FINITE DIFFERENCE APPROACH FOR OPTIMUM COMPRESSIVE STRENGTHS OF GRP COMPOSITES

BY

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ABSTRACT:

The Paper presents finite difference method as computational approach for optimization of compressive failure response of Glass Fibre Reinforced Polyester (GRP). A two dimensional multiple linear regression model obtained by subjecting replicated samples of GRP composites to compressive failure was found to be a two dimensional Laplace function through transformation to partial differential equation(PDE). By passing the Finite difference model of the function through nine interior mesh points of a composite region a system of nine by nine linear equations was developed and solved by Leibman method (Gauss-Seidel iteration). The Leibman method algorithms result was optimized by Visual Basic (VB) language Programme. The finite difference method gave the ultimate compressive strength as 154 MPa. The 3-D surface plot of the model also showed that the compressive failure response of the composite is elliptical.

Keywords: optimum, compressive strengths, GRP, homogeneous PDE, finite difference, reinforced plastics,

1 INTRODUCTION

Composites with their high strength to weight ratio have become very important in many technological applications such as in aerospace, automobile and medical industries. One of the key factors which make composites attractive for engineering applications is the possibility of property enhancement through fibre reinforcement. Composites are widely used today in aerospace and automobile industries. Currently in the USA industries utilize over 100,000 tonnes of reinforced plastics out of a total consumption of over one million tones, Crawford (1998).

However Christensen and DeTeresa (1997) reported the compressive failure of fibre composite materials as the most limiting property. Also Crawford (1998) reported that the compressive failure in the fibre direction is much less in magnitude than the corresponding tensile stress at failure. The objective of this work is to model the optimum compressive failure parameter as strength, so that the structural components buckling instability could be controlled.Kyriakides etal (1998) consistently reported the compressive strength of typical fibre reinforced matrix composites to be only 50 to 60 percent of their tensile strength. The importance of complete knowledge of composites properties is reported by Shati etal (1991).

Black and Adams.(1981) reported need for ensuring that the design stress be less than yield stress for ductile material and less than the ultimate stress for brittle material. The design properties of composite elements are very important because of increasing demand for design of light weight structures. In the study of compression members, buckling which is the compressive failure of slender or thin section subjected to axial compression is important because it occurs before the elastic limit of the material. Koshal (1998) reported the tensile strength of GRP composites as 303 MPa while Benhan and Warknock (1981) reported 300 MPa.

Budiansky (1994), Chung and Weitzman (1994), Kyriakides(1994)and Hsu etal (1998) used idealized macro-buckling mechanical models of fibre reinforced composites to establish that the compressive strengths of fibre composites subjected to compressive loads are only about 50% to 60% of their ultimate strength intension, while this paper attempts to use finite difference method to optimize compressive failure response of GRP composites working under normal conditions using Ihueze (2005) multiple regression model which was transformed to two-dimensional Laplace equation to enable analysis by Finite difference methods.

The Finite Difference Method and was used to model compressive failure response of GRP composite material while the Leibman method and Visual Basic iterative programme were used to obtain solution for the compressive failure response model of GRP composite material. Elliptical function was found to fairly represent the compressive failure behaviour of GRP.

2 THEORETICAL ANALYSIS AND REVIEW

2.1 Some Engineering Phenomena.

Sundaram etal (2003) reported that engineering phenomena can be broadly be grouped into three kinds, namely wave phenomenon, diffusion phenomenon and potential phenomenon. However some complex engineering phenomenon may be a combination of these, so that a second order (Linear or quasi linear) partial differential equation in two independent variables may be classified as hyperbolic, parabolic and elliptic equations respectively. By following Zill and Cullen (1989) method in classifying partial differential equation using the relation of linear second order PDE as

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + D\frac{\partial u}{\partial x} + E\frac{\partial u}{\partial y} + Fu = G \quad (1)$$

where A, B, C, D, E, F, G are functions of x and y

If $G(x, y) = 0$, the equation is **Homogeneous** .If **$G(x, y) \neq 0$**, the equation is **Nonhomogeneous**

The homogeneous equation can further be analyzed if $B^2 - 4AC > 0$, as **Hyperbolic**, $B^2 - 4AC = 0$, as **Parabolic** and $B^2 - 4AC < 0$, as **Elliptic**

2.1.1 Possible Functions for Approximation

- **Laplace Equation**

Elliptical function of simple wave function classically has been expressed as

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad (2)$$

This is a laplace equation in two variables and homogeneous linear partial differential equation with constant coefficient which could be solved analytically or by Numerical methods.

