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KOLAWOLE . O. AIYESIMOJU

A NON-ITERATIVE PROCEDURE FOR COLEBROOK-WHITE ESTIMATION OF WATER DISTRIBUTION PIPE DIAMETERS

By

Kolawole .O. Aiyesimoju

Department of Civil Engineering
University of Lagos
Lagos, Nigeria

ABSTRACT

The most accurate of the numerous semi-empirical and fully empirical friction loss relationships for turbulent flows in pipes is the Colebrook-White formula. Unfortunately, its use in estimating the pipe size requires a number of iterations. This paper develops a relationship for computing the diameter of pipes based on the Colebrook-White Formula, which does not require any iteration but still estimates pipe diameters over the entire range of flows in the water supply industry within an error of about 2%. A comparison with existing non-iterative procedures confirms its simplicity and preferrability.

1. INTRODUCTION

Estimation of pipe diameters needs to be done accurately since an overestimation implies a high initial cost of conduit installation whilst underestimation will lead to functional inadequacies.

Laminar flows are governed by the Hagen-Poiseuille equation, which shows a linear relationship between head loss, pipe discharge and pipe diameter. This enables pipe diameters to be estimated accurately and without iteration for any given head loss and discharge. Attention here will be focused on turbulent flows which are governed by more complex relationships.

Aiyesimoju [1] has compared the accuracy of the most commonly used ones of the numerous semi-empirical and fully empirical friction loss relationships for turbulent flows in water supply practice and confirmed that the Colebrook-White formula yields the best results. The problem is that computation of pipe diameters with the formula involves iteration. Numerous graphical aids such as Moody's Chart [2] and Asthana's Diagram [3] which were developed to simplify using this formula, are however often not available and when they are, the errors introduced in scaling the graphs are often significant.

Recent work in this area has been about assessment or proper use of existing methods. For example, Cristensen et al. [4] was concerned only with the proper use of the Hazen-Williams formula whilst Khatibi et al. [5] focussed mainly on the estimation of friction factors for flow in nearly flat tidal channels. Although Aiyesimoju [6] obtained an accurate equation for estimating turbulent head losses in water distribution pipes which avoids iteration, the equation does not avoid iteration if it is to be used to estimate the pipe diameter.

The aim of this paper is to develop a similar relationship that will enable computing required pipe diameters for turbulent flows over the entire range of flow of practical interest in water supply practice, without any iteration. The developed relationship as well as four previously developed relationships that are generally applicable in practical situations, and that do not involve any iteration, will be compared with the most accurate Colebrook-White formula. These four other methods are 1) Ranga Raju and Garde equation 2) Prabhata and Akalank equation 3) Manning's formula and 4) Hazen-Williams formula

The comparison will be in the context of several examples of flow situations designed to reflect extremes of practical flow situations as follows (see Aiyesimoju [1]).

1. Pipe equivalent roughness height ε between 1 μm (less than the value for drawn tubing) and 1 cm (corresponding to riveted steel).

2. Pipe diameter D between 2cm and 2m.

3. Pipe relative roughness ε/D up to 0.033 (limit of Nikuradse's original data, upon which Colebrook-White formula is based [2].

4. Flow Reynold's Number R above 4000 (lower limit for turbulent pipe flow).

5. Maximum velocity V_{max} of water which is the fluid of interest, will be limited to 10m/s here (usually less than 3m/s in practice [2]) which implies maximum Reynolds's Number R_{max} in a pipe of diameter D, is $R_{max} = V_{max}*D/\nu = 10D/\nu$.

6. Minimum kinematic viscosity ν_{min} of water will be taken as 3E-7 m^2/s (a little below that for water at 100^{o}C) and maximum ν_{max} will be taken as 2E-6 m^2/s (a little more than that for water at 0^{o}C).

