A NOVEL INSULATING CERAMIC POWDER DEVELOPED FOR APPLICATION IN THE MATERIALS INDUSTRY

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ABSTRACT

Finite element stress analysis relevant for the study of welded hollow rectangular 'K' Joints is presented. Thin shell theory and iso-parametric formulation were employed to obtain equilibrium equations. Thereafter, the effect of brace spacing was investigated by varying the spacing between the two braces and making computer runs. The results obtained for the present study show a gradual deterioration in strength as the spacing between the two braces was increased.

Due to geometrical differences between the rectangular hollow section and tubular joints, it was considered not necessary to compare results of present work directly with published ones on tubular joints. However, the observed trend between tubular and the rectangular hollow section 'K' joints studied is similar. The present investigation and others show that lapped rectangular 'K' joints are stronger than un-lapped joints of same cross-sections. Such results will form a data bank for a strength reliability analysis of steel truss bridges.

NOTATIONS

U^e	*element strain energy*
W^e	*external work done by the element during deformation*
u	*x nodal displacement*
v	*y nodal displacement*
X	*Global coordinate axis in the x direction*
Y	*Global coordinate axis in the*
y	*direction*
N_i	*element shape functions*
$[B]$	*element strain matrix*
$[D]$	*element material matrix*
$[J]$	*jacobian matrix*
q	*distributed pressure over element*

$[Q^e]$	*element total load per boundary*
$[K^e]$	*element stiffness matrix*
$[K^s]$	*assembled system stiffness matrix*
σ_f	*stress from finite element analysis*
σ_n	*calculated stress from formula*
Q	*assembled system load vector*
$\{\delta\}$	*element displacement vector*
$\{\Delta\}$	*system displacement vector*
ξ	*non-dimensional coordinate*
β	*non-dimensional coordinate*
Π	*system total potential energy functional*
σ	*stress*
ε	*strain*

INTRODUCTION

Lapped joints generally, and lapped tubular joints in particular are known to be stronger than un-lapped joints following a heuristic reasoning. However, for the purpose of bracing, it is not always feasible or even possible to brace a structural system with lapped joints through out. Hence, a strong case arises for using un-lapped tubular or square hollow 'K' joints which are in many cases welded on site due to ease of construction and transportation. As usual, one of the problems with welded steel construction and transportation is the development of stress concentrations or "hot spot" stresses. The "hot spots" affect the fatigue strength of both lapped and un-lapped joints in offshore structures and steel bridges.

Potvin et al [13] investigated the distribution of stress concentration in tubular joints using the finite element method but did not consider the effect of brace spacing on rectangular hollow joints. Paul et al [12] carried out a similar study but they too did not consider the effect of brace spacing on the distribution of stress concentrations. Gibstein [4] carried out a parametric study of T tubular joint, which precludes the study of brace spacing, and no rectangular hollow section joints were used in the study. Wordsworth and Smedley [16] and Toprac and Beale[15]also investigated the effect of stress concentrations in un stiffened tubular T,Y and K joints but did not investigate the effect of brace spacing on stress distribution in the tubular T,Y and K joints considered in their respective studies. Jiki [6,7,8] also investigated the effect of brace spacing on stress concentrations in tubular K joints, but did not however, examine the same effect on rectangular K joints.

Due to the importance of joints to the offshore oil industry and due to the persistent failure of offshore structures due to fatigue failure of joints, more research is still going on to understand better the behavior of offshore steel joints. For instance, Soh and Soh [14] have reported their findings of a study of 'Hot Spot' stresses of tubular K joints subject to combined in plane and out of plane loads. They modified existing parametric equations in the literature and obtained reduced peak 'Hot Spot' stresses. Unfortunately they did not study 'Hot Spot' stresses in rectangular hollow K joints. Cheng and Zang [3] introduced the concept of joint flexibility modeling to study the behavior of multi planar tubular joints. They argue that joints in practice are between pin-pin and fixed – fixed and a better model is that which introduces some kind of flexibility into the joint. Unfortunately, they too did not include the behavior of rectangular hollow section (RHS) K joints in their study. Jubran and Coffer[9,10] employed the super computer technology and the Finite element method to model in plane and out of plane Joints. Their treatment is similar to the one presented by the International Institute for Welding[5] because they included plasticity and partial yielding in the analysis of the joints they investigated. They also proposed relevant design equations of practical significance in their study. Unfortunately they too did not consider the effect of brace spacing in their modeling and proposed design equations. Similarly Parker[11] increased the strengths of HSS joints with in filled concrete but did not look at the effect of brace spacing on the strengths of the HSS connections considered.