- **Poisson's Equation.**

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x,y) \quad (3)$$

This is a nonhomogeneous linear equation which may be a combination of a wave phenomenon and diffusion phenomenon, Sundaram (2003).

2.2 Compressive Model and Transformations.

Ihueze (2005) multiple linear regression model was transformed to two-dimensional Laplace equation to enable analysis by Finite difference method and method of separation of variables.

The model was expressed as:

$$S = 154.0432 - 2.6797x_{11} - 11.5726x_{22} \quad (4)$$

where, S = critical stress, x_{11} = slenderness ratio, x_{22} = height or thickness of section

- **Compressive failure as a Homogeneous function**

By expressing (4) as

$$u = 154.0432 - 2.6797x - 11.5726y \quad (5)$$

and by partial differentiation of function with respect to variables,

$$\frac{\partial u}{\partial x} = -2.6797 \qquad = -11.5726 \quad (6)$$

$$\frac{\partial^2 u}{\partial x^2} = 0 \quad (7)$$

$$\frac{\partial u}{\partial y} = 11.5726 \quad (8)$$

$$\frac{\partial^2 u}{\partial y^2} = 0 \quad (9)$$

By adding (7) and (9)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad (10)$$

Equation (10) represents a Laplacian function, u in two variables x and y.

Other compressive responses of GRP composites are found in the work of Argon (1972), Cains etal (1994), Enetanya and Ihueze (2004), Poulsen etal (1997),Volger and Kyriakides (1997) and Zang and Latour (1993).

3 METHODOLOGY AND MODELLING

Ihueze (2005) compressive failure response model of GRP composites was approximated to two- dimensional Laplace equation by transformation. The Finite Difference Method and Method of Separation of Variables were used to model compressive failure response of GRP composite material, while the Leibman method, Visual Basic iterative programme and method of separation of variables product solution models were used to obtain solution for the compressive failure response model of GRP composite material. The Flowchart for finite difference modeling and solution is presented in Figure 1.

3.1 Finite Difference Approximation of Function

The approximation function for single valued function y(x) is established following Taylor's approximation method as follows:

$$y(x + h) = y(x) + y^{1}(x)h + y^{11}(x)\, h^2/2 + y^{111}\, h^3/3! + \ldots\ldots\ldots\ldots \quad (11)$$

Also

$$y(x - h) = y(x) - y^{1}(x)h + y^{11}(x)\, h^2/2 - y^{111}(x)\, h^3/3! + \ldots. \quad (12)$$

When h, step size is small, higher powers of h ie h^2, h^3, h^4 …. could be neglected so that

$$y^{1}(x) = 1/h\,[y(x + h) - y(x)] \quad (13)$$

$$y^{1}(x) = 1/h\,[y(x) - y(x - h)] \quad (14)$$

By subtracting (11) and (12),

$$y^{1}(x) = 1/2h\,[y(x + h) - y(x - h)] \quad (15)$$

By adding (11) and (12),

$$y^{11}(x) = 1/h^2\,[y(x + h) - 2y(x) + y(x - h)] \quad (16)$$

By considering a meshed rectangular region of mesh size h shown in Figure1 and employing Taylor's expansion approximation, the difference quotients derived for the single valued function y(x) is employed for Laplacian equation of (2),

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} = 0$$

If u is the function of two variables, x and y Following Taylor's expansion approximation as the central differences can be expressed

u(x+ h, y) –2u (x y) + u (x – h, y) and

u(x, y + h) –2u (x,y) + u (x, y - h)

So that following (16) we can write

$\partial^2 u/\partial x^2 = 1/h^2$

∂^2 [u (x+h y)–2u (x,y)+u(x–h, y] (17)

$\partial^2 u/\partial x^2 = 1/h^2$ [u (x,y+h)-2u(x,y)+ u(x,y,-h (18)

By adding (17) and (18)

