APPLICATION OF OSADEBE'S ALTERNATIVE REGRESSION MODEL IN OPTIMISING COMPRESSIVE STRENGTH OF PERIWINKLE SHELL – GRANITE CONCRETE

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ABSTRACT

Osadebe's Regression method of concrete design was used to formulate a model. The model is a mathematical equation of dependent variable – Response - and independent variables – Predictors. The model equation had several coefficients, whose values were determined by carrying out few laboratory tests. At the end of the laboratory tests, the compressive strengths were used to complete the model equation. More laboratory test points were carried out to test the adequacy of the model using student's t – test statistics. The test statistics found the model adequate for predicting the compressive strength periwinkle shell – granite concrete when the mix ratio is known and vice versa.

INTRODUCTION

Several mix design methods exits. British method, American method, Road Note 4 method are some of the known methods. Each of these empirical methods requires trial mixes to come up to a design that will give a desired concrete property. Time spent from the design and crushing of the trial mixes is about a minimum of seven days. When the trial mixes failed to give what was expected another design and trial mixes would be made. It is obvious here that the empirical method is not only laborious but also time consuming and resource wasting. These problems attracted the attention of the Authors to formulate a statistical experiment model to care of the time wasted, labour and resource wasting.

Mix design of concrete is a means of producing the most economical and durable concrete that meet with certain properties as consistency, strength and durability by properly and systematically combining the ingredients at relative proportions (Neville (1983)). Two categories of mix design method exist. One of them is the empirical method and the other is the statistical method (Simon and others (1997)).

With empirical methods one will rely on information from historic data or laid down rules (codes) for mixture proportioning. Historic data is just about the rule of thumb. It will only work for the materials that were used for it in the past. The method will not incorporate new material other than the ones used in the past. The use of trial mixes and repeated mixes (further trials) to enable all specified criteria to be met is inevitable. Hence, it will be difficult if not impossible to achieve an optimal mixture for desired criterion with historic information method of concrete mixture design. The use of laid down rules or codes in concrete mix design is an improvement over the historic information method. This is so because there is a stepwise (though rigorous) approach and it can accommodate a wider range of materials. Some common laid down rules in concrete mix design are Road Note No.4, ACI standards 211.1 – 77 and 211.3 – 75 and 1975 British method. (Simon and others (1997), Road Research (1950), ACI Committee 211 (1974) and Teychenne and others (1975)).

Statistical experimental method is the use of theory of statistics and some specified laboratory results from practical experiments to formulate a mathematical model (equation), which will later be used to predict concrete mix ratio when a desired target strength is known. The long run advantage will always pay off or compensate the initial input and justifies it. Unlike the empirical method, there won't be any further trial mixes once the model has been formulated. The limitation to this method is the model is always particularized and can not be used as a generalized model. For example, if a model is formulated for periwinkle aggregate concrete, it can not be used to predict the concrete mixture ratio involving aggregate other the periwinkle shells. Some statistical experimental methods are Regression design (Obam (2006), Simplex design (Scheffe (1958), Scheffe (1963), Obam (1998), Axial design, Mixture experiments involving process variables, Mixture model with inverse terms (Draper and others (1997) and K-model (Draper and others (1999).

OSADEBE REGRESSION DESIGN

Osadebe (2003) showed that a set of parameters, X_1, X_2, X_3, . . ., X_N known as predictors can be used to predict the probable value of a dependent variable, Y at a particular degree of certainty. This means that so long as the values of the predictors are known the corresponding value of the dependent variable can be predicted with some degree of certainty. In this method few points of observation will be used to formulate the model. He assumed that the response function, F(z) is continuous and differentiable with respect to its predictors, Z_i. By making use of Taylor's series the response function could be expanded in the neighborhood of a chosen point Z_0.

$F(z) = \sum F^m(Z_0)(Z_i - Z_0)^m / m!$ .... eq. 1

$0 \le m \le \infty$

Expanding eq. 1 up to second order gives

$F(Z) = F^0(Z_0)(Zi - Z_0)^0 / 0! + \sum F^I(Z_0)$
$(ZI - Z_0)^I / 1! + \sum F^{II}(Z_0)(Zi - Z_0)^2$
$/ 2! + \sum F^{II}(Z_0)(Zi - Z_0)$
$(Zj - Z_0) / 2!$ eq.2

$1 \le i \le j \le 5$

Equations 2 can be rewritten as

$F(Z) = F^0(Z_0) + \sum F^I(Z_0)(Zi - Z_0) + \sum F^{II}(Z_0)$
$(Zi - Z_0)^2 / 2! + \sum F^{II}(Z_0)(Zi - Z_0)$
$(Zj - Z_0) / 2!$ eq.3