2. PIPE DIAMETER ESTIMATION METHODS

2.1 Colebrook-White Formula

The best known friction loss relationship for turbulent flows is the D'arcy-Weisbach formula

$$S_f = f\frac{V^2}{2gD} \tag{1}$$

where S_f is the friction slope (head loss h over pipe length L), V is flow velocity, D is pipe diameter, g is acceleration due to gravity and f is the friction factor. There are numerous relationships for the friction factor but the best is the Colebrook-White formula,

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{2.51}{R\sqrt{f}}\right) \tag{2}$$

where ε is the pipe roughness height (a measure of the typical height of protrusions from the pipe material surface), D is pipe diameter and R is pipe Reynold's number. The Colebrook-White formula is based on the results of experiments on sand-roughened pipes [2], with appropriate modification of the results to reflect observations of commercial pipes. The well known Moody's Chart [2] is simply a graphical presentation of this. If Q is pipe discharge, ν is the fluid kinematic viscosity and g is assumed as 9.8 m/s^2, then

$$V = \frac{4Q}{\pi D^2} \tag{3}$$

and

$$R = \frac{VD}{\nu} = \frac{4Q}{\pi\nu D} = \frac{1.273Q}{\nu D} \tag{4}$$

From Eqn. 1,

$$f = \frac{2gDS_f}{V^2}$$

and using Eqn. 3 above gives

$$f = \frac{12.1D^5S_f}{Q^2} \tag{5}$$

Substituting Eqns. 4 and 5 in Eqn. 2 yields

$$Q = -6.957D^{5/2}\sqrt{S_f}\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{0.567\nu}{\sqrt{D^3S_f}}\right) \tag{6}$$

Obviously, determining pipe diameter D with the above formula is cumbersome and thus numerous simpler, less accurate formulas, in some cases with limited range of applicability, have been developed as follows.

2.2 Ranga Raju and Garde's method

RangaRaju and Garde [7] proposed the following equations for pipe sizing

a) For $\varepsilon^3 gS_f/\nu^2 \leq 10^{-2}$

$$\frac{Q}{\nu\varepsilon}\left[\frac{\varepsilon}{D}\right]^{2.633} = 3.39\left[\frac{\varepsilon^3 gS_f}{\nu^2}\right]^{0.54} \tag{7a}$$

which can be rewritten as

$$D = \frac{\left[\frac{Q}{\nu}\right]^{0.38} \varepsilon^{0.62}}{1.59\left[\frac{\varepsilon^{3} g S_{f}}{\nu^{2}}\right]^{0.205}} \quad (7b)$$

b) For $\varepsilon^3 gS_f/\nu^2 > 10^{-2}$

$$\frac{Q}{\nu\varepsilon}\left[\frac{\varepsilon}{D}\right]^{2.633} = 2.85\left[\frac{\varepsilon^{3} g S_{f}}{\nu^{2}}\right]^{0.502} \quad (7c)$$

which can be rewritten as

$$D = \frac{\left[\frac{Q}{\nu}\right]^{0.38} \varepsilon^{0.62}}{1.488\left[\frac{\varepsilon^{3} g S_{f}}{\nu^{2}}\right]^{0.191}} \quad (7d)$$

2.3 Prabhata and Akalank's method

Prabhata and Akalank [8] proposed the following equation

$$D^* = 0.66(\varepsilon^{*1.25} + \nu^*)^{0.04} \quad (8)$$

where D^* is the non-dimensional diameter $D(gS_f/Q^2)^{0.2}$

ε^* is the non-dimensional roughness $\varepsilon(gS_f/Q^2)^{0.2}$

ν^* is the non-dimensional roughness $\nu(1/gS_fQ^3)^{0.2}$

2.4 Manning's formula

The widely used Manning's formula

$$V = \frac{1}{n} R^{2/3} S_f^{\ 1/2} \quad (9a)$$

can be rewritten for pipes flowing full as

$$D = \left(\frac{Qn}{0.283 S_f^{\ 1/2}}\right)^{3/8} \quad (9b)$$

where n is Manning's roughness coefficient and R is the hydraulic radius of pipe cross-section.