$\partial^2 u / \partial x^2 + \partial^2 u / \partial y^2 = 1/h^2$

[u(x+h,y)+u(x,y+h)+u(x,y-h)-4u(x,y)]=0

u(x,y)=+u(x,y+h)+u(x,y-h)-4(x,y)=0 (19)

(19)can be simplified by substituting in the following relations

$u(x, y) = u_{ij}$, $u(x + h,y) = u_{i+1,j}$, $u(x,y + h)$ $u_{i,j+1}$, $u(x, y - h) = u_{i,j-1}$ to obtain

$$u_{i+1,j}+u_{i,j+1} + u_{i-1,j} + u_{i,j-1} - 4u_{ij} = 0 \quad (20)$$

If the points of inter sections of mesh points are expressed as

Pij = P (ih, jh)

where

i, j = integers of the lines designating horizontal and vertical mesh points.

(20) could be rearranged to read

$$u_{ij} = \frac{1}{4}\left(u_{i+1,j} + u_{i,j+1} + u_{i-1,j} + u_{i,j-1}\right) \quad (21)$$

(21), means that the value of four neighboring points must be evaluated to give the five-point approximation of Laplace equation.

3.1.1 Derivation of Equations of Function

- Problem Statement:

Obtain the finite difference solution of the Laplacian function,

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} = 0, \text{ subject to } 0 < x < 2, 0 < y < 2$$

Procedures for Solving Systems of Equations of Function

- Apply the finite difference function, u_{ij} at the nine interior mesh points.
- Apply boundary or Dirichlet condition to the nine interior nodes equations (31-39)of Figure3a.
- Establish matrix equation of the nine equations above
- Solve the system of equations if diagonal dominance prevails by Gauss-seidel iteration
- Optimize solution by VB Language programme for Gauss –Seidel algorithms
- Compare result with any known classical report. This procedure is as presented in Figure1

- The boundary conditions specified a square region of length 2

By choosing the number of interval, n = 4

Mesh size, h = L/n = 2/4 = 1/2

Interior mesh points = $(n-1)^2 = (4-1)^2 = 9$

The problem is solved with Dirichlet conditions as shown in Figure.3

- **Evaluation of u_{ij} at Interior Mesh Points.**

The interior mesh points are by Figure 3, P11, P12, P13, P21, P22, P23, P31, P_{32}, and P33,

So that the five points equation for function at 9 interior points are by (21) and considering the coordinates of P11, P12, P13, P21, P22, P23, P31, P_{32}, and P33

$$u_{11} = 1/4\ (u_{21} + u_{12} + u_{01} + u_{10}) = 1/4\ (u_{21} + u_{12} + 308.0864) \quad (22)$$

$$u_{12} = 1/4\ (u_{22} + u_{13} + u_{02} + u_{11}) = 1/4\ (u_{22} + u_{13} + u_{11} + 154.0432) \quad (23)$$

$$u_{13} = 1/4\ (u_{23} + u_{14} + u_{03} + u_{12}) = 1/4\ (u_{23} + u_{12} + 308.0864) \quad (24)$$

$$u_{21} = 1/4\ (u_{31} + u_{22} + u_{11} + u_{20}) = 1/4\ (u_{31} + u_{22} + u_{11} + 154.0432) \quad (25)$$

$$u_{22} = 1/4\ (u_{32} + u_{23} + u_{12} + u_{21}) \quad (26)$$

$$u_{23} = 1/4\ (u_{33} + u_{24} + u_{13} + u_{22}) = 1/4\ (u_{33} + u_{13} + u_{22} + 154.0432) \quad (27)$$

$$u_{31} = 1/4\ (u_{14} + u_{32} + u_{21} + u_{30}) = 1/4\ (u_{32} + u_{21} + 308.0864) \quad (28)$$

$$u_{32} = 1/4\ (u_{42} + u_{33} + u_{22} + u_{31}) = 1/4\ (u_{33} + u_{22} + u_{31} + 154.0432) \quad (29)$$

$$u_{33} = 1/4\ (u_{33} + u_{34} + u_{22} + u_{32}) = 1/4\ (u_{22} + u_{32} + 308.0864) \quad (30)$$

By setting $x_1 = u_{11}$, $x_2 = u_{12}$, $x_3 = u_{13}$, $x_4 = u_{21}$, $x_5 = u_{22}$, $x_6 = u_{23}$, $x_7 = u_{31}$, $x_8 = u_{32}$, $x_9 = u_{33}$.

