CLARIFICATIONS BETWEEN THE LAW OF LINEARITY AND THE SUPERPOSITION THEOREM

By

U. T. ITAKETO

Department of Electrical/Electronics & Computer Engineering
University of Uyo,
Uyo, Nigeria

ABSTRACT

The superposition theorem is presented, as stated by various authors. One of he authors unknowingly states the law of linearity in place of the superposition theorem. It is stressed in the paper that the superposition theorem is based on the law of linearity which is a general engineering law. The paper emphasizes that when stating the superposition theorem, the system to which it is applicable should be mentioned as it can be formulated for any type of system provided it obeys the law of linearity.

Keywords: linearity; law; superposition; theorem.

1. INTRODUCTION

In several textbooks of electrical engineering, there have been some mix-ups in the presentation and explanation of the superposition theorem. Most of the authors present the theorem as the law of linearity whereas it is not. The law of linearity is not the superposition theorem and the superposition theorem is not the law of linearity. Rather, the superposition theorem is based on the law of linearity. The law of linearity can be expressed by the following mathematical axioms:

Axiom 1: A map of L of one vector space V into another vector space W is said to be *additive* if:

$L(v_1 + v_2) = L(v_1) + L(v_2)$ .....(1) *Ref(1)*

for all vectors $v_1, v_2 \in V$, where the addition on the left is in V, while that on the right is in W.

Axiom 2: In the law of linearity, it is true that L:V----→W is homogeneous if and only if (iff) $L(kv) = kL(v)$..... (2), for all scalars $k \in K$ and all vectors $v \in V$.

Definition: If $L: V \dashrightarrow$ is *both additive and homogeneous*, it can be stated that *L is linear* or that it is *a linear transformation from V to W* [2]

2. Various Presentations Of The Superposition Theorem

Ralph J. Smith, in his book "Electronics: Circuits and Devices," states the superposition theorem as follows: "If the cause and effect are linearly related, then the total effect of several causes acting simultaneously is equal to the sum of the effects of the individual causes acting one at a time."[3] By the author of this paper, the above statement is the law of linearity and not the superposition theorem. The superposition theorem is however based on the law of linearity. Fink [4] , states the law of linearity, which is also known as the superposition principle, (note: principle, **NOT** theorem), as follows: "In any *linear* system (that is one in which the effect is proportional to the cause), each cause acting separately produces its separate effect. When several causes act simultaneously, the resultant effect is the sum of the effects produced by each cause acting separately." If this principle holds for any system, then that system is linear.[4]

The following authors present the superposition theorem different from Smith quoted above. They are: A. H. Morton,[5] E. Hughes,[6] and J. Shepherd et al.[7] They all state the superposition theorem as follows: "In a linear network containing more than one source of electromotive force (e.m.f.), the resultant current in any branch is the algebraic sum of the currents that would be produced by each e.m.f. acting alone, while all the other sources of e.m.f. are replaced by their respective internal resistances."[5],[6],[7] Also, Markus [8] states it as follows: "The current that flows at a point in a linear network during simultaneous application of a number of voltages throughout the network is the sum of the component currents at the point which would be produced by the individual voltages acting separately. Similarly, the voltage between any two points under such conditions is the sum of the voltages that would be produced between these two points by the individual voltages acting separately. [8]

The superposition theorem, as stated above by different authors, assumes linearity conditions as would be noticed from the statement: "In a linear network....."

3. Discussion

The superposition theorem is based on the law of linearity. That theorem can be developed for any kind of system, be it engineering, economic, financial, management, etc, provided the considered

events are linearly related. Thus when stating the superposition theorem, one should state the system for which the system is applicable to. That theorem, as presented by Morton, Hughes, Shepherd and Markus above, is for linear electrical circuits. The theorem can equally be developed for mechanical systems such as two pumps installed in parallel. In such scenario, provided they satisfy the law of linearity, the total effect of the two pumps combined would be the same as each of the pumps operating individually while the other is replaced by the internal viscous friction it has experienced (analogous to internal electrical resistance of a current source) then the two effects added up. Hence it can be seen that the law of superposition can be fashioned for any kind of system, provided they are linearly related.

4. Conclusion

A distinction has been set between the law of linearity and the superposition theorem. Though they appear similar, they aren't the same. The law of linearity is a general engineering law and the theorem of superposition depends on it. Scholars of electrical engineering (and engineering generally), should go back to the books and assess these two laws in order to decipher their intents and meanings. Superposition theorem, when stated, should be specifically referred to a particular system. Statements about the superposition theorem have been mixed up for years and should be corrected.

REFERENCES

1. Smyrl, J. L. "Introduction to University mathematics," Hodder & Stoughton Ltd., Mill Road, Dunton Green, Sevenoaks, Kent, (2006), 259-265.

2. Padulo, L & Arbib, M. A. "Systems Theory," Saunders Company, Philadelphia, USA (1974).

3. Smith, R. J. : "Electronics: Circuits and Devices," John Wiley & Sons, Inc; New York, (1990), 46-48.

4. Fink, D. G. & Mckenzie, A. A. " Electronics Engineers' Handbook," Mcgraw-Hill Book Company (1975), New York, USA.

5. Morton, A. H. : "Advanced Electrical Engineering," Pitman Publishing Ltd, 39 Parker Street, London, WC2B 5PB (1979), 23-25.

6. Hughes, E, : "Electrical Technology," Longman Group Ltd, London (1975), 14-15.

7. Shepherd, J.,et al: "Higher Electrical Engineering," Pitman Publishing Ltd, 39 Parker street, London, WC2B 5PB (1979), 50-51.

8. Markus, J. "Electronics Dictionary, " Mcgraw-Hill Book Company (1978), New York, USA.