$I \le i \le j \le 5$

Assuming the origin, Z_0 is equal to zero, then

$F^0(0) = b_0$ ....eq. 4

$F^I(0)_{(0 \le i \le 5)} = b_{0\,(0 \le i \le 5)}$ .. ..eq. 5

$F^{II}(0)_{(0 \le i \le 5)} = b_{0\,(0 \le i \le 5)}$ ....eq. 6

$F^{II}(0)_{(0 \le i \le j \le 5)} = b_{0\,(0 \le i \le j \le 5)}$ .... eq. 7

Substituting eqs. 4, 5, 6, and 7 into eq. 3 will give

$F(Z) = b_0 + \sum b_i Z_i + \sum b_{ii} Z_I^2 + \sum b_{ij} Z_i Zj$ ..eq.8

$1 \le i \le j \le 5$

the actual mixture components are designated Si. The fractional portion of the components as a ratio of the total components, S is designated Zi. That is

$S = \sum Si$ eq.9

$Zi = Si / s$ eq.10

$\sum Zi = 1$ eq.11

Where $1 \le i \le 5$

Multiplying eq.11 by b_0 and Zi will respectively give

$b_0 = b_0 Z_1 + b_0 Z_2 + b_0 Z_3 + b_0 Z_4 + b_0 Z_5$....eq12

$Z_i = Z_1 Z_i + Z_2 Z_i + ... + Z_i^2 + ... + Z_i Z_5$..eq13

Where $1 \le i \le 5$

Rearranging eq. 13 gives

$Z_i^2 = Z_i - Z_1 Z_i - Z_2 Z_i - .... - Z_i Z_5$ ....eq 14

$1 \le i \le j \le 5$

Factorizing eq. 15 will give

$Y = \sum \alpha I\, Zi + \sum \alpha ij\, ZiZj$ eq.16

Where $\alpha_I = b_0 + bi + bii$ eq.17

$\alpha ij = bij - bii - bjj$ eq.18

different points of observation will have different responses with predictors at constant coefficients. Nth observation will have Y^n responses corresponding with Z_i^n predictors. This means that eq. 16 supposed to be vector equation, putting it in matrix form will give

$[Y^n] = [Z_i^n][\alpha]$ eq.19

$[\alpha]^T = [\alpha_1\ \alpha_2\ \alpha_3\ \alpha_4\ \alpha_5\ \alpha_5\ \alpha_{12}\ .\ .\ .$
$\alpha_{25}\ \alpha_{34}\ \alpha_{35}\ \alpha_{45}]$ eq.20

$[Y^n]^T = [Y^1\ Y^2\ Y^3\ Y^4\ Y^5\ Y^{12} . . .$
$Y^{25}\ Y^{34}\ Y^{35}\ Y^{45}]$ eq.21

$[Z_i^k] = [Z_1^k\ Z_2^k\ Z_3^k\ Z_4^k\ Z_5^k\ Z_{12}^k\ . . .$
$Z_{25}^k\ Z_{34}^k\ Z_{35}^k\ Z_{45}^k$ eq.22

Where $1 \le n \le 15$ and $1 \le k \le 5$

From eq.19 the coefficient of the regression is

$[\alpha] = [Z_i^n]^{-1}[Y^n]$ eq.23

Eq.16 is the equation of regression model. Eq. 23 gives the values of coefficient of eq. 16. In summary the regression model equation is given as:

$$Y = \alpha_1 Z_1 + \alpha_2 Z_2 + \alpha_3 Z_3 + \alpha_4 Z_4 + \alpha_5 Z_5 + \alpha_{12} Z_1 Z_2 + \alpha_{13} Z_1 Z_2 + \alpha_{13} Z_1 Z_3 + \alpha_{14} Z_1 Z_4 + \alpha_{15} Z_1 Z_5 + \alpha_{24} Z_2 Z_4 + \alpha_{25} Z_2 Z_5 + \alpha_{34} Z_3 Z_4 + \alpha_{35} Z_3 Z_5 + \alpha_{45} Z_4 Z_5. \qquad \text{...eq.24}$$

CRUSHING STRENGTH TEST OF CUBES

The materials used for this test include:

- The periwinkle shell, which were free from deleterious matters and has a specific gravity of 1.35, compacted bulk density of 500kg/m^3 and non-compacted bulk density of 400 kg/m^3.
- The Otamiri River sand, which were free from deleterious matters and has a specific gravity of 2.60 and bulk density of 1675kg/m^3 (compacted) and 1429kg/m^3 (non-compacted).
- The granite, were also free from deleterious matters. The compacted bulk density of the granite is 1613kg/m^3 and the non-compacted bulk density is 1398kg/m^3. Both the sand and granite are in accordance with BS 1881 (1970).
- The cement used for this study was "Eagle" cement, a brand of Ordinary Portland Cement and conforms to BS 12 (1978). The water used for this work was taken from bore-hole tap from Federal University of Technology, Owerri.