Apart from offshore structures, hollow section steel joints are used in steel truss bridges (highway and rail bridges), where fatigue failure of these structures have been reported (see Ayyub et al [1], Ayyub and Ibrahim [2]) . According to the 1986 Federal Highway Administration (FHWA) statistics, there are 574,729 bridges on the highway system in United States. There are 11,013 steel truss bridges (sinple span and continuous span) on the interstate, major and minor highway systems inventoried by the FHWA. More than 80 % of the inventoried truss bridges on American highways are structurally deficient and / or functionally obsolete.

In order to salvage these obsolete bridges, Ayyub et al [1] have recommended The following: (1) to increase the redundancy of the bridges by pre stressing them, (2) to carry out a reliability study to ascertain the safety level of these bridges. (3) to carry out a fatigue reliability analysis to ascertain the safety level of the joints in the truss bridges. To carry out the implementation of the above salvage measures, Ayyub et al [1] carried out structural analysis of pre stressed bridge strusses to obtain forces in the members of the pre stressed bridge strusses. The reliability analysis of the pre stressed bridge strusses was carried out in Ayubb and Ibrahim [2]. Unfortunately, Ayubb and his co workers [1,2] have not reported their findings of studies on fatigue reliability of hollow square/rectangular K joints. However, information on strengths of above-mentioned joints is useful for fatigue reliability analyses to ascertain the safety levels of the deteriorated truss bridges. Unfortunately, information on stress analysis of these hollow joints is scarce.

Thus, the purpose of this study is to provide a data bank on stress analysis of a rectangular hollow section K joint that would be useful for fatigue reliability analysis of such joints

EQUILIBRIUM EQUATIONS..

The usual curved thin shell iso-parametric element was used for the present investigation[6, 7]. The shape of the element is shown in Fig. 2 and the element loading is shown in fig.3. Using potential energy formulation and the Rayleigh-Ritz process, the equilibrium equations were derived as follows:

The element total potential energy functional is given as:

$$\pi^e = U^e - W^e \qquad (1)$$

in which U^e is the strain energy of the element

W^e is the external work done by the element during deformation

The strain energy of the 8 noded element is given as:

$$U^e = \frac{1}{2}\iiint_V \sigma_{ij}\,\varepsilon_{ij}\,dvol \qquad (2)$$

in which σ_{ij} is direct stress

ε_{ij} is direct strain

Using iso-parametric formulation, the displacements are interpolated as:

$$u = \sum_{i=1}^{8} N_i(\alpha\beta) u_i \qquad (3)$$

$$v = \sum_{i=1}^{8} N_i(\alpha\beta) v_i \qquad (4)$$

The 8 node (curved) isoparametric element is mapped into the normalized square space through the following transformations:

$$x=\sum_{i=1}^{8} N_i(\alpha,\beta) x_i \tag{5}$$

$$y=\sum_{i=1}^{8} N_i(\alpha,\beta) y_i \tag{6}$$

where N_i are the shape functions corresponding to node I with Cartesian coordinates (x_i, y_i) in x,y system of axis and non dimensional coordinates (α_i, β_i) coordinate system where $\alpha_i, \beta_i = \pm 1$. For corner nodes and zero for mid nodes. Then the shape functions are given as:

$$N_i = [(1+\alpha\alpha_i)(1+\beta\beta_i) - (1-\alpha^2)(1+\beta\beta_i) - (1-\beta^2)(1+\alpha\alpha_i)\frac{\alpha_i^2\beta_i^2}{4} + (1-\alpha^2)(1+\beta\beta_i)(1-\alpha_i^2)\frac{\beta_i^2}{2} + (1-\beta^2)(1+\alpha\alpha_i)(1-\beta_i^2)\frac{\alpha_i^2}{2}] \tag{7}$$