The following 9 x 9 System of equations is obtained for nine mesh nodes of Figure 3

$$x_1 = 1/4\ (x_4 + x_2 + 308.0864) \quad (31)$$

$$x_2 = 1/4\ (x_5 + x_3 + x_1 + 154.0432) \quad (32)$$

$$x_3 = 1/4\ (x_6 + x_2 + 308.0864) \quad (33)$$

$$x_4 = 1/4\ (x_7 + x_5 + x_1 + 154.0432) \quad (34)$$

$$x_5 = 1/4\ (x_8 + x_6 + x_2 + x_4) \quad (35)$$

$$x_6 = 1/4\ (x_9 + x_3 + x_5 + 154.0432) \quad (36)$$

$$x_7 = 1/4\ (x_8 + x_4 + 308.0864) \quad (37)$$

$$x_8 = 1/4\ (x_9 + x_5 + x_7 + 154.0432) \quad (38)$$

$$x_9 = 1/4\ (x_6 + x_8 + 308.0432) \quad (39)$$

To establish the diagonal dominance necessary for the application of Gauss-Seidel iteration method the following matrix equation is established for (31) – (39) as

$$\begin{pmatrix} -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -4 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -4 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -4 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{pmatrix} = \begin{pmatrix} -308.0864 \\ -154.0432 \\ -308.0864 \\ -154.0432 \\ 0 \\ -154.0432 \\ -308.0864 \\ -154.0432 \\ -308.0864 \end{pmatrix} \qquad (40)$$

This is an example of sparse and banded matrix. The absolute values of all the diagonal elements are respectively not less than the sum of the absolute values of the remaining elements of their rows; so that we have diagonal dominance of the coefficient matrix hence Gauss-Seidel iteration could be applied.

4 COMPUTATIONS AND ANALYSIS

4.1 Numerical Solution of FD Model

Gauss-Seidel iteration and VB iterative programme were used to obtain solution for the system of linear equations of (31) – (39).

- **Gauss-Seidel Iteration**

By taking the initial guesses as:

$x_1 = x_2 = x_3 = x_4 = x_5 = x_6 = x_7 = x_8 = x_9 = 0$

1st Iteration

By putting values in (31) – (39), x_1`= 77.0216, x_2 = 95.0730, x_3 = 91.4632, x_4 = 57.7662, x_5 = 28.8831, x_6 = 68.5974, x_7 = 91.4632, x_8 = 68.5974, x_9 =.3203

2nd Iteration

By putting above values in (31) – (39), x_1`= 105.9047, x_2 = 57.7662, x_3 = 117.9394, x_4 =72.5736, x_5 = 63.1818, x_6 = 111.6212, x_7 = 112.3144, x_8 = 109.6950, x_9 =.132.4182

By similar procedures as above, 9 iterations that led to convergence are presented in Table1.

• **VB Iterative Programme listing.**

To still optimize FD results, VB Programme was developed as listed in Fig 6. The result of this programme is found in Table3. This programme solves the system (31) – (39) by iteration as listed in Fig.6.

4.2 3-D Graphics

The 3-D Graphics were produced with excel spread sheet package using Table 4 and presented in figures 7.

5 DISCUSSION OF RESULTS.

The finite difference method showed that the compressive failure response may be approximated by potential phenomenon or elliptical function such as the two dimensional Laplace function.

Though solution of PDE may lead to infinite number of solutions, the optimum compressive strength of GRP composites have been evaluated as 154MPa. Table 1 and Table 2 of Guass-Seidel iterations for finite difference solution gave the approximate compressive strength of GRP as 154MPa

The 3 -D plots of Fig.7 also showed that stress distribution in composites is described by mixed phenomena which could be approximated by an elliptical function, the centre of the GRP composite having the lowest strength. The strength of GRP composite at the centre or along its neutral axis is lower than its compressive strength of 154MPa. This means that composites under compressive loading could fail within loading less than its elastic limit stress. The 3-D graphics of Fig.7 also shows the maximum and minimum compressive strengths as 154MPa and 153MPa.respectively.