The materials were batched by mass in their dried state. The bunker where the materials were mixed was wetted to avoid absorbing the mixing water. Mixing was done manually using spade and hand trowel. The moulds used for the concrete cubes were 150mm x150mm. After mixing properly to a consistent state the concrete was cast into the moulds. The concrete was tamped very well to ensure proper compaction. The cube moulds were covered with polythene papers and allowed to stand for 24 hours. After 24 hours, the cubes were de-moulded and transferred immediately to curing water at room temperature. The concrete cubes stayed in the curing water for 27 days to make up the 28 days after which they were tested for compressive strength using universal tensile / compressive testing machine to the requirement of Bs 1881: part 115: 1986. The Saturated Surface Dry (SSD) density was also measured. The mixture proportions of the tested samples are as shown in table 2.

RESULT

CONCRETE CUBE STRENGTH

The 28th days' cubes Strengths of the periwinkle shell-granite concrete and the corresponding concrete SSD densities are shown in table 3. The regression model was used to predict the 28th day compressive strength of periwinkle shell – granite concrete when the mix ratios are known and vice versa. This is shown in table 4.

Substituting the values of Y^n (cubes strengths from experimental test results of table 3) into equation 23 will give the coefficients of the Regression model as:

$\alpha_1 = 251.3756$: $\alpha_2 = -475.231$: $\alpha_3 = -7.39914$:

$\alpha_4 = 39.23829$: $\alpha_5 = 57.4016$: $\alpha_6 = 125.83$:

$\alpha_7 = -8.8288$ $\alpha_8 = -64.1667$: $\alpha_9 = -76.4536$:

$\alpha_{10} = 71.1391$: $\alpha_{11} = 47.16307$: $\alpha_{12} = 46.30922$: $\alpha_{13} = -6.49375$

$\alpha_{14} = -4.49886$: $\alpha_{15} = -2.7628$

Substituting these coefficients into equation 24 will give the final Regression equation as

$$Y = 251.3756Z_1 - 475.231Z\ 7.39914Z_3+39.23829Z_4 + 57.4016Z_5 + 125.83Z_1Z_2 - 48.8288Z_1Z_3 - 64.1667Z_1Z_4 - 76.4536Z_1Z_5 + 71.1391Z_2Z_3 + 47.16307Z_2Z_4 + 46.30922\,Z_2Z_5 - 6.49375Z_3Z_4 - 4.49886Z_3Z_5 - 2.7628Z_4Z_5 \quad \text{----------- eq.25}$$

TEST FOR GOODNESS OF FIT OF REGRESSION MODEL

It is expected that the results of the model will be about 95% accurate. If it is not, it will not be accepted as being adequate. Statement of statistical hypothesis is as follows:

- Null Hypothesis (H_0): There is no significant difference between the Model results and the experimental results.
- Alternative Hypothesis (H_1): There is a significant difference between the Model results and experimental results.
- The risk is that 5% or below of the Model's results will be incorrect.

Total Variance of the Replicate is given by Cramer (1946) as $Sy^2 = \sum S_i^2 / (N - 1)$

$= 74.23 / (30 - 1) \quad = 2.56$

Therefore, the Standard Error of the Replicates is the square root of the above Variance.

$S_y = \sqrt{S_y^2} = \sqrt{2.56} \quad = 1.60$

STUDENT'S T – TEST

Aknazoroval (1982),the table 6 shows the statistics analysis of Model Results. Akhnazorova and Kafarov (1982). T-values from appendix is given as $t_{\infty/N}$ (V). This is equal to $t_{0.05/15}(14) = t_{.0033}(14)$

From Appendix, $t_{.0033}$ (14) = 3.06 (by interpolation). Therefore, t from table is equal to 3.06. This is greater than each of the t values calculated in table 5.14, hence the Null Hypothesis is accepted.

CONCLUSION

Periwinkle shell-granite concrete can make lightweight concrete that has minimum strength of 17 N mm^{-2} and maximum SSD density of 1830KN m^{-3}. Regression (Osadebe) Model is adequate and reliable in use for predicting the compressive strength of periwinkle shell-granite concrete when the mixture proportions are known and vice versa.