In terms of the shape functions N_i the strain matrix [B] is given as :

$$[B]=\begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 \\ 0 & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial x} & \frac{\partial N_i}{\partial y} \end{bmatrix} \tag{8}$$

in which

$$\begin{Bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{Bmatrix} = [J]^{-1} \begin{Bmatrix} \frac{\partial N_i}{\partial \alpha} \\ \frac{\partial N_i}{\partial \beta} \end{Bmatrix} \tag{9}$$

and [J] is the Jacobian matrix given as :

$$[J]=\begin{bmatrix} \frac{\partial x}{\partial \alpha} & \frac{\partial y}{\partial \alpha} \\ \frac{\partial x}{\partial \beta} & \frac{\partial y}{\partial \beta} \end{bmatrix} \tag{10}$$

In terms of the strain matrix of equation (8) the element strain vector is given as :

$$\{\varepsilon\} = [B]\begin{Bmatrix} u_i \\ v_i \end{Bmatrix} \tag{11}$$

The element strain energy from equation (2) can also be written in terms of material matrix [D] as:

$$U=\frac{1}{2}\iiint_V \{\varepsilon_{ij}\}^T [D]\{\varepsilon_{ij}\} dvol \tag{12}$$

Substitution of equation (11) into equation (2) and assuming a unit thickness for the element gives:

$$U=\frac{1}{2}\iint_A [B]^T \begin{Bmatrix} u_i \\ v_i \end{Bmatrix}^T [D][B]\begin{Bmatrix} u_i \\ v_i \end{Bmatrix} dA \tag{13}$$

$$= \frac{1}{2}\iint_A [B]^T \{\delta\}^T [D][B]\{\delta\} dA \tag{14}$$

For the present investigation the element shown in Fig. 2 is loaded with a uniform Pressure q over the element boundary between nodes.

The external work done by the load during displacement of the boundary is given as :

$$W^e = \int_b q\{\delta\}^T db \qquad (15)$$

Substitution of equations (14) and (15) into equation (1) gives the total potential energy of the element as :

$$\pi^e = \frac{1}{2}\int\int_A [B]^T\{\delta\}^T[D][B]\{\delta\}dA - \int_b q\{\delta\}^T db \qquad (16)$$

$$= \frac{1}{2}\{\delta\}^T[K^e]\{\delta\} - Q^e\{\delta\} \qquad (17)$$

in which $[K^e]$ is the element stiffness matrix

Q^e is the element boundary loads such that :

$$[K^e] = \int_{-1}^{1}\int_{-1}^{1}[B]^T[D][B]\det[J]\, d\alpha\, d\beta \qquad (18)$$

$$Q^e = 2q \qquad (19)$$

In practice the element stiffness matrix of equation (18) is obtained by performing a numerical integration using Gauss integration rule [6,7] .

The Rayleigh-Ritz process gives equilibrium equations as :

$$\frac{\partial \pi^e}{\partial\{\delta\}} = [K^e]\{\delta\} - Q^e = 0 \qquad (20)$$

After assembly of element stiffness matrices and applied load into system stiffness matrix and system load vector, the system potential energy functional is given as :

$$\Pi = \frac{1}{2}\{\Delta\}^T[K^s]\{\Delta\} - \{\Delta\}^T\{Q\} \qquad (21)$$

in which $\{\Delta\}$ is system node displacement vector

$[K^S]$ is assembled system stiffness matrix

$\{Q\}$ is assembled system load vector

Again application of the Rayleigh- Ritz process leads to the system equilibrium equations as:

$$\frac{\partial \Pi}{\partial\{\Delta\}} = [K^s]\{\Delta\} - \{Q\} \qquad (22)$$

SOLUTION OF SYSTEM EQUILIBRIUM EQUATIONS.