5 CONCLUSIONS

The finite difference method with approximating function as Laplace function representing compressive failure response of GRP composite strength has been successfully applied with the following deductions:

- The compressive failure response of GRP composites can be represented by a Laplacian homogeneous function.
- The finite difference grid or meshing method enables evaluation of function at interior nodes when the values of at least four neighbouring nodes are known.
- Gauss-Seidel iteration method provides a converging algorithm for evaluation of a Laplacian function
- For a homogeneous GRP composite the value of the function at any interior point is the same
- Given the coordinate of an interior point of FDM (mesh), (i,j) as in Fig. 7 ,of

Laplacian function, $u_{i,j}$ of two dimension and other four surrounding boundary points coordinates, (i+1,j), (i-1,j), (i,j+1) and (i,j-1) the value of the function, $u_{i,j}$ at the interior node is estimated as

$$u_{ij} = 1/4\,(u_{i+1,\,j} + u_{i,\,j+1} + u_{i-1,\,j} + u_{i,\,j-1})$$

- Elliptical function is found to fairly represent the compressive failure behaviour of GRP
- The compressive strength of GRP composites is about 154 MPa

Table 1: Finite Difference Solution Results by Traditional Gauss -Seidel Iteration

variable / Iteration	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
1	77.0216	57.7662	91.4632	57.7662	28.8831	68.5974	91.4632	68.5974	111.320
2	105.9047	95.0736	117.9394	72.5736	63.1818	111.6212	112.3144	109.9650	132.418
3	118.9334	113.5245	133.3080	112.1182	111.8072	132.8942	132.5424	132.7028	143.420
4	108.4323	126.8977	141.9696	126.7063	129.8002	142.3085	141.8739	142.2845	148.169
5	140.4226	141.5589	147.9884	141.5350	141.9217	148.0308	147.9765	148.0278	151.036
6	147.7951	147.9371	151.0136	147.9341	147.9825	151.0188	151.0121	151.0185	152.530
7	150.9894	151.0072	152.5281	151.0068	151.0128	152.5288	152.5279	152.5287	153.286
8	152.2251	152.5273	152.5273	151.0068	152.5280	153.2857	153.2856	153.2857	153.664
9	153.2853	153.2855	153.6644	153.6644	153.2856	153.5697	153.6644	153.6644	153.830

Table 2: Visual Basic Programme Output

Iteration / Variable	1	2	3	4	5	6	7	8
x_1	77.0216	105.9047	124.5584	136.8186	145.1207	149.5431	151.7883	152.9152
x_2	154.0410	154.0421	154.0426	154.0429	154.0431	154.0431	154.0432	154.0432
x_3	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432
x_4	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432
x_5	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432
x_6	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432
x_7	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432
x_8	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432
x_9	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432

Iteration / Variable	9	10	11	12	13	14	15	16
x_1	153.4791	153.7611	153.9022	153.9727	154.0079	154.0256	154.0344	154.0388
x_2	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432
x_3	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432
x_4	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432
x_5	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432
x_6	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432
x_7	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432
x_8	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432
x_9	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432

