.DETERMINING THE COMPOSITION OF ALLOYS AT A EUTECTIC POINT USING EQUATION OF A STRAIGHT LINE

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ABSTRACT

A eutectic point indicates the composition of an alloy when cooling is complete at isothermal conditions. Simple linear equation was developed and applied to an imaginary Y-axis to evaluate the eutectic composition by solving the various equations that were developed. Solution of the equations revealed that the eutectic composition for the case study Pb-Sn, Bi-Cd and AL-Si alloy systems are 39.89% Pb, 60.11% Sn, 58.01%, Bi, 41.99% Cd and 90.94% AL, 9.06% Si respectively. These values are very close to experimental results.

It is concluded that equation of straight line could be used to predict/determine the eutectic composition of binary alloy.

INTRODUCTION

When two metals which are completely soluble in the liquid state but completely insoluble in the solid state solidify, they do so by crystallizing out as alternate layers of the two pure metals. Thus a laminated type of structure is obtained and is termed a eutectic. This eutectic mixture is always of a fixed composition for the two metals concerned and it melts at a temperature below the melting points of either of the metals (Higgins). Lead and tin may form a eutectic alloy. The melting point of lead is 400°C while the melting point of tin is 327°C. If increasing amounts of lead are added to tin, the melting point of tin is progressively depressed along a line ZE (Fig. 1). Similarly if increasing amounts of tin are added to lead the melting point of lead decreases progressively along a line WE (Fig.1). These depressions of freezing point lines meet at a point called the eutectic point. For a tin-lead alloy system, the eutectic point is 38% wt.Pb and 62% wt.Sn.

The eutectic point is usually determined in the laboratory by experiments. This practical method is associated with high cost of labour and material as well as experimental errors. The aim of this paper is to eliminate the above disadvantages by using equation of straight line to theoretically determine the eutectic composition of simple binary alloy systems.