2.5 Hazen-Williams formula

This is widely used in the water supply industry and is as follows

$$V = 0.85 C_{H-W} R^{0.63} S_f^{\ 0.54} \quad (10a)$$

where $C_{H\text{-}W}$ is the Hazen-Williams coefficient. For pipes flowing full above yields

$$D = \left(\frac{3.586Q}{C_{H-W} S_f^{\ 0.54}}\right)^{0.38} \quad (10b)$$

2.6 Proposed method

It should be noted that Eq. 2 is implicit in f but experience suggests that the value of f on the left hand side of the equation is not very sensitive to the value of f on the right hand side. This suggests that reasonably accurate friction factors can be obtained by assuming a constant value for the f on the right hand side (say f_o). The best value of f_o over the entire range of flow of practical interest can be determined in this study. This implies that Equation 2 can be approximated as Equation 11a

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{2.51}{R\sqrt{f_o}}\right) \qquad (11a)$$

Substituting Eqs 4 and 5 in Eq. 11a and rearranging, yields

$$\frac{Q}{3.479D^{2.5}\sqrt{S_f}} = -2\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{2.51}{R\sqrt{f_o}}\right) \qquad (11b)$$

To avoid iterating for D, its value on the right hand side of Eq. 11b will be replaced by an estimate D' thus yielding

$$\frac{Q}{3.479D^{2.5}\sqrt{S_f}} = -2\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{2.51}{R\sqrt{f_o}}\right) \qquad (12)$$

A good estimate for D' can be obtained assuming a friction factor of f_o in Eq. 5, thus

giving Eq. 13 $f_o = \dfrac{12.1D'^5 S_f}{Q^2}$ (13)

which implies D' can be estimated as

$$D' = \left(\frac{f_o Q^2}{12.1 S_f}\right)^{0.2} \qquad (14)$$

And from Eq. 4, an approximation can be obtained for the Reynolds Number, for use on the right hand side of Eq. 12, as

$$R = \frac{1.273Q}{\nu D'}$$

Subsituting the above in Eq.12 yields Equations 15a and 15b.

$$D = \left(\frac{Q}{-6.957\sqrt{S_f}\log_{10}\left(\frac{\varepsilon}{3.7D'} + \frac{1.972\nu D'}{Q\sqrt{f_0}}\right)}\right)^{0.4} \qquad (15a)$$

which from Eq. 14 can be rewritten as

$$D = \frac{D'}{1.32 f_o^{0.2}\left(-\log_{10}\left(\frac{\varepsilon}{3.7D'} + \frac{1.972\nu D'}{Q\sqrt{f_0}}\right)\right)^{0.4}} \qquad (15b)$$

3. TEST PROBLEMS

Aiyesimoju [1] has designed eight test problems which cover up to the extreme range of practical flow situations in water supply practice. These are applicable here and will be adopted. They can be characterized as follows (where κ is ε/D).

1) low κ; low R, large D
2) low κ; large R, large D
3) large κ; low R, large D
4) large κ; large R, large D
5) low κ; low R, small D
6) low κ; large R, small D

7) large κ, low R, small D

8) large κ, large R, small D

The problem in each case is to determine the pipe diameters for the different extreme flow conditions listed in Table 1. The performance of a method in a test problem will be measured by the ratio (relative to Colebrook-White method) within which it is able to estimate the diameter for that test problem.

4. DISCUSSION AND CONCLUSION

From the results for all the test cases summarized in Table 1, Ranga Raju's method predicted diameters within a ratio of accuracy ranging from 1.006 to 1.077. This ratio ranged from 1.008 to 1.027 for Prabhata and Akalank's method, 1.005 to 1.320 for Manning's formula, 1.001 to 1.162, for Hazen-William's formula and from 1.001 to 1.014 for the proposed method. A kinematic viscosity coefficient of 3E-7 only is shown in the table. However increasing this value up to the maximum of 2E-6 had no noticeable effect on the results.