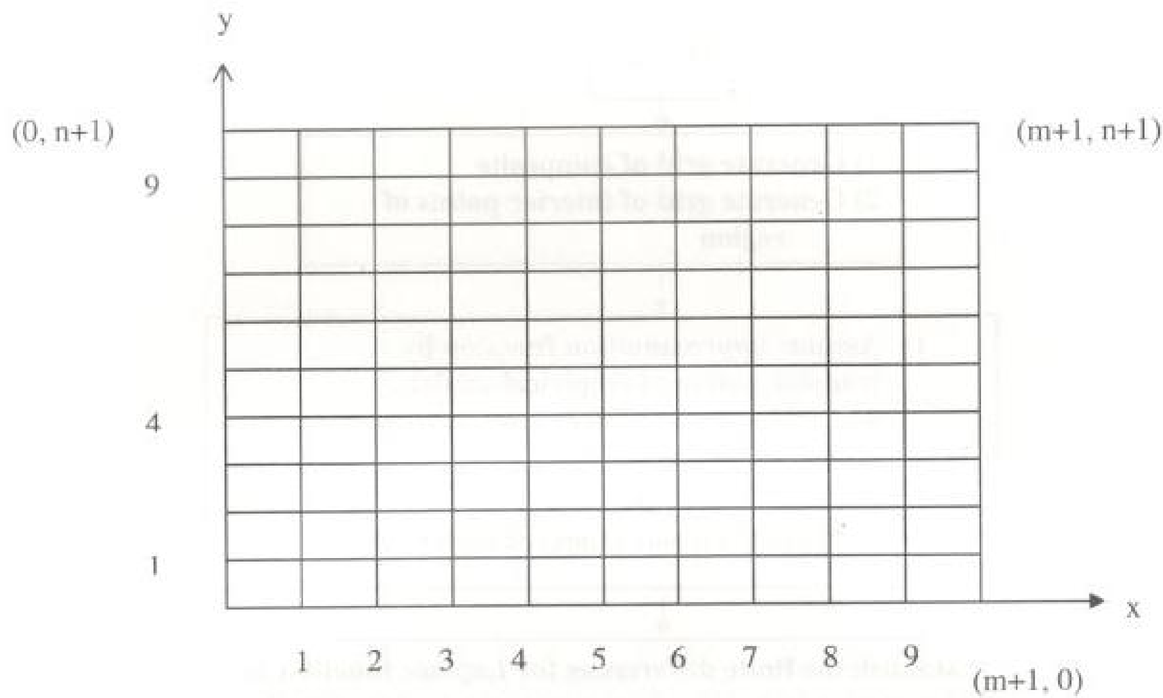


Fig 2: Typical Grid System for FD Solution of Functions of Two Variables

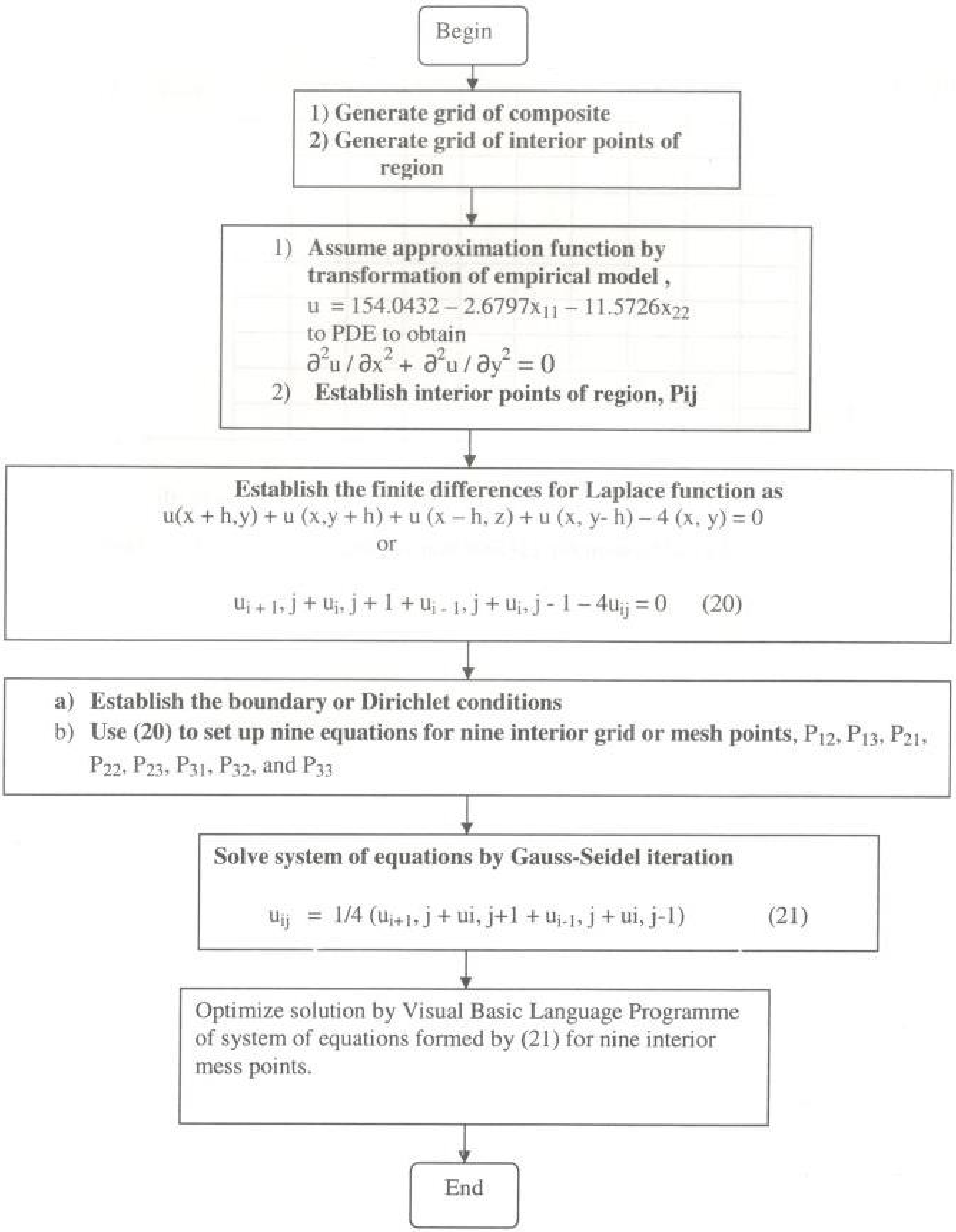
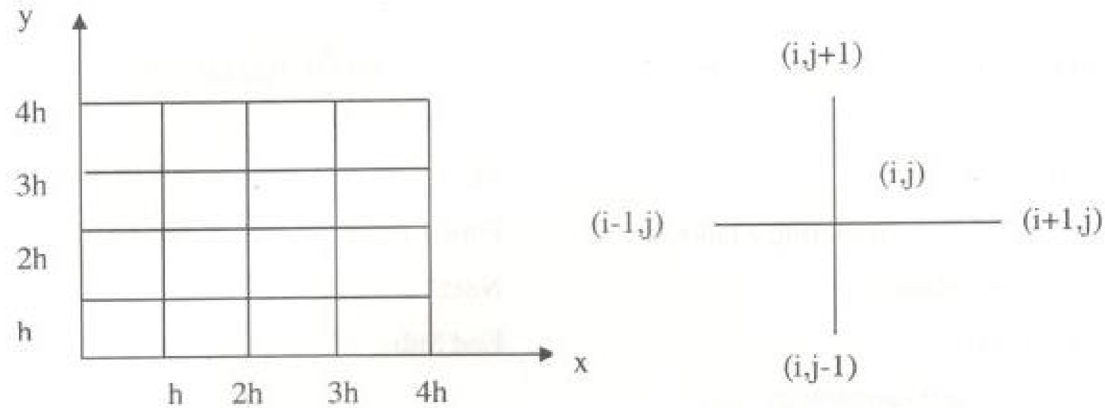

Begin
1) Generate grid of composite
2) Generate grid of interior points of region
1) Assume approximation function by transformation of empirical model ,
$u = 154.0432 - 2.6797x_{11} - 11.5726x_{22}$
to PDE to obtain
$\partial^2 u / \partial x^2 + \partial^2 u / \partial y^2 = 0$
2) Establish interior points of region, Pij
Establish the finite differences for Laplace function as
$u(x + h,y) + u (x,y + h) + u (x - h, z) + u (x, y- h) - 4 (x, y) = 0$
or
$u_{i+1}, j + u_i, j + 1 + u_{i-1}, j + u_i, j - 1 - 4u_{ij} = 0$ (20)
a) Establish the boundary or Dirichlet conditions
b) Use (20) to set up nine equations for nine interior grid or mesh points, P_{12}, P_{13}, P_{21}, P_{22}, P_{23}, P_{31}, P_{32}, and P_{33}
Solve system of equations by Gauss-Seidel iteration
$u_{ij} = 1/4 (u_{i+1}, j + ui, j+1 + u_{i-1}, j + ui, j-1)$ (21)
Optimize solution by Visual Basic Language Programme of system of equations formed by (21) for nine interior mess points.
End

Fig 1: Flowchart for FD modeling and solution



a) Regional mesh, **b) Interior and boundary points**

Fig. 3: Finite Difference Model (FDM)

Where, m = number of intervals for coordinating x variable
n = number of intervals for coordinating y variable

