0.000	7.101	0.484	1.000	1.090	0.026

The optimum value of Strength predictable by this model is 7.102079 N/sq.mm

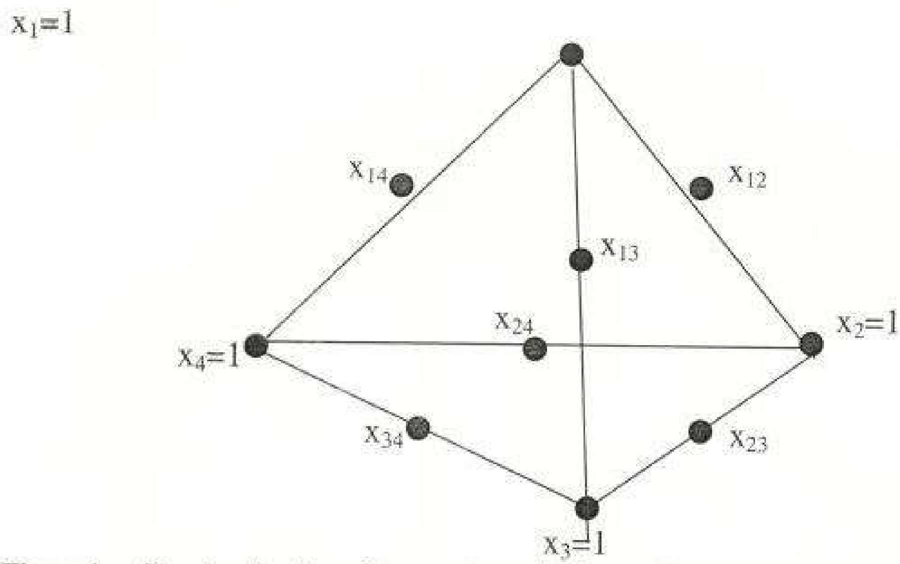

Figure1: Simplex Lattice of a quarternal mixture for a quadratic polynomial regression Model

Table 1: Weight Percent for Raw Materials in the Formulations

	WEIGHT PERCENT %		
	Formulation A	Formulation B	Formulation C
Soyabean Oil	51.38	50.38	49.13
Lithium hydroxide	0.025	0.025	0.020
Glycerol	12.87	12.45	14.50
Pentaerythritol	1.45	2.00	1.47
Phthalic anhydride	30.35	31.00	30.00
Benzoic acid	0.40	0.40	0.48
Xylene	3.525	3.745	4.40
TOTAL	**100**	**100**	**100**

Table 2: Variation of Acid Value with Time

TIME/HRS	ACID VALUE (mgKOH/g)		
	Formulation A	Formulation B	Formulation C
1	27.00	30.90	33.80
2	19.30	26.00	26.00
3	15.80	21.80	21.10
4	13.80	18.50	17.50
5	13.00	17.30	15.60
6	12.50	16.50	14.20
7	12.20	16.40	13.40

Table 3: Variation of Kinematic Viscosity with Time

TIME/HRS	KINEMATIC VISCOSITY		
	Formulation A	Formulation B	Formulation C
1	8.5	10.0	13.5
2	10.0	15.5	21.0
3	14.0	21.0	29.0
4	19.0	29.0	44.0
5	25.0	41.0	63.0
6	34.0	53.0	80.0
7	41.0	65.0	96.0

Table 4: Final Result Table

PARAMETER	Formulation A	Formulation B	Formulation C
Acid Value, MgKOH/G	12.0	16.5	13.0
Viscosity	43.0	72.0	105
Gardner's Colour	$G_5 - G_6$	$G_{11} - G_{12}$	$G_5 - G_6$
Stability	Good	Good	Good