The system equilibrium equations for the present problem were solved after assembly and application of boundary conditions using the Gauss elimination algorithm [6,7]. The technique of sub structuring was used in the solution process so as to reduce the large core memory requirement in the banded form of storage system adopted. The gain in using this form of solution technique is that less core memory was used at each instance of the elimination and back substitution process. After the solution was carried out the results were presented element by element. The results of the solutions are presented in figures 4-8 of the present work.

INVESTIGATION OF STRESS CONCENTRATIONS

Due to symmetry, only half the ' K' joint was employed to investigate the stress concentrations in the present study as shown in Fig. 1. The finite element discretization and loading of the joint for the purpose of stress analysis are also shown in Fig.1. In order to use the Gauss elimination process, element node numbering system was adopted [6, 7]. Thereafter, computer runs were made and both normal or direct stresses as well as shear stresses were obtained for each element. Gauss points with highest element direct and shear stresses from the finite element analysis were marked as stress concentrations. Stress concentrations were recorded both in the chord and the two braces of the 'K' joint. The results for the chord are shown in Figs. 5 and 6 for Gauss point type 1 for direct stresses. The gap between the two braces was kept constant at 50 mm and stress concentration factor (S_{CF}) was obtained as :

$$S_{CF} = \frac{\sigma_F}{\sigma_N} \quad (23)$$

in which σ_F is finite element stress in the longitudinal axis of the chord and σ_N is average stress in the longitudinal axis given as :

$$\sigma_N = \frac{P}{A} + \frac{M}{Z} \quad (24)$$

in which P and M are applied axial load and moment respectively and A and Z are area and elastic modulus of the section respectively .However, the result for the 50 mm gap is not shown here because it is not critical. Critical cases are those for 60 mm and 70 mm gaps and are shown here in Figs. 4 to 8.

GAP STUDY

The effect of gap between braces (brace spacing) on the stress concentrations in the joint was investigated by sampling or marking some elements and studying the behavior of the stress concentration factors in the elements as gap between the two braces was increased from zero gap to 70 mm gap. The results of the effect of brace spacing or gap on the chord are shown in Figs. 5 and 6 and, those of the effect of corner center chord location on the stress concentration factors are shown in fig 7. The effect of shear stresses on the chord for zero gap and 70 mm gap are compared in Fig. 8. Accuracy of FE and experimental tests is shown in fig 4. Differences between the two solutions are within 10%. This is because a coarse mesh was used in the FE solution.

DISCUSSION

Gauss point type 1 was used in the sample study. Plots of stress concentration factors (S_{CF}) against the element Gauss points for the center and corner of the chord were made and critical values are shown in Figs. 5 and 6.

The results for zero gap up to 30 mm and even 50 mm gap follow a similar pattern and the stress concentration factors are not critical and are therefore not shown here. However, from the gap of 60 mm up to 70 mm , as shown in Figs. 5 and 6, the pattern has drastically changed and at the middle of the range of the Gauss points, the stress concentration factors , apart from having higher values, have increased from negative to positive values for corner of the chord while the center of the chord still displayed negative values. The change from negative to positive stresses indicates failure or overstress at the points or the elements where the change occurs.

The results for the braces are not shown here because they are not critical .The effect of geometry (corner/ center chord location) on stress concentration factors is recorded in fig 7. which shows that for a gap of 70mm the chord experiences a severe over-stress due to both bending and shear. This fact is confirmed in fig 8.

The results of shear stresses for zero gap and 70 mm gap are shown in Fig. 8 and they indicate high shear stress concentration factors of about 45 which occur on element sample point 5 (Fig. 3). There is clearly a marked difference as far as zero gap and 70 mm gap are concerned. The result obtained in the present study using rectangular hollow K joint is very similar to that reported by Jiki [7] using a tubular K joint model. This further supports the findings by Jiki [7] that lapped un-stiffened tubular K joints are stronger than un-lapped un stiffened tubular K joints.