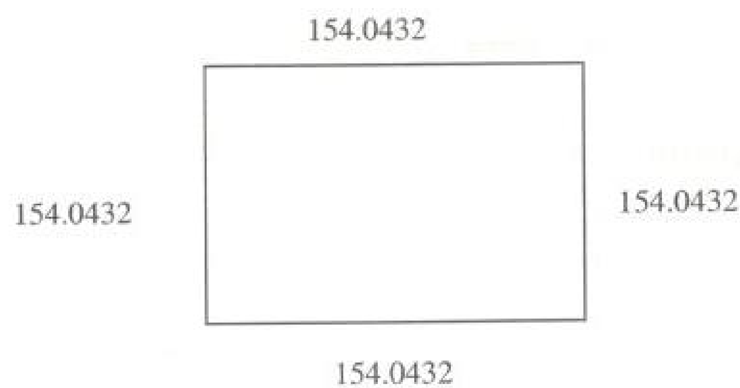


Fig 4: Dirichlet Boundary Conditions

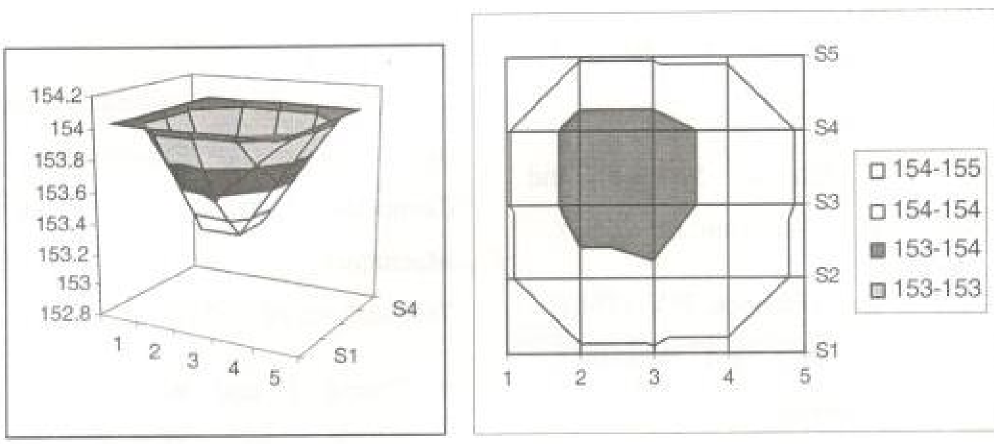
153.6644	153.5697	153.8301
153.2855	153.2856	153.6644
153.2853	153.2855	153.6644

Fig.5: Compressive Stress Distribution in 9 interior meshes predicted by Gauss –Seidel algorithm.

• **Visual Basic Programme Listing.**

```
Private Sub Command1_Click ()
Rem ITERATION
Dim a(20, 20)          'declaring a table to
hold iteration values
Dim x(1 To 9)
Rem initializing 9 variables as zero
For i = 1 To 9
 x(i) = 0
Next i
Form1.Print
"================================
================================
=="
Form1.Print "ITERATION"
Form1.Print
For t = 1 To 9    'controls number of
variables
For u = 1 To 16     'controls number of
iterations
x(1) = 0.25 * (x(4) + x(2) + 308.0864)
x(2) = 0.25 *(x(5)+ x(3) + x(1) + 154.0432)
x(3) = 0.25 * (x(6) + x(2) + 308.0864)
x(4)=0.25 *(x(7)+ x(5) + x(1) + 154.0432)
x(5) = 0.25 * (x(8) + x(6) + x(2) + x(4))
x(6)= 0.25 *(x(9) +x(3)+ x(5) + 154.0432)
x(7) = 0.25 * (x(8) + x(4) + 308.0864)
x(8)= 0.25 *(x(9)+x(5) + x(7) + 154.0432)
x(9) = 0.25 * (x(6) + x(8) + 308.0864)
 a(t, u) = x(t)
Form1.Print Format$(a(t, u),
"#000.0000"); "    ";
Next u
Form1.Print
Next t
End Sub
```

Figure 6: Visual Basic Programme Listing



a) 3-D Plot   b) surface plot

Fig: 7, 3-D: Graphics of Gauss-Seidel iteration function results

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