Table 1: Values of the Actual mixture proportions and the corresponding Fractional portions for a system of $\sum Z = 10$

N	S1	S2	S3	S4	S5	RESPONSE	Z1	Z2	Z3	Z4	Z5
1	0.67	1	1.7	2	0.5	$Y^{(1)}$	1.14	1.7	2.9	3.41	0.85
2	0.56	1	1.6	1.8	0.8	$Y^{(2)}$	0.97	1.74	2.78	3.13	1.39
3	0.5	1	1.2	1.7	1	$Y^{(3)}$	0.93	1.85	2.22	3.15	1.85
4	0.7	1	1	1.8	1.2	$Y^{(4)}$	1.23	1.75	1.75	3.16	2.11
5	0.75	1	1.3	1.2	1.5	$Y^{(5)}$	1.3	1.74	2.26	2.09	2.61
12	0.62	1	1.65	1.9	0.65	$Y^{(12)}$	1.07	1.72	2.84	3.26	1.12
13	0.59	1	1.45	1.85	0.75	$Y^{(13)}$	1.05	1.77	2.57	3.28	1.33
14	0.69	1	1.35	1.9	0.85	$Y^{(14)}$	1.19	1.73	2.33	3.28	1.47
15	0.71	1	1.5	1.6	1	$Y^{(15)}$	1.22	1.72	2.58	2.75	1.72
23	0.53	1	1.4	1.75	0.9	$Y^{(23)}$	0.95	1.79	2.51	3.14	1.61
24	0.63	1	1.3	1.8	1	$Y^{(24)}$	1.1	1.75	2.27	3.14	1.75
25	0.66	1	1.45	1.5	1.15	$Y^{(25)}$	1.15	1.74	2.52	2.6	2
34	0.6	1	1.1	1.75	1.1	$Y^{(34)}$	1.08	1.8	1.98	3.15	1.98
35	0.63	1	1.25	1.45	1.25	$Y^{(35)}$	1.13	1.79	2.24	2.6	2.24
45	0.73	1	1.15	1.5	1.35	$Y^{(45)}$	1.27	1.75	2.01	2.62	2.36

Z^n matrix

	Z1	Z2	Z3	Z4	Z5	Z1Z2	Z1Z3	Z1Z4	Z1Z5	Z2Z3	Z2Z4	Z2Z5	Z3Z4	Z3Z5	Z4Z5
1	1.14	1.7	2.9	3.41	0.85	1.938	3.306	3.887	0.969	4.93	5.797	1.445	9.889	2.465	2.899
2	0.97	1.74	2.78	3.13	1.39	1.688	2.697	3.036	1.348	4.837	5.446	2.419	8.701	3.864	4.351
3	0.93	1.85	2.22	3.15	1.85	1.721	2.065	2.93	1.721	4.107	5.828	3.423	6.993	4.107	5.828
4	1.23	1.75	1.75	3.16	2.11	2.153	2.153	3.887	2.595	3.063	5.53	3.693	5.53	3.693	6.668
5	1.3	1.74	2.26	2.09	2.61	2.262	2.938	2.717	3.393	3.932	3.637	4.541	4.723	5.899	5.455
1,2	1.07	1.72	2.84	3.26	1.12	1.84	3.039	3.488	1.198	4.885	5.607	1.926	9.258	3.181	3.651
1,3	1.05	1.77	2.57	3.28	1.33	1.859	2.699	3.444	1.397	4.549	5.806	2.354	8.43	3.418	4.362
1,4	1.19	1.73	2.33	3.28	1.47	2.059	2.773	3.903	1.749	4.031	5.674	2.543	7.642	3.425	4.822
1,5	1.22	1.72	2.58	2.75	1.72	2.098	3.148	3.355	2.098	4.438	4.73	2.958	7.095	4.438	4.73
2,3	0.95	1.79	2.51	3.14	1.61	1.701	2.385	2.983	1.53	4.493	5.621	2.882	7.881	4.041	5.055
2,4	1.1	1.75	2.27	3.14	1.75	1.925	2.497	3.454	1.925	3.973	5.495	3.063	7.128	3.973	5.495
2,5	1.15	1.74	2.52	2.6	2	2.001	2.898	2.99	2.3	4.385	4.524	3.48	6.552	5.04	5.2
3,4	1.08	1.8	1.98	3.15	1.98	1.944	2.138	3.402	2.138	3.564	5.67	3.564	6.237	3.92	6.237
3,5	1.13	1.79	2.24	2.6	2.24	2.023	2.531	2.938	2.531	4.01	4.654	4.01	5.824	5.018	5.824
4,5	1.27	1.75	2.01	2.62	2.36	2.223	2.553	3.327	2.997	3.518	4.585	4.13	5.266	4.744	6.183