DESCRIPTION OF THE WORK

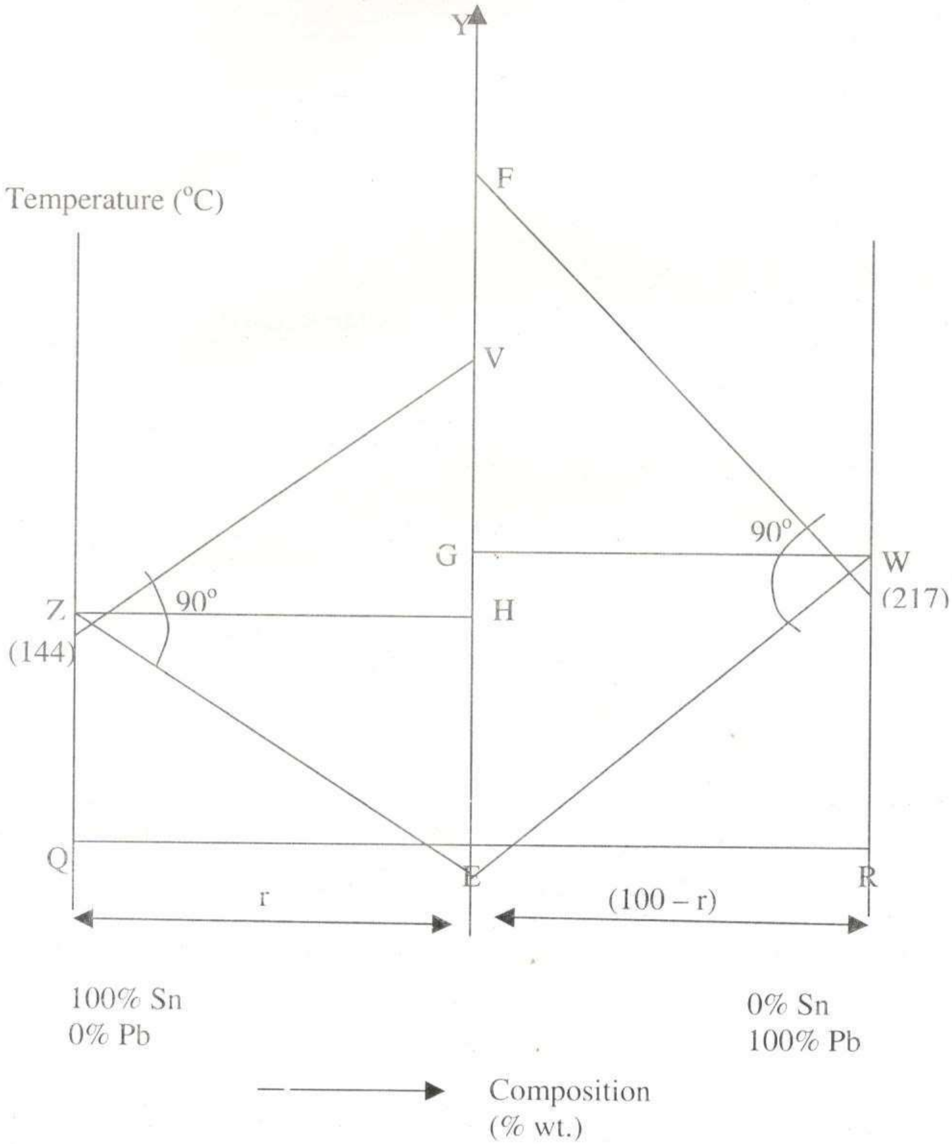

Fig. 1: Thermal equilibrium diagram of Sn-Pb alloy system showing the reference Y-axis and the intercepting straight lines.

In fig. 1, WE and ZE are the liquids lines for lead and tin respectively. As the liquidus lines must start, one from the melting point of lead (400°C) and the other from the melting point of tin (327°C), the two lines WE and ZE) are drawn to meet at an assumed eutectic point E – personal reasoning. The eutectic temperature is 183°C. It is assumed that the eutectic temperature line QER (Fig. 1) is zero on the temperature scale. The basis for this assumption is that if 183°C is represented by 0°C and subtracted from the melting points of lead and tin to get lower values, the result we shall get shall be the same. The aim of this assumption is to get the origin for the reference Y-axis.

Consequently, W (melting point of lead) is represented by the value 400-183 = 217°C and Z (melting point of tin) by the value 327-183 = 144°C. The two liquidus lines are joined from points 217°C and 144°C to meet at the assumed point E. From E, draw a reference Y-axis line EY. From W, draw a line WF perpendicular to WE to cut EY at F. From Z, draw a line ZV perpendicular to ZE to cut EY at V.

The scale of the composition axis is taken to be 1cm = 1% so that it is 100cm long. Also the scale of the temperature axis is taken to be 1cm=1°C.

The temperature axis is the Y-axis and the composition axis is the x axis.

Therefore, from Fig.1, if QE = rcm, ER will be (100-r) cm since QR = 100cm (representing 100%).

Let gradient of WE = $m = \dfrac{217}{100-r}$ (i)

Let gradient of ZE = $n = \dfrac{144}{r}$ (ii)

To find the relationship between n and m,

$$\frac{n}{m} = \frac{144}{r} \times \frac{100-r}{217} = \frac{14400-144r}{217r}$$

$\therefore n(217)r = m(14400-144r)$

or

$$n = \frac{m(14400-144r)}{217r} \qquad \text{.............(iii)}$$

EQUATIONS The equation of a straight line is given by: $Y = Px + C$ (Ogum)

Where

P = gradient of the line

C = its intercept on Y-axis

Replacing P with M,

the gradient of a straight line is given as

$$m = \frac{\text{increase in y value}}{\text{increase in x value}}$$

or

$m = \dfrac{dy}{dx}$ (Stroud)

Limiting value

If the gradient of a straight line is m, the gradient of another straight line perpendicular to it $= \dfrac{-1}{m}$

The equation of line WE is given by:

$$Y = mx + 0 \quad \text{.....(iv)}$$

(Since 0 is its intercept on the Y-axis)

The equation of line WF is given by:

$$Y = \frac{-x}{m} + 217 + K \quad \text{.....(v)}$$

Where K = FG

The equation of line ZE is given by:

$$Y = \frac{m(14400-144r)x}{217r} + 0 \quad \text{.....(vi)}$$

The equation of line ZV is given by:

$$Y = \frac{-(217r)x}{m(14400-144r)} + 144 + J \quad \text{.....(vii)}$$

Where

$$J = VH$$

Subtracting (vii) from (v),

$$0 = \frac{-x}{m} + \frac{(217r)x + 73}{m(14400-144r)} + K-J \quad \text{.....(viii)}$$