CONCLUSION

From the above results it can be said that lapped un-stiffened rectangular hollow K joints are stronger than un lapped un stiffened rectangular hollow K joints. The results obtained in the present study using rectangular hollow (un stiffened) K joints are in agreement with those reported earlier by Jiki [7] for un lapped un stiffened tubular (circular) K joints.

In both studies, both high punching shear stresses from braces and bending of the chord seem to be critical when the spacing between braces exceed 50 mm. Therefore the braces in un stiffened rectangular hollow section K joints should be spaced not more than 60mm apart with brace/chord ration of at least ½ for better results.

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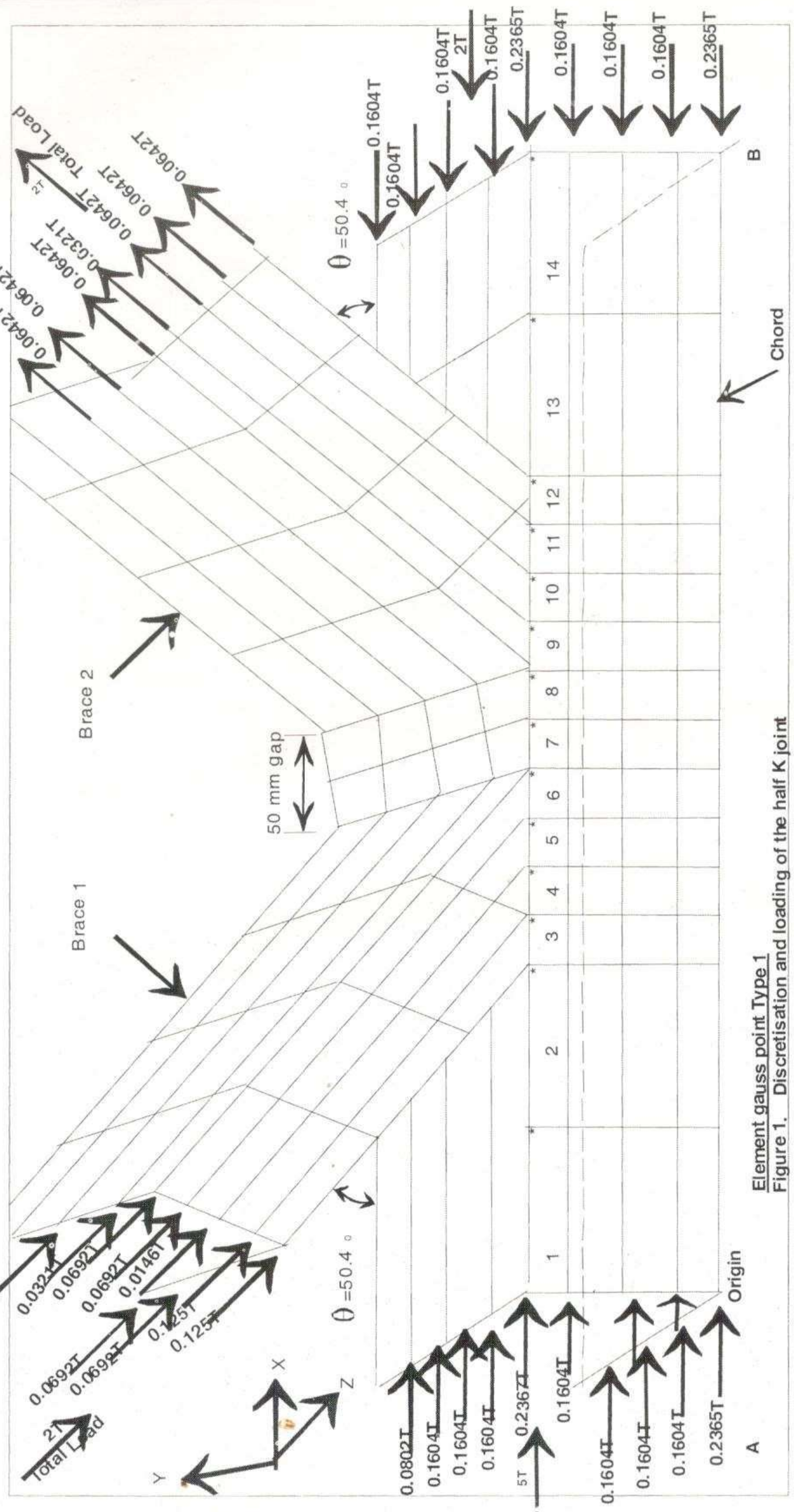