INVERSE of Z^n matrix

	Z1	Z2	Z3	Z4	Z5	Z1Z2	Z1Z3	Z1Z4	Z1Z5	Z2Z3	Z2Z4	Z2Z5	Z3Z4	Z3Z5	Z4Z5
1	405.3	981.8	437.6	168.6	33.79	-1256	583.5	-372	341.8	-1114	675.3	-412	-535	180.9	-121
2	-887	-2425	-1053	-432	-86.3	2920	-1732	1114	-727	3154	-1923	946.4	1287	-570	414.6
3	-4.45	44.45	-6.83	-29.6	-6.82	-5.37	-14.9	50.92	-32.5	-24.7	-92.8	56.74	79	-39.6	26.63
4	52.99	181.7	74.3	66.23	28.89	-202	140.2	-129	107.5	-254	235	-175	-146	115.7	-96.6
5	130	149.8	7.782	11.49	-11.2	-318	160.4	-112	25.57	-129	104.2	15.49	-28.3	-15.2	8.479
1,2	275.7	770	269.9	109.5	23.19	-931	576	-361	229.8	-980	589.1	-306	-331	195.6	-131
1,3	-86.7	-234	-84.9	-32.2	-4.77	285.9	-149	81.49	-65.8	266.6	-140	83.21	99.48	-46.2	26.92
1,4	-119	-316	-124	-51.3	-14.6	389.6	-218	146.7	-113	387.3	-249	147.1	155.8	-77	55.25
1,5	-144	-340	-125	-49.6	-7.41	444	-234	149.4	-106	399.7	-245	126.5	154.6	-71	47.24
2,3	116.3	293.2	142.5	80.51	15.73	-375	235.2	-173	112.5	-399	293.3	-152	-215	98.6	-74.7
2,4	94.36	244.2	110.2	28.84	1.146	-300	169.6	-91.7	49.44	-312	152.3	-47.4	-105	20.71	-14.9
2,5	68.91	260.4	138.2	52.84	15.96	-260	158.6	-101	84.26	-364	211.7	-131	-160	83.42	-58.8
3,4	-7.1	-23.6	-11.8	-9.42	-3.09	25.78	-18.5	13.57	-9.23	34.25	-20.5	11.34	15.43	-8.46	11.28
3,5	-8.51	-13.1	-5.45	-5.34	-0.32	20.54	-12.9	13.06	-5.85	17.55	-14	5.521	8.898	-3.65	3.556
4,5	-1.91	-0.54	0.485	0.077	-0.77	2.903	-0.54	-1.51	2.906	-0.63	2.134	-3.18	-1.18	1.255	0.488

Table 2: Experimental concrete mixture proportions

Mixture Label	Water (kg)	Cement (kg)	Sand (kg)	Periwinkle (kg)	Granite (kg)	Mixture Label	Water (kg)	Cement (kg)	Sand (kg)	Periwinkle (kg)	Granite (kg)
N1	2.32	3.46	5.88	6.91	1.73	AC1	2.11	3.46	4.77	6.32	3.04
N2	1.94	3.46	5.53	6.22	2.76	AC2	2.14	3.46	5.01	5.81	3.28
N3	1.73	3.46	4.15	5.88	3.46	AC3	2.32	3.46	4.84	5.88	3.46
N4	2.42	3.46	3.46	6.22	4.15	AC4	2.28	3.46	4.49	5.81	3.63
N5	2.59	3.46	4.49	4.15	5.18	AC5	2.18	3.46	4.42	5.63	3.91
N12	2.14	3.46	5.7	6.57	2.25	AC6	2.21	3.46	4.7	5.88	3.46
N13	2.04	3.46	5.01	6.39	2.59	AC7	2.04	3.46	5.01	6.32	2.8
N14	2.38	3.46	4.67	6.57	2.94	AC8	2.04	3.46	5.11	6.12	2.9
N15	2.45	3.46	5.18	5.53	3.46	AC9	2.25	3.46	4.99	6.22	3.11
N23	1.83	3.46	4.84	6.05	3.11	AC10	2.21	3.46	4.49	6.12	3.32
N24	2.18	3.46	4.49	6.22	3.46	AC11	2.07	3.46	4.39	5.91	3.62
N25	2.28	3.46	4.49	5.18	3.97	AC12	2.07	3.46	4.53	6.19	3.28
N34	2.07	3.46	3.8	6.05	3.8	AC13	2.14	3.46	4.6	6.32	3.08
N35	2.18	3.46	4.32	5.01	4.32	AC14	2.18	3.46	4.87	6.39	2.94
N45	2.52	3.46	3.97	5.18	4.67	AC15	2.11	3.46	4.32	6.19	3.42