Table 3: Design Matrix for {4,2} lattice

No. of points N	x_1	x_2	x_3	x_4	$y_{exp.}$
1	1	0	0	0	y_1
2	0	1	0	0	y_2
3	0	0	1	0	y_3
4	0	0	0	1	y_4
5	½	½	0	0	y_{12}
6	½	0	½	0	y_{13}
7	½	0	0	½	y_{14}
8	0	½	½	0	y_{23}
9	0	½	0	½	y_{24}
10	0	0	½	½	y_{34}

Table 4: Design Matrix for Control Test Points

N	x_1	x_2	x_3	x_4	$y_{exp.}$
1	¼	¼	¼	¼	C1
2	½	¼	0	¼	C2
3	¼	0	¾	0	C3
4	1/3	0	1/3	1/3	C4
5	1/5	2/5	0	2/5	C5
6	¼	1/8	½	1/8	C6

Table 5: Actual components (z) of{4,2} lattice

N	x_1	x_2	x_3	x_4	Response y	z_1	z_2	z_3	z_4
1	1	0	0	0	y_1	0.48	1	1	0.025
2	0	1	0	0	y_2	0.5	1	1.5	0.03
3	0	0	1	0	y_3	0.65	1	2.0	0.040
4	0	0	0	1	y_4	0.6	1	1.8	0.035
5	½	½	0	0	y_{12}	0.49	1	1.25	0.0275
6	½	0	½	0	y_{13}	0.565	1	1.5	0.0325
7	½	0	0	½	y_{14}	0.54	1	1.4	0.03
8	0	½	½	0	y_{23}	0.575	1	1.75	0.035
9	0	½	0	½	y_{24}	0.55	1	1.65	0.0325
10	0	0	½	½	Y_{34}	0.625	1	1.9	0.0375

Table 6: Actual Components for Control Test Points

1	¼	¼	¼	¼	C1	0.5575	1	1.575	0.0325
2	½	¼	0	¼	C2	0.515	1	1.325	0.0288
3	¼	0	¾	0	C3	0.6075	1	1.75	0.0363
4	1/3	0	1/3	1/3	C4	0.5766	1	1.5998	0.0333
5	1/5	2/5	0	2/5	C5	0.536	1	1.52	0.031
6	¼	1/8	½	1/8	C6	0.5825	1	1.6625	0.0344

Table 8: Flexural Strength Test Results and Replication Variance of Response

Expt. No. (N)	Repetition	Response y_r (N/mm^2)	Response Symbol	$\sum_{r=1}^{M_i} y_r$	$\bar{y}$	$\sum_{r=1}^{M_i} y_r^2$	S_i^2
1	1A 1B	7.5 6.6	Y_1	14.1	7.05	99.81	0.405
2	2A 2B	6.8 5.9	Y_2	12.7	6.35	81.05	0.405
3	3A 3B	3.2 3.8	Y_3	7.0	3.5	24.68	0.18
4	4A 4B	6.1 6.5	Y_4	12.6	6.3	79.46	0.08
5	5A 5B	7.1 6.9	Y_{12}	14.0	7.0	98.02	0.02
6	6A 6B	4.6 6.3	Y_{13}	10.9	5.45	60.85	1.445
7	7A 7B	6.1 7.0	Y_{14}	13.1	6.55	86.21	0.405
8	8A 8B	5.8 6.2	Y_{23}	12.0	6.0	72.08	0.08
9	9A 9B	6.6 5.7	Y_{24}	12.3	6.15	76.05	0.405
10	10A 10B	4.1 4.9	Y_{34}	9.0	4.5	40.82	0.32
11	11A 11B	6.8 7.0	C1	13.8	6.9	95.24	0.02
12	12A 12B	6.4 5.9	C2	12.3	6.15	75.77	0.125
13	13A 13B	4.5 4.9	C3	9.4	4.7	44.26	0.08
14	14A 14B	6.3 4.8	C4	11.1	5.55	62.73	1.125
15	15A 15B	6.2 5.6	C5	11.8	5.9	69.8	0.18
16	16A 16B	5.1 5.9	C6	11.0	5.5	60.82	0.32
							$\sum$=5.595