Element gauss point Type 1

Figure 1. Discretisation and loading of the half K joint

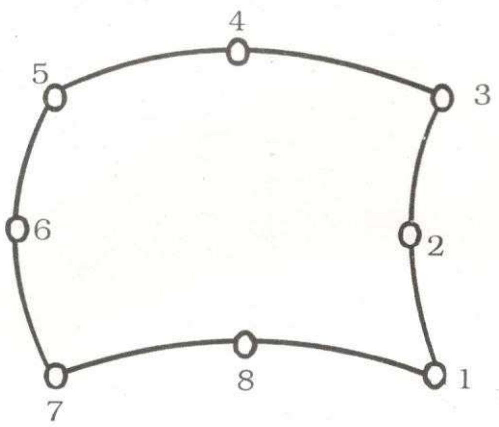

Figure 2. 8 noded isoparametric element.

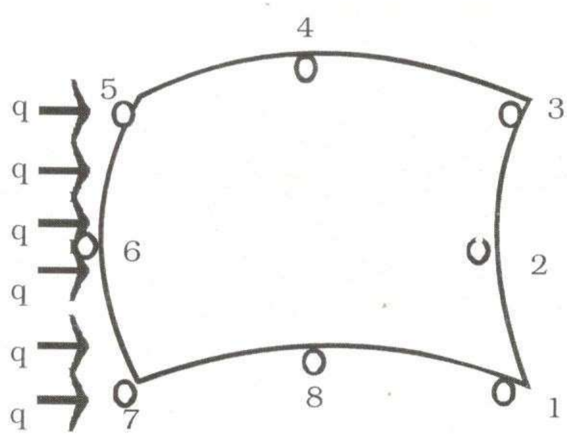

Figure 3. Applied pressure on element

Figure 4. Comparison of F.E. and Exp.Solutions for 50 mm gap.