Table 3: Experimental test result of 28th days' concrete cubes strengths

S/N	Points of obser-vation	SSD Density Replicate 1	SSD Density Replicate 2	SSD Density Replicate 3	Cube SSD Density [KN / M3]	strength Replicate 1	strength Replicate 2	strength Replicate 3	Cube strength Y [N / mm2]
1	1	1344.31	1455.392	1358.28	1385.994	5.82	5.95	7.55	6.44
2	2	1402.51	1518.391	1417.08	1445.993	7.99	9.34	9.19	8.84
3	3	1288.75	1395.254	1302.15	1328.719	8.72	9.86	10.37	9.65
4	4	1300.85	1408.346	1314.37	1341.188	5.74	6.79	6.52	6.35
5	5	1485.96	1608.721	1501.39	1532.021	10.58	12.88	11.64	11.7
6	12	1373.41	1486.891	1387.68	1415.993	6.47	7.43	7.58	7.16
7	13	1316.53	1425.323	1330.22	1357.356	6.7	7.04	8.49	7.41
8	14	1322.58	1431.869	1336.33	1363.591	5.47	5.98	6.79	6.08
9	15	1415.13	1532.057	1429.83	1459.008	7.32	7.96	9.11	8.13
10	23	1345.63	1456.823	1359.62	1387.356	8.31	9.54	9.84	9.23
11	24	1351.68	1463.369	1365.73	1393.591	6.71	7.36	8.31	7.46
12	25	1444.23	1563.556	1459.23	1489.007	8.82	11.12	9.46	9.8
13	34	1294.8	1401.799	1308.26	1334.953	6.95	8.76	7.45	7.72
14	35	1387.35	1501.987	1401.77	1430.37	9.21	11.61	9.87	10.23
15	45	1393.4	1508.534	1407.88	1436.605	7.61	9.6	8.16	8.46
16	AC1	1166.35	1262.751	1178.48	1202.526	9.38	11.83	10.06	10.42
17	AC2	1312.8	1421.283	1326.45	1353.509	8.14	10.26	8.72	9.04
18	AC3	1383.41	1412.658	1482.83	1426.299	8.77	8.32	4.9	7.33
19	AC4	1424.11	1454.222	1526.45	1468.261	9.44	8.96	5.27	7.89
20	AC5	1543.57	1576.206	1654.47	1591.415	15.33	14.54	8.56	12.81
21	AC6	1204.96	1230.445	1291.61	1242.338	12.89	12.22	7.19	10.77
22	AC7	1313.41	1341.185	1407.82	1354.14	9.1	8.63	5.08	7.6
23	AC8	1363.82	1392.663	1461.85	1406.112	9.7	7.94	6.67	8.1
24	AC9	1408.47	1438.253	1509.69	1452.139	8.44	6.91	5.8	7.05
25	AC10	1301.42	1328.944	1394.98	1341.782	8.68	7.11	5.97	7.25
26	AC11	1323.44	1351.427	1418.57	1364.48	9.62	7.88	6.62	8.04
27	AC12	1423.21	1453.307	1525.49	1467.338	9.53	7.8	6.55	7.96
28	AC13	1416.13	1446.076	1517.9	1460.037	9.74	7.98	6.7	8.14
29	AC14	1525.47	1557.73	1635.08	1572.762	12.62	10.33	8.67	10.54
30	AC15	1485.97	1517.387	1592.74	1532.032	13.19	10.8	9.07	11.02

Table 4 : Experimental Test, Regression (Osadebe) Model Results

S / N	Experimental Test Results	Osadebe Model Compresive Strength (N/mm^{-2}) Results	S / N	Experimental Test Results	Osadebe Model Compresive Strength (N/mm^{-2}) Results
N1	6.44	6.61	AC1	10.42	10.95
N2	8.84	8.35	AC2	9.04	9.43
N3	9.65	9.70	AC3	7.33	7.30
N4	6.35	6.46	AC4	7.89	7.13
N5	11.7	11.70	AC5	12.81	10.26
N12	7.16	6.73	AC6	10.77	12.17
N13	7.41	7.47	AC7	7.6	7.67
N14	6.08	5.99	AC8	8.1	7.98
N15	8.13	8.44	AC9	7.05	6.24
N23	9.23	9.25	AC10	7.25	7.65
N24	7.46	7.02	AC11	8.04	8.21
N25	9.8	9.35	AC12	7.96	8.43
N34	7.72	8.10	AC13	8.14	7.20
N35	10.23	10.28	AC14	10.54	7.32
N45	8.46	8.04	AC15	11.02	9.78

Table 5 Regression Model's result verses Experimental Test results

LEGEND $S_I^2 = 1/(n_I-1) * [\sum y_I^2 - 1/n_I *(\sum y_I)^2]$, $n_I = 3$

S/N	Replicate values Y_i	$\bar{Y}$	Y_i^2	$\sum Y_i$	$\sum Y_i^2$	$(\sum Y_i)^2$	S_i^2
	5.82		33.87				
N1	5.95	6.44	35.40	19.32	126.28	373.26	0.93
	7.55		57.00				