Subtracting (vi) from v

$$0 = \frac{-x}{m} - \frac{m(14400-144r)x}{217r} + 217 + K \quad \text{.....(ix)}$$

Subtracting (ix) from (viii),

$$0 = \frac{(217r)x}{m(14400-144r)} + \frac{m(14400-144r)x}{217r} - 14 \quad \text{...(x)}$$

Subtracting (vii) from (iv),

$$0 = mx + \frac{(217r)x}{m(14400-144r)} - 144-J \quad \text{.....(xi)}$$

Subtracting (xi) from (x)

$$0 = \frac{m(14400-144r)x}{217r} - mx \quad \text{.....(xii)}$$

Dividing (xii) by mx

$$0 = \frac{(14400-144r)}{217r} - 1 \quad \text{.....(xiii)}$$

Multiplying (xiii) by 217r,

$$0 = 14400 - 144r - 217r \quad \text{.....(xiv)}$$

$$\therefore 316r = 14400$$

$$r = 39.89cm = QE$$

Since 1cm = 1%, 39.89cm = 39.89%

E is 39.89cm away from 0% Pb

$\therefore$ eutectic composition at E

= 39.89% Pb, 60.11% Sn.

The experimental value is 38% Pb, 62% Sn (Higgins). The difference between the experimental and the analytical values should be attributed to experimental errors. Using this method for other simple binary eutectic alloy systems, the results show that the method is correct.

Other eutectic alloys systems considered are the bismuth-cadmium and aluminum-silicon alloys systems.

For the bismuth-cadmium alloy system, the freezing points of bismuth and cadmium are 271 and 321°C respectively. The eutectic temperature is 140°C and the experimental composition of the eutectic mixture is 60% Bi, 40% Cd. For the silicon-aluminium alloy

system, the freezing point of aluminum is 659°C (Holderness and Lambert), the eutectic temperature is 577°C and the experimental composition of the eutectic mixture is 11.6% Si, 88.4% Al (Higgins).
The freezing point of silicon is 1410°C (Hill and Holman).

The governing equation for the calculation of the composition of a simple binary alloy at a eutectic point by analytical method is equation (xiv).

$0 = 14400 - 144r - 217r$ ……………..(xiv)

This can be re-written as

$0 = A(100) - Ar - Br$

where

r = Composition of the metal that has the higher freezing point.

A = Freezing point of the metal that has the lower freezing point minus eutectic temperature.

B = Freezing point of the metal that has the higher freezing point minus eutectic temperature.

Therefore, for Bi – cd alloy system,

$A = (271-140) - = 131$, $A(100) = 13100$

$B = 321-140 = 181$

$\therefore 0 = 13100 - 131r - 181r$

$\therefore r = 41.99\%$

= Composition of cadmium.

Composition of bismuth = 100 – 41.99 = 58.01%

For the silicon – aluminum alloy system,

$A = (659 - 577) = 82$, $A(100) = 8200$

$B = 1410 - 577 = 833$

$\therefore 0 = 8200 - 82r - 833r$

$\therefore r = 8.96\%$

= Composition of silicon

$\therefore$ Composition of aluminum

= 100% – 8.96%

= 91.04%

RELEVANCE OF THE RESEARCH

The research is used to eliminate labour and material cost that is associated with the experimental method of determining the composition of alloys at a eutectic point. Another usefulness of the research is that experimental errors are not involved. The method therefore, gives a more accurate result.

ANALYSIS OF RESULT

The results of all alloy considered are given in table 1 below.

Alloy system	Experimental values	Analytical values	% variation $\frac{\text{Exp. Value - Analy. Val.x100}}{\text{Experimental value}}$
Sn-Pb	62%Sn	60.11% Sn	Sn = 3%
	38% Pb	39.89% Pb	Pb = -5%
Bi-cd	60% Bi	58.01%Bi	Bi = 3.3%
	40% cd	41.99%cd	Cd = -5%
Si-Al	11.6%Si	8.96% Si	Si = 22.76%
	88.4%Al	91.04% AL	Al = -2.99%

Table 1: Comparism of experimental and analytical results.

CONCLUSION

Analysis of the result shows that equation of straight line can be used to determine the composition of simple binary alloy systems.

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