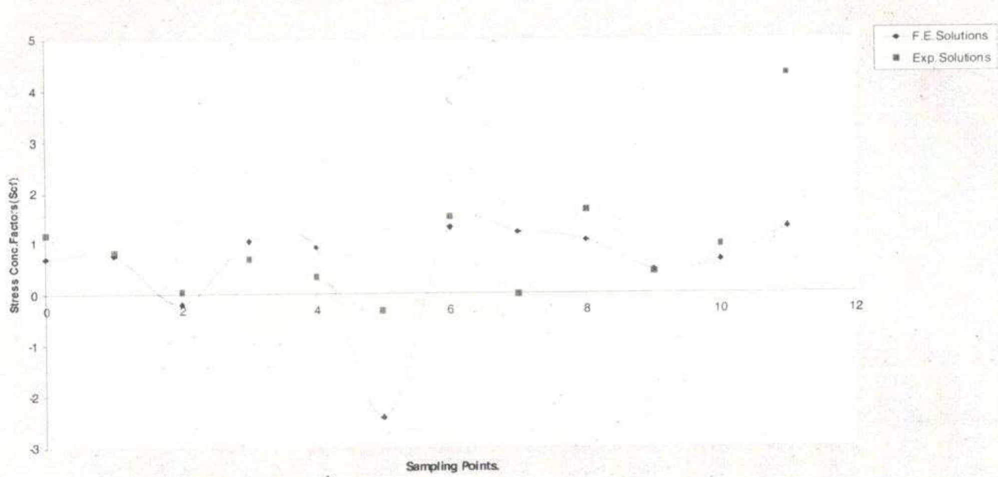

Figure 5. Effect of increasing gap on stress con factors(Scf) of top Chord

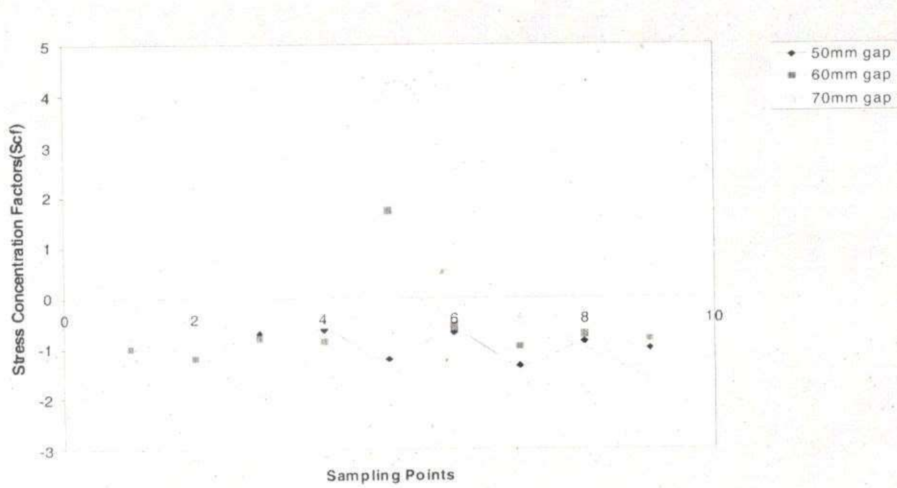

Figure 6. Effect of increasing gap on Stress con. Factors(Scf) at corner of Top Chord

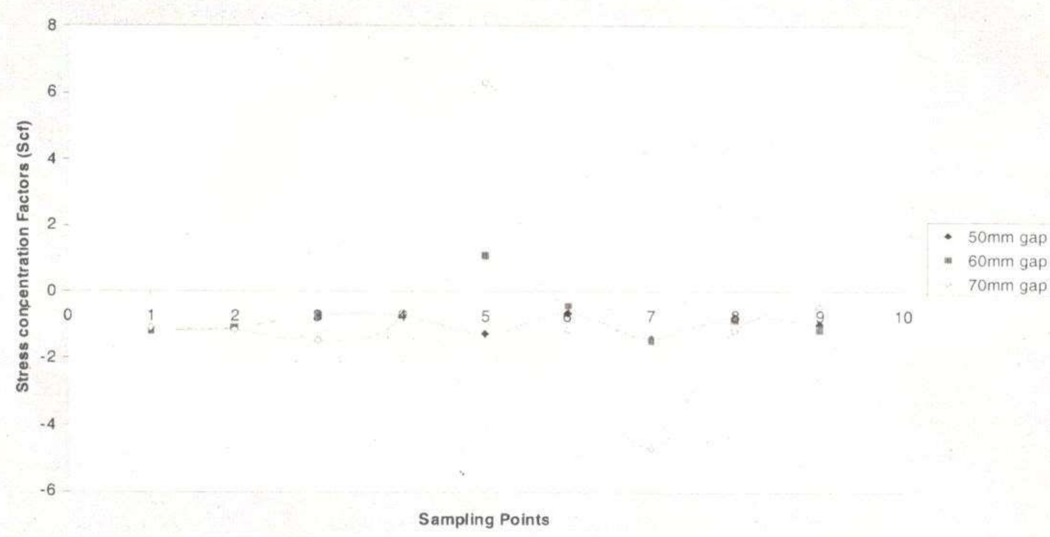

Figure 7. Effect of geometry on Stress conc.Factors(Scf) for 70 mm gap Joint.

Fig. 8. Comparison of shear Stress con.Factors(Scf) for zero and 70mm gap