Similarly,

N2	9.34	8.84	87.24	26.52	235.53	703.31	0.55
N3	9.86	9.65	97.22	28.95	280.79	838.10	0.71
N4	6.79	6.35	46.10	19.05	121.56	362.90	0.30
N5	12.88	11.7	165.89	35.1	413.32	1232.01	1.33
N12	7.43	7.16	55.20	21.48	154.52	461.39	0.36
N13	7.04	7.41	49.56	22.23	166.53	494.17	0.90
N14	5.98	6.08	35.76	18.24	111.79	332.70	0.44
N15	7.96	8.13	63.36	24.39	199.94	594.87	0.82
N23	9.54	9.23	91.01	27.69	256.89	766.74	0.66
N24	7.36	7.46	54.17	22.38	168.25	500.86	0.65
N25	11.12	9.8	123.65	29.4	290.94	864.36	1.41
N34	8.76	7.72	76.74	23.16	180.54	536.39	0.87
N35	11.61	10.23	134.79	30.69	317.03	941.88	1.54
N45	9.60	8.46	92.16	25.37	216.66	643.64	1.06
AC1	11.83	10.42	139.95	31.27	329.14	977.81	1.60
AC2	10.26	9.04	105.27	27.12	247.57	735.49	1.20
AC3	8.32	7.33	69.22	21.99	170.15	483.56	4.48
AC4	8.96	7.89	80.28	23.67	197.17	560.27	5.21
AC5	14.54	12.81	211.41	38.43	519.69	1476.86	13.70
AC6	12.22	10.77	149.33	32.3	367.18	1043.29	9.71
AC7	8.63	7.6	74.48	22.81	183.09	520.30	4.83
AC8	7.94	8.1	63.04	24.31	201.62	590.98	2.31
AC9	6.91	7.05	47.75	21.15	152.62	447.32	1.76
AC10	7.11	7.25	50.55	21.76	161.54	473.50	1.85
AC11	7.88	8.04	62.09	24.12	198.46	581.77	2.27
AC12	7.80	7.96	60.84	23.88	194.56	570.25	2.24
AC13	7.98	8.14	63.68	24.42	203.44	596.34	2.33
AC14	10.33	10.54	106.71	31.62	341.14	999.82	3.93
AC15	10.80	11.02	116.64	33.06	372.88	1092.96	4.28
						TOTAL =	**74.23**

Table 6 : T – Statistical Test of Osadebe Regression Model

LEGEND $\Delta Y = \tilde{Y}p - \tilde{Y}m$, $\varepsilon = \sum ai^2 + \sum aij^2$, $t = \Delta Y * \sqrt{(n_I)} / S_Y / \sqrt{(1+\varepsilon)}$, $n_I = 3$

N	CN	I	J	ai	aij	ai^2	aij^2	ε	Ỹp	Ỹm	ΔY	abs(ΔY)	t
		1	2	-0.13	0.25	0.0156	0.0625						
		1	3	-0.13	0.25	0.0156	0.0625						
		1	4	-0.13	0.25	0.0156	0.0625						
		1	5	-0.13	0	0.0156	0.0000						
1	AC1	2	3	-0.13	0.25	0.0156	0.0625						
		2	4	-0.13	0.25	0.0156	0.0625						
		2	5	-0.13	0	0.0156	0.0000						
		3	4	-0.13	0.25	0.0156	0.0625						
		3	5	-0.13	0	0.0156	0.0000						
		4	5	-0.15	0	0.0225	0.0000						
		5	-	0	0	0.0000	0.0000						
					Σ	**0.1629**	**0.3750**	0.5379	10.42	10.95	0.53	0.53	**0.463**

Similarly

2	AC2				Σ	**0.1560**	**0.3750**	0.5310	9.04	9.43	0.39	0.39	**0.341**
3					Σ	**0.1404**	**0.3750**	0.5154	7.33	7.3	-0.03	0.03	**0.026**
4					Σ	**0.1248**	**0.3750**	0.4998	7.89	7.13	-0.76	0.76	**0.672**
5					Σ	**0.1092**	**0.3750**	0.4842	12.81	10.26	-2.55	2.55	**2.266**
6					Σ	**0.1584**	**0.2560**	0.4144	10.77	12.17	1.4	1.4	**1.274**
7					Σ	**0.1360**	**0.4320**	0.5680	7.6	7.67	0.07	0.07	**0.061**
8					Σ	**0.1360**	**0.4320**	0.5680	8.1	7.98	-0.12	0.12	**0.104**
9					Σ	**0.1216**	**0.4320**	0.5536	7.05	6.24	-0.81	0.81	**0.703**
10					Σ	**0.1072**	**0.4320**	0.5392	7.25	7.65	0.4	0.4	**0.349**
11					Σ	**0.0928**	**0.4320**	0.5248	8.04	8.21	0.17	0.17	**0.149**
12					Σ	**0.1120**	**0.4320**	0.5440	7.96	8.43	0.47	0.47	**0.409**
13					Σ	**0.1200**	**0.4320**	0.5520	8.14	7.2	-0.94	0.94	**0.817**
14					Σ	**0.1280**	**0.4320**	0.5600	10.54	7.32	-3.22	3.22	**2.79**
15					Σ	**0.0980**	**0.4368**	0.5348	11.02	9.78	-1.24	1.24	**1.08**