The closer the ratio of accuracy is to unity the better the prediction, which indicates clearly that the proposed method achieves better results than the other methods. The accuracy of Prabhata and Akalank's method is also good hence suggesting that these two methods may in fact be applicable over a wider range of flow conditions than is applicable in the water supply industry in the proposed method, f_o corresponds to a value of 0.035 which gave the least value of the worst ratio of accuracy (of 1.014) for all the eight test problems, on which basis equation 16 is recommended as a simple, accurate and non-iterative procedure for Colebrook-White estimation of water distribution pipe diameters

$$D = \frac{1.481D'}{\left(-\log_{10}\left(\frac{\varepsilon}{3.7D'} + \frac{10.54\nu D'}{Q}\right)\right)^{0.4}} \quad (16)$$

where

$$D' = 0.311\left(\frac{Q^2}{S_f}\right)^{0.2} \quad (17)$$

TABLE 1 - Test Cases and the computed pipe diameters and Ratios of Accuracy for different methods

Test	Colebrok-White (Ideal) Flow Parameters								Ranga Raju		Prabhata and Akalank		Manning's			Hazen-William's			Proposed	
Case No.	Rough. height (m) ε	Pipe Dia. (m) D	Relative Rough. $\kappa = \varepsilon/D$	Reynold's Number R	atic Visco-sity ν	Discha-rge (m^3/s) Q	Friction Factor f	Friction Slope S_f	Pipe Dia. Eqs. 7 D	Ratio of Accu-racy	Pipe Dia. Eq. 8 D	Ratio of Accu-racy	Mann. Rough coeff. n	Pipe Dia. Eq. 9b D	Ratio of Accu-racy	Haz. Will. coeff. C_{HW}	Pipe Dia. Eq. 10b D	Ratio of Accu-racy	Pipe Dia. Eq. 16 D	Ratio of Accu-racy
1	1.E-06	2.00	5.00E-07	3.98E+03	3.E-07	1.9E-03	4.0E-02	3.63E-10	1.86E+00	1.077	1.95E+00	1.024	0.01	1.60E+00	1.253	143	1.96E+00	1.020	2.01E+00	1.004
2	1.E-06	2.00	5.00E-07	1.00E+06	3.E-07	4.7E-01	1.2E-02	6.69E-06	2.02E+00	1.012	2.02E+00	1.011	0.01	2.01E+00	1.005	143	2.14E+00	1.068	1.98E+00	1.012
3	1.E-02	2.00	5.00E-03	3.98E+03	3.E-07	1.9E-03	4.5E-02	4.07E-10	1.94E+00	1.029	1.96E+00	1.021	0.019	1.99E+00	1.006	86	2.32E+00	1.162	2.01E+00	1.006
4	1.E-02	2.00	5.00E-03	1.00E+06	3.E-07	4.7E-01	3.0E-02	1.75E-05	2.07E+00	1.034	2.05E+00	1.023	0.019	2.14E+00	1.068	86	2.13E+00	1.063	2.00E+00	1.001
5	1.E-06	0.02	5.00E-05	3.98E+03	3.E-07	1.9E-05	4.0E-02	3.64E-04	1.90E-02	1.053	1.95E-02	1.024	0.01	2.13E-02	1.064	143	2.00E-02	1.001	2.01E-02	1.004
6	1.E-06	0.02	5.00E-05	1.00E+06	3.E-07	4.7E-03	1.3E-02	7.26E+00	2.04E-02	1.019	2.02E-02	1.008	0.01	2.64E-02	1.320	143	2.14E-02	1.071	1.98E-02	1.010
7	1.E-02	0.30	3.33E-02	3.98E+03	3.E-07	2.8E-04	6.6E-02	1.79E-07	2.96E-01	1.015	2.92E-01	1.027	0.019	3.12E-01	1.039	86	3.24E-01	1.081	3.04E-01	1.014
8	1.E-02	0.30	3.33E-02	1.00E+06	3.E-07	7.1E-02	6.0E-02	1.02E-02	2.98E-01	1.006	2.97E-01	1.010	0.019	3.18E-01	1.060	86	2.80E-01	1.071	3.03E-01	1.010

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