APPENDIX
PERCENTAGE POINTS OF THE t - DISTRIBUTION
One Tail

v	0.100	0.050	0.025	0.010	0.005	0.001
1	3.078	6.314	12.706	31.821	63.657	636.620
2	1.886	2.920	4.303	6.965	9.925	31.600
3	1.638	2.353	3.182	4.541	5.841	12.940
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.870
6	1.440	1.943	2.447	3.143	3.707	5.960
7	1.415	1.895	2.365	2.998	3.499	5.410
8	1.397	1.860	2.306	2.896	3.355	5.040
9	1.383	1.833	2.262	2.821	3.250	4.780
10	1.372	1.812	2.228	2.764	3.169	4.590
11	1.363	1.796	2.201	2.718	3.106	4.440
12	1.356	1.782	2.179	2.681	3.055	4.320
13	1.350	1.771	2.160	2.650	3.012	4.220
14	1.345	1.761	2.145	2.624	2.977	4.140
15	1.341	1.753	2.131	2.602	2.947	4.070
16	1.337	1.746	2.120	2.583	2.921	4.020
17	1.333	1.740	2.110	2.567	2.898	3.970
18	1.330	1.734	2.101	2.552	2.878	3.920
19	1.328	1.729	2.093	2.539	2.861	3.880
20	1.325	1.725	2.086	2.528	2.845	3.850
21	1.323	1.721	2.080	2.518	2.831	3.820
22	1.321	1.717	2.074	2.508	2.819	3.790
23	1.319	1.714	2.069	2.500	2.807	3.770
24	1.318	1.711	2.064	2.492	2.797	3.750
25	1.316	1.708	2.060	2.485	2.787	3.730
26	1.315	1.706	2.056	2.479	2.779	3.710
27	1.314	1.703	2.052	2.473	2.771	3.690
28	1.313	1.701	2.048	2.467	2.763	3.670
29	1.311	1.699	2.045	2.462	2.756	3.660
30	1.282	1.699	2.040	2.326	2.750	3.650
40	1.282	1.680	2.020	2.326	2.700	3.550
50	1.282	1.680	2.010	2.326	2.680	3.500
100	1.282	1.660	1.980	2.326	2.630	3.390
inf	1.282	1.645	1.960	2.326	2.580	3.290

Source: The West African Examinations Council (1982). *Mathematical and Statistical Tables and Formulae*. University Press Limited. Ibadan

Vitalis Obioma Nwachukwu (2005). *Principles of Statistical Inference*. Peace Publishers LTD. Owerri

REFERENCE

1. ACI Committee 211(1974) *"Recommended practice for selecting proportions for no-slump concrete"*. Journal of America Concrete Institution, 71, pp. 153 – 70.

2. Akhnazorova, S. and Kafarov (1982) "Experiment Optimization in Chemistry and Chemical Engineering". MIR Publishers Moscow. Pp 241 – 280.

3. British Standards Institution BS 12 (1978) *"Ordinary and rapid hardening Portland cement"*. London. pp. 38.

4. British Standards Institution BS 1881 (1970) *"Methods of testing concrete"*. London. Parts 1 – 6.

5. British Standards Institution BS 1881 (1986) *"Methods of testing concrete"*. London.

6. Draper, N. R. and Pukelsheim, F. (1999) *" Keifer Ordering of Simplex Designs for First and Second Degree mixture Models"*. Journal of Statistical Planning and Inference, vol. 79, 325 – 348.

7. Draper, N. R. and Pukelsheim, F. (2000) *"Keifer Ordering of Simplex Designs for Second degree Mixture models with four or more ingredients"*. Annuals of Statistics.

8. Draper, N. R. and St. John, R. C. (1977) *" A mixture Model with Inverse Terms"*. Techno metrics, 17, 37 – 46.

9. Neville, A. M. (1983) *"Properties of Concrete"*. English Language Book Society and Pitman Publishing, London.

10. Obam, S. A (1998), *"A Model for Optimisation of Strength of Palm Kernel Shell aggregate Concrete"*. A Master's thesis submitted in Civil Engineering Department University of Nigeria, Nsukka.

12. Road Research (1950) *"Design of concrete mixes"*. D. S. I. R. Road Note No.4. London.

13. Scheffe H. (1958) *"Experiments with mixtures"*. Journal of Royal statistics Society series B, 20, 344-360.

14. Scheffe, H. (1963) *"Simplex-centroid designs for experiments with mixtures"*. Journal of Royal Statistics Society Series B, 25, 235-236.

15. Simon, M. J., Lager green, E. S. and K. A. Snyder (1997) *"Concrete mixture optimization using statistical mixture Design methods"*. In proceedings of the PCI/FHWA International Symposium on High Performance Concrete, New Orleans, PP. 230-244.

16. Teychenne, D. C., Franklin, R. E. and Erntroy, H. (1975) *"Design of Normal Concrete mixes"*. Department of Environment, pp